Informal Unemployment and Education*

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Abstract

This paper develops a four sector equilibrium search and matching model with informal sector employment opportunities and educational choice. We show that underground activities reduce educational attainments if informal employment opportunities mainly are available for low educated workers. More zealous enforcement policy will in this case improve educational incentives as it reduces the attractiveness of remaining a low educated worker. Characterizing the optimal enforcement policies, we find that relatively more audits should be targeted towards the sector employing low educated workers, elsewise a too low stock of educated workers is materialized.

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1 Introduction

Researchers have been puzzled by the fact that observed tax evasion, despite low audit rates and fairly modest fines, is substantially lower than what is predicted by theory. Andreoni et al (1998), argue that this discrepancy is most likely explained by non-economic factors, such as morality, guilt, and shame. However, Kleven et al (2011) whom conduct a large field experiment in Denmark, suggests that this discrepancy is explained by the degree of third party reporting. As incomes for individuals are not self-reported, rather reported by a third party such as the employer, it is difficult, and thus costly, to evade taxes. These costs, both due to third-party reporting, or even morality, guilt or shame, tend to reduce the profitability of evading taxes and limits the size of the informal sector, although the expected punishment fees are low relative to taxes.

In this paper we argue that these types of costs may explain why highly educated workers to a lesser extent evade taxes and work informally than low educated workers. If highly educated workers to a smaller extent work in industries which handles cash-payments and to a larger extent are subject to third party reporting, it will be more difficult, and thus more costly, for these workers to evade taxes.

This is consistent with data. Evidence indicates that manual workers, or workers with a lower level of formal education, to a substantially larger degree face informal employment opportunities compared to highly educated workers. Pedersen (2003), using the same questionnaire design for Germany, Great Britain, Denmark, Norway, and Sweden confirms that skilled blue collar workers carry out more informal market activities than others. Figure 1 shows the extent of informal activities in the five countries by industry. Most informal work are carried out in the construction sector, followed by the agricultural sector, hotels and restaurants.

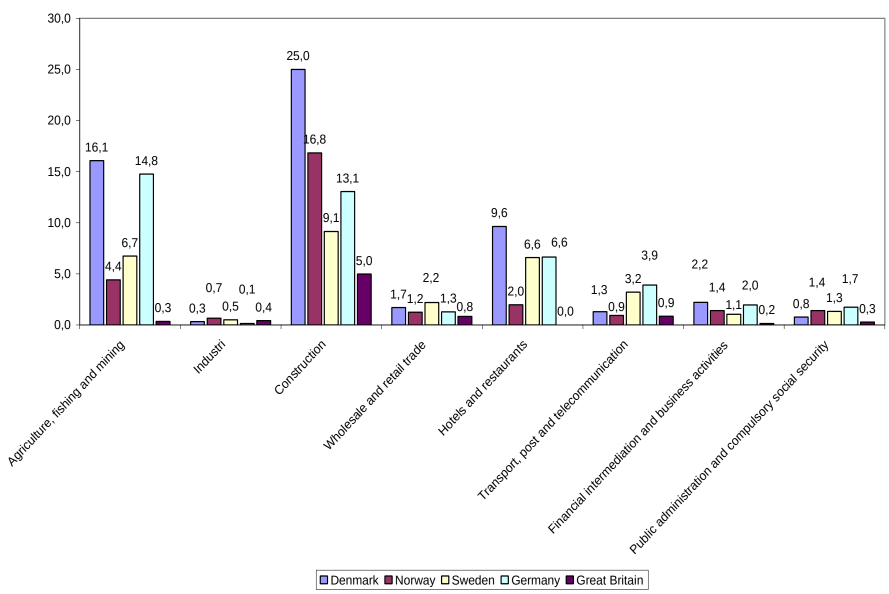


Figure 1: Fraction of informal sector work by industry. Pedersen 2003.

Furthermore, performing logistic regressions for the five countries, Pedersen (2003) confirms that the likelihood of informal market activities falls with the length of education. In addition, Boeri and Garibaldi (2002) show for Sicily, that mainly workers at the lower end of the skill distribution engage in informal activities.

The fact that mainly low educated workers seem to work in the informal sector suggests that the choice of educational attainment is potentially distorted. Informal employment opportunities foregone with education may simply reduce the incentives for workers to acquire education. The aim of this paper is to investigate the equilibrium impact of underground activities on labour market outcomes and educational attainment, as well as to characterize the optimal enforcement policy. Although harsher punishment policies may correct for a too low stock of educated workers, total unemployment may increase with such policy. In fact, we have little guidance from research to what extent formal sector jobs replace jobs in the underground economy as those jobs disappear with harsher informal sector punishment.

For this purpose, we develop a four sector general equilibrium model featuring matching frictions on the labour market. Unemployed workers search for jobs in both a formal and an informal sector, and workers decide whether or not to acquire higher education based on their ability levels.

In order to isolate the mechanisms and increase the transparency of the model, we keep the differences between the sectors at a minimum. The only dissimilarity between the formal and the informal sector is that taxes are not paid in the latter. Instead of paying taxes, informal sector firms have to pay a fine in case they are hit by an audit and detected as tax cheaters. In addition, firms in the informal sector are assumed to face concealment costs. In our model, we let concealment costs capture costs associated with concealing taxable income due to third party reporting or even moralilty, guilt or shame. The costs reduce the profitability of evading taxes and limits the size of the informal sector although the expected punishment fees are low relative to taxes. In line with Kleven et al (2010), we also let these costs be higher the more income that is hidden from the tax authorities.

Early theoretical analyses of tax evasion are provided by Allingham and Sandmo (1972) and Srinivasan (1973), where under-reporting of income is modelled as a decision made under uncertainty.[1] There has, however, been a switch in research focus from individual decision making on the extent of evasion, towards a modelling strategy where firms under-report their true profits, sales and wages paid, as advanced economies nowadays make extensive use of third-party information reporting.[2] There has also been a movement towards considering the case where firms or workers do not report at all. This characterizes the informal sector in focus in this paper.

The present paper extends the recent strand of the tax evasion literature, which departs from the assumption of imperfectly competitive labour markets, by incooperating involuntary unemployment through the inclusion of search friction. See for example Fugazza and Jacques (2004), Kolm and Larsen (2006), and Boeri and Garibaldi (2002).[3] As opposed to these previous studies, we do not need to rely on asymmetries between the formal and the informal sector, such as heterogeneity in morality, in order to generate existence of both a formal and an informal sector.

We find that underground activities reduce the incentives to acquire higher education if informal employment opportunities mainly are available to low educated workers. More zealous enforcement policies will in this case improve educational incentives as it reduces the attractiveness of remaining

[1] Subsequent papers have since then enhanced the basic model of individual behavior by, for example, incorporating endogenous labour supply decisions. See for example Sandmo (1981) for an early contribution of endogenous labour supply and underreporting of income.

[2] Also equilibrium models with tax evasion have been developed. For example see the early study by Cremer and Gahvari (1993) and the recent study by Tonin (2010).

[3] Some papers have also investigated informal employment from a non western economic point of view. Albrecht et al (2009) considers the impact of payroll taxes and severance pay on unemployment in the presence of an informal sector from a Latin American perspective. The informal sector can be seen as an unregulated sector which is not affected by payroll taxes and other formal policies.

a low educated worker. However, if also highly educated workers to a large extent are exposed to informal employment opportunities, the incentives to acquire higher education may fall with stricter enforcement policies as underground work pays off better to workers with high productivity. Moreover, we find that the actual unemployment rate most likely increases with stricter enforcement policies, although the official unemployment rate falls. Finally, characterizing the optimal enforcement policies, we find that relatively more audits should be targeted towards the sector employing low educated workers, elsewise the outcome is a too low stock of educated workers.

The paper is organized as follows. In section 2 the model is set-up. Section 3 offers a comparative statics analysis of an increase in the expected punishment fee. Simulations are presented in Section 4, whereas Section 5 considers optimal policy, and finally Section 6 concludes.

2 The model

This section develops a four sector general equilibrium model with formal and informal sector employment opportunities and educational choice. Workers differ in the ability to acquire education. Abilities, e, are uniformly distributed between 0 and $1, e \in [0,1]$, and the cost of higher education, $c(e)$, is decreasing in ability. Thus, workers with a high level of ability will find it more than worthwhile to attain higher education, whereas workers with low ability will not. Workers not attaining higher education will from now on be referred to as manual workers.

Both manual and highly educated workers allocate search effort optimally between the formal and the informal sector. Once matched with a firm, they bargain over the wage. The economy thus consists of four sectors; the formal and informal sector for manual workers, (denoted F, m and I, m), and the formal and informal sector for highly educated workers (denoted F, h and I, h).

2.1 Matching

Manual and highly educated workers search for jobs in both a formal and an informal sector. For simplicity, we assume that only unemployed workers search for jobs. This is a simplification, i.e. we do not acknowledge that the connection to the labour market given by working in the formal sector may bring about job opportunities not available while unemployed. The matching functions for the four categories of jobs are given by $X_l^j = \left(v_l^j\right)^{\frac{1}{2}} \left(\left(\sigma_l^j\right)^{\gamma} u_l\right)^{\frac{1}{2}}$, where X_l^j is the sectorial matching rate, v_l^j, is the sectorial vacancy rate, and u_l, is the unemployment rate, $j = F, I$ and $l = m, h$. The rates are defined as the numbers relatively to the labour force of manual and highly educated workers, respectively. The exponents in the matching function is set to be equal to a half in order to simplify the welfare analysis where we derive the optimal tax and punishment system when we have imposed the traditional Hosios condition. In that case we can disregard congestion externalities as the elasticity of the expected duration of a vacancy is equal to the bargaining power of workers in a symmetric Nash bargaining situation.[4]

Workers allocate search effort optimally across the formal and the informal sector. A worker with educational level l will direct σ_l^F units of search for a formal sector job, and σ_l^I units of search for an informal sector job. Thus workers with different levels of education may differ in their alloca-

[4]As we consider that educated workers direct their search towards jobs exclusively for educated workers, we need not be concerned with inefficiencies due to hold up problems (see Acemoglu and Shimer, 1999).

tion of search time across a formal and informal sector. Each worker's total search intensity is, however, exogenously given and normalized to unity, i.e. $\sigma_l^F + \sigma_l^I = 1,\ l = m, h$. The parameter $\gamma < 1$ captures that the effectiveness of search falls with search effort, i.e., the first unit of search in one sector is more effective than the subsequent units of search. This could capture that different search methods are used when searching for a job in a market. The more time that is used in order to search in a market, the less efficient search methods have to be used.

The transition rates into informal and formal sector employment for a particular worker i, are $\lambda_{li}^I = \left(\sigma_{li}^I\right)^\gamma \left(\theta_l^I\right)^{\frac{1}{2}}$ and $\lambda_{li}^F = \left(1 - \sigma_{li}^I\right)^\gamma \left(\theta_l^F\right)^{\frac{1}{2}}$, where $\theta_l^I = v_l^I / \left(\left(\sigma_l^I\right)^\gamma u_l\right)$ and $\theta_l^F = v_l^F / \left(\left(1 - \sigma_l^I\right)^\gamma u_l\right)$ are labour market tightness, $l = m, h$, measured in effective search units. The rates at which vacant jobs become filled are $q_l^j = \left(\theta_l^j\right)^{-\frac{1}{2}}$, $j = F, I$, $l = m, h$.

2.2 Value functions

Let U_l, E_l^F, and E_l^I denote the expected present values of unemployment and employment for manual and highly educated workers. The value functions for worker i then reads:

$$
\begin{aligned}
rU_{li} &= R + \lambda_{li}^F(E_l^F - U_{li}) + \lambda_{li}^I(E_l^I - U_{li}) - I\,(l)\,c(e_i), l = m, h, && (1)\\
rE_{li}^F &= R + w_{li}^F + s(U_l - E_{li}^F) - I\,(l)\,c(e_i), l = m, h, && (2)\\
rE_{li}^I &= R + w_{li}^I + s(U_l - E_{li}^I) - I\,(l)\,c(e_i), l = m, h, && (3)
\end{aligned}
$$

where r is the exogenous discount rate, w_l^j is the sector wage, and s is the exogenous separation rate. R is a lump-sum transfer that all individuals receive from the government which reflects that the government has some positive revenue requirements.[5] Highly educated workers pay the individual educational costs $c(e_i)$, where e_i is the worker's ability, $e_i \in [0,1]$, $c'(e_i) < 0$ and $c''(e_i) > 0$. The indicator function $I(l), l = m, h$ takes the value zero for manual workers and the value one for highly educated workers, hence $I(m) = 0$ and $I(h) = 1$.[6] For simplicity, we disregard unemployment benefits.

Let J_l^F and V_l^F represent the expected present values of an occupied job and a vacant job in the formal sector, respectively. The arbitrage equations for the formal sector of a job paying the wage w_{li}^F and a vacant job are then

$$rJ_{li}^F = y_l - w_{li}^F(1+z) + s(V_l^F - J_{li}^F), l = m, h, \tag{4}$$

$$rV_l^F = q_l^F(J_l^F - V_l^F) - ky_l, l = m, h, \tag{5}$$

where z is the payroll tax rate. Vacancy costs are denoted ky_l. Analogous notation for the informal sector yields:

$$rJ_{li}^I = y_l - w_{li}^I(1 + p\alpha + \kappa_l) + s(V_l^I - J_{li}^I), l = m, h, \tag{6}$$

$$rV_l^I = q_l^I(J_l^I - V_l^I) - ky_l, l = m, h, \tag{7}$$

where p is the auditing rate which captures the probability of being detected employing a worker in the informal sector, and α is the associated firm punishment fee rate. The concealment costs, $\kappa_l, l = h, m$, capture that it is costly

[5] Everyone receives this transfer. The government cannot exclude the informal sector workers as the government does not know who the informal sector workers are (if it did, it could punish all of them).

[6] It is assumed to be costless to become a manual worker, but that workers who get a higher education find it costly to do so. This is a normalisation and has no consequences for the results. Moreover, we assume that the educational cost is a cost to acquire and maintain skill. This is a simplifying assumption and is not important for the results. The assumption enables us to use a model without having workers continuously being born and dying. Such a model would, however, generate the same qualitative expressions.

to hide income from the tax authorities. The costs could, for example, capture what Kleven et al (2011) refer to as third party reporting. When there is third party reporting of income, such as the firm reporting the wage payments directly to the tax authorities, this has to be agreed upon also by the worker, which is costly. These concealment costs could also be other direct costs associated with concealing evasion, as well as morality costs associated with evading taxes.

If firms hiring highly educated workers have a harder time concealing their activities than firms hiring manual workers, then $\kappa_h > \kappa_m$. This is the case if, for example, third party reporting is more common for highly educated or, as assumed in Kleven et al (2011), the marginal costs of evasion increases with the amount of income evaded. Although this is likely to be the case, we do not a priori impose any restriction on the values of $\kappa_l, l = h, m$.

In order to improve the transparency of the model, we disregard taxation, expected punishment and concealment costs on the worker side. This is of no importance for the results.

The unemployed worker i allocates search between the two sectors, σ_{li}^{I}, in order to maximize the value of unemployment, rU_{li}. A necessary condition for an interior solution is that $\gamma < 1$, which holds by assumption. The first order condition can be written as:

$$\frac{\left(1-\sigma_{li}^{I}\right)^{1-\gamma}}{\left(\sigma_{li}^{I}\right)^{1-\gamma}} = \left(\frac{\theta_l^F}{\theta_l^I}\right)^{\frac{1}{2}} \frac{E_l^F - U_{li}}{E_l^I - U_{li}}, l = m, h. \tag{8}$$

Workers allocate their search between sectors to equalize the net marginal returns to search effort across the two sectors.

2.3 Wage determination

When a worker and firm meet they bargain over the wage, w_{li}^{j}, taking economy wide variables as given. The first order conditions from the Nash bargaining with equal bargaining power for workers and firms, can be written as:

$$J_l^F = \left(E_l^F - U_l\right)(1+z), l = m, h, \tag{9}$$

$$J_l^I = \left(E_l^I - U_l\right)(1+p+\kappa_l), l = m, h, \tag{10}$$

where we have imposed symmetry and the free entry condition, $V_l^j = 0$, $j = F, I$, $l = m, h$.

We can now derive an equation determining how search is allocated between the formal and the informal sectors in a symmetric equilibrium by substituting (9) and (10) into (8) and using that $J_l^F = \frac{ky_l}{q_l^F}$ and $J_l^I = \frac{ky_l}{q_l^I}$ from (5) and (7) together with free entry. This yields:

$$\frac{\left(1-\sigma_l^I\right)^{1-\gamma}}{\left(\sigma_l^I\right)^{1-\gamma}} = \frac{\theta_l^F}{\theta_l^I}\psi_l, \tag{11}$$

where

$$\psi_l = \frac{1+p\alpha+\kappa_l}{1+z}$$

is the cost wedge between the informal sector and the formal sector. When workers allocate their search between the formal and the informal sectors in equilibrium, they account for the wedge, ψ_l, and for the formal relative to the informal sectorial tightness, θ_l^F/θ_l^I. It follows that relatively more search will be directed towards the formal sector if expected punishment plus concealment costs are higher than the tax payments, i.e. if $\psi_l > 1$, and if formal sector tightness exceeds informal sector tightness, i.e., $\theta_l^F/\theta_l^I > 1$. And vice versa when $\psi_l < 1$ and $\theta_l^F/\theta_l^I < 1$.

By use of equation (1)-(7) and (11) in equations (9) and (10), equilibrium producer wages, ω_l^j, are given by:

$$\omega_l^F = w_l^F (1+z) = \frac{1}{2} y_l \left(1 + k \frac{\theta_l^F}{(1-\sigma_l^I)^{1-\gamma}}\right), l = m, h, \quad (12)$$

$$\omega_l^I = w_l^I (1 + p\alpha + \kappa_l) = \frac{1}{2} y_l \left(1 + k \frac{\theta_l^I}{(\sigma_l^I)^{1-\gamma}}\right), l = m, h. \quad (13)$$

An increase in tightness, θ_l^j, makes it easier for an unemployed worker to find a job, and at the same time harder for a firm to fill a vacancy. This improves the worker's relative bargaining position, resulting in higher wage demands. An increase in search, will instead increase the firm's relative bargaining position. This follows as firms then will find it easier to match with a new worker in case of no agreement. The improved bargaining position for firms moderates wage pressure.

2.4 Labour market tightness

Labour market tightness for the formal sector and the informal sector are determined by equation (4),(5), (6) and (7) using the free entry condition and the wage equations (12) and (13):

$$k(r+s)\left(\theta_l^F\right)^{\frac{1}{2}} = \frac{1}{2}\left(1 - \frac{k\theta_l^F}{(1-\sigma_l^I)^{1-\gamma}}\right), l = m, h, \quad (14)$$

$$k(r+s)\left(\theta_l^I\right)^{\frac{1}{2}} = \frac{1}{2}\left(1 - \frac{k\theta_l^I}{(\sigma_l^I)^{1-\gamma}}\right), l = m, h, \quad (15)$$

By use of the equilibrium search allocation equation in (11), where $\frac{\theta_l^I}{(\sigma_l^I)^{1-\gamma}} = \frac{\theta_l^F}{(1-\sigma_l^I)^{1-\gamma}}\psi_l$, in (15) it becomes clear that the wedge, ψ_l, is the crucial factor determining the size of the formal sector in relation to the informal sector.[7] When $\psi_l > 1$, and thus expected punishment plus concealment costs are higher than payroll taxes, informal sector producer wages are higher than formal sector producer wages. In this case it is relatively more attractive for firms to enter the formal sector, which makes formal sector tightness exceed informal sector tightness. Hence, we obtain that $\theta_l^F > \theta_l^I$ and $\sigma_l^I < \frac{1}{2}, l = m, h$ if $\psi_l > 1$. And vice versa when $\psi_l > 1$.

As the formal sector exceeds the informal sector in size in most western countries, it is most realistic to consider the case when $\psi_l > 1$. This implies considering the situation when the expected punishment rate plus concealment costs exceed the tax rate, i.e., $p\alpha + \kappa_l > z$, which does not seem unrealistic given a broad interpretation of concealment costs. In fact, as discussed in the introduction, positive concealment costs $\kappa_l > 0$ such that $p\alpha + \kappa_l > z$ could potentially explain the puzzle of why we observe a relatively small informal sector although we, at the same time, observe rather low audit rates and fairly modest fines, i.e., $p\alpha < z$. However, we do not a priori impose any restrictions on the size of $\psi_l, p\alpha$ or κ_l when deriving the results in this paper. When discussing results that depends on the size of ψ_l, however, we focus the discussion on, what we believe is the most realistic case.

2.5 Education

When workers decide whether to acquire higher education or remain a manual worker, they compare the value of unemployment as an educated worker to

[7] Relative tightness is determined by: $\theta_l^F/\theta_l^I = \left(\frac{1-k\theta_l^F(1-\sigma_l)^{\gamma-1}}{1-k\theta_l^F(1-\sigma_l)^{\gamma-1}\psi_l}\right)^2 \begin{array}{l} > 1 \text{ if } \psi_l > 1 \\ = 1 \text{ if } \psi_l = 1 \\ < 1 \text{ if } \psi_l < 1 \end{array}$

the value of unemployment as a manual worker. Workers with low ability find it too costly in terms of effort to acquire higher education, whereas high ability workers find it more than worthwhile to do so since they face lower costs of education. The marginal worker has an ability level, $\hat{e}$, which makes him just indifferent between acquiring higher education and remaining a manual worker. We write the condition determining the ability level of the marginal worker as:

$$rU_h(\hat{e}) = rU_m. \tag{16}$$

By using equations (1)-(3), it is clear that workers proceed to higher education if the expected income gain of education exceed their cost of education. However, as wages are endogenous, we can use equations (1) and (16) together with the first order conditions for wages, and equations (5), (7), (11), together with the free entry condition. This gives the following rewriting of condition (16):

$$c(\hat{e}) = \frac{k}{1+z}(y_h o_h - y_m o_m), \tag{17}$$

where $o_l = \theta_l^F / \left(1 - \sigma_h^I\right)^{1-\gamma}, l = h, m$. Equation (17) gives $\hat{e}$ as a function of the endogenous variables θ_l^F and σ_l^I, $l = m, h$. Workers with $e \leq \hat{e}$, choose not to acquire education, whereas workers with $e > \hat{e}$ acquire education. Hence, $\hat{e}$ and $1 - \hat{e}$ constitute the manual and educated labour forces, respectively.

The right hand side in equation (17) is the expected income gain of attaining education. This gain needs to be positive in order for, at least some, worker to proceed to higher education. The fact that productivity is higher for highly educated workers, which gives rise to an educational wage premium, provides incentives for higher education. However, higher education may potentially also be associated with losses in expected income. For example, if concealment costs are higher for highly educated workers, i.e.,

$\kappa_h > \kappa_m$, relatively more attractive informal employment opportunities for manual workers will be foregone in case of higher education. This reduces the incentives for education.[8]

Clearly, in order to study the non-trivial case, where at least some workers proceed to higher education, it is necessary to assume that there is a net gain in expected income of higher education. Thus, we need to assume that productivity differences between manual and highly educated workers are sufficiently high, i.e., $y_h/y_m > o_m^F/o_h^F$. Moreover, to guarantee a non-trivial interior solution where at least some, but not all, individuals choose to acquire education, the individual with highest ability face a very low costs of education, more specifically $c(1) = 0$, and the individual with the lowest ability face very high cost of education, i.e., $\lim_{e\to 0} c(e) = \infty$. See the appendix for the proof of existence of $\hat{e} \in (0,1)$.

2.6 Employment and Unemployment

The equations determining the employment rates in the formal sector and the informal sector, n_l^F, n_l^I, and the unemployment rates, $u_l, l = m, h$, are given by the flow equilibrium equations and the labour force identity.[9] The official unemployment rate u_l^o, is given by $u_l^o = u_l + n_l^I$. Solving for the employment and unemployment rates yield:

$$n_l^I = \frac{\lambda_l^I}{s + \lambda_l^I + \lambda_l^F}, n_l^F = \frac{\lambda_l^F}{s + \lambda_l^I + \lambda_l^F}, l = m, h, \tag{18}$$

$$u_l = \frac{s}{s + \lambda_l^I + \lambda_l^F}, u_l^o = \frac{s + \lambda_l^I}{s + \lambda_l^I + \lambda_l^F}, l = m, h. \tag{19}$$

[8]See the appendix for the proof of that $o_h < o_m$ when $\kappa_h > \kappa_m$.

[9]For highly educated workers $\lambda_h^I u_h \hat{e} = s n_h^I \hat{e}$, $\lambda_h^F u_h \hat{e} = s n_h^F \hat{e}$, and $n_h^F + n_h^I = 1 - u_h$, and for manual workers $\lambda_m^I u_m (1-\hat{e}) = s n_m^I (1-\hat{e})$, $\lambda_m^F u_m (1-\hat{e}) = s n_m^F (1-\hat{e})$, and $n_m^F + n_m^I = 1 - u_m$

A comparison of the unemployment rates for manual and highly educated workers requires assumptions about the size of the concealment costs. If concealments cost are higher for educated workers, i.e., $\kappa_h > \kappa_m$, the official unemployment rate is always lower for highly educated workers than for manual workers, i.e., $u_h^o < u_m^o$. This is also what is observed in data. However, if furthermore, $\psi_l > 1, l = h, m$ and hence the informal sector is smaller than the formal sector, the actual unemployment rate is higher for the highly educated workers, $u_h > u_m$, i.e. in this case, manual workers have a lower actual unemployment rate than highly educated workers.

The following proposition summarizes the results.

Proposition 1 *The official unemployment rate is lower for highly educated workers, $u_h^o < u_m^o$, if they face higher concealment costs, $\kappa_h > \kappa_m$. The actual unemployment rate is higher (lower) for highly educated workers, $u_h > u_m$ ($u_h < u_m$), if they face higher concealment costs, $\kappa_h > \kappa_m$, and these concealment costs are high (low) enough to induce $\psi_l > 1$ ($\psi_l < 1$), $l = m, h$.*

For proofs of all Propositions see the Appendix. The actual and official total number of unemployed workers are given by:

$$\begin{aligned} U_{TOT} &= \hat{e}u_m + (1 - \hat{e})u_h, \\ U_{TOT}^o &= \hat{e}u_m^o + (1 - \hat{e})u_h^o. \end{aligned}$$

3 Comparative statics

This section is concerned with the impact of more severe punishment of informal activities on labour market performance and educational attainment. We only consider fully financed changes in enforcement policies. Hence, changes in the audit rate and the punishment fees are always followed by adjustments in the tax rate so as to balance the government budget constraint given by:

$$\hat{e} \sum_{j=F,I} n_m^j w_m^j z + (1 - \hat{e}) \sum_{j=F,I} n_h^j w_h^j p\alpha = R, \tag{20}$$

where R is the exogenous revenue requirement.

From (20) it follows that an increase in the audit rate or the punishment fee, p or α, or an increase in the tax rate, z, will, for a given tax base, always increase government revenues. The tax base may, however, fall and thereby reduce revenues. Throughout the analysis we will assume that we are located on the positively sloped side of the "Laffer curves". This implies that the direct effect of taxation and punishment on government revenues will always dominate the impact on revenues due to that the tax base may be reduced. An increase in the audit or punishment rate then always calls for a reduction in the tax rate in order to regain a balanced government budget. A fully financed increase in the punishment of the informal sector then induces ψ_l to increase both because $p\alpha$ increases and because z falls.

In the government budget restriction in (20), potential auditing costs are left out. To include auditing costs will not affect any of the results derived in the positive analysis below. However, it affects the welfare analysis as it tends to favour costless taxation and punishment fees at the expense of auditing. The implications for the case of auditing costs is shown in the appendix .

3.1 Sector Allocation

The effects on the allocation of search and employment across the formal and the informal sector are summarized in the following proposition.

Proposition 2 *A fully financed increase in the audit rate, p, or the punishment fee, α, will reallocate search intensity and employment towards the formal sector, i.e., σ_l^I falls, n_l^F increases, and n_l^I falls.*

More zealous enforcement will make informal work less attractive, inducing unemployed workers to reallocate their search effort towards the formal sector. In addition, when search is reallocated towards the formal sector, the wage bargaining position strengthens for firms in the formal sector whereas it falls for firms in the informal sector. The lower producer wages in the formal sector stimulates formal firms to open vacancies, while at the same time, informal firms are discouraged to open new vacancies as they now face higher producer wages. As a consequence of that both vacancies and search effort is reallocated towards the formal sector, the formal sector employment rate increases at the expense of informal employment.

3.2 Unemployment Rates

As became clear in proposition 2, employment in the formal sector increases at the expense of employment in the informal sector following more severe punishment of the informal sector. While this is somewhat expected, it is a prior not clear what would happen to the unemployment rates. We have the following results:

Proposition 3 *A fully financed increase in the audit rate, p, or in the punishment fee, α, will always cause the official unemployment rate (u_l^o) to fall, whereas the actual unemployment rate (u_l) increases if $\psi_l > 1$ (falls if $\psi_l < 1$).*

The actual unemployment rates increase with more severe punishment of informal work if $\psi_l > 1$. The reason for this is that the large concealments costs discourages workers from searching, and firms from opening vacancies, in the informal sector. In fact, too few firms and too little search are allocated into the informal sector from an efficiency point of view. Increased punishment of the informal sector will encourage further reallocation of search and workers away from the informal sector, where relatively efficient search methods are used, towards the formal sector. Total search efficiency then falls, inducing unemployment to increase.

The fact that search becomes less efficient when reallocated towards the formal sector also has an impact on unemployment working through wage formation and tightness. As search is reallocated towards the formal sector, the wage demand is moderated in the formal sector and exaggerated in the informal sector. As search efficiency in the formal sector increases by less than search efficiency in the informal sector is reduced, the informal sector wage push will dominate the formal sector wage moderation. Thus, the incentives to open up a vacancy in the formal sector subseeds the disincentives to open up a vacancy in the informal sector; formal sector tightness will increase by less than informal sector tightness falls when $\psi_l > 1$.

The opposite holds if $\psi_l < 1$. In this case too much search, and too many firms, are allocated into the informal sector as there is a relative cost advantage of producing underground. Total search efficiency would then improve when the government tries to combat the informal sector.

The official unemployment rate always falls with more harsh punishment of informal activates as workers to a larger extent becomes formally employed. In this unemployment measure, workers in the informal sector where counted as unemployed to start with.

3.3 Education

From (17) it is clear that more severe punishment of the informal sector affect the number of educated workers as such policy increases ψ_l. This effect is further reinforced if the tax rate is reduced in order to assure a balanced government budget as the increase in ψ_l is reinforced by a reduction in z. However, a reduced payroll tax rate will also have a direct effect on the stock of educated workers. More specifically, a reduction in the tax rate, z, for a given wedge, will increase the number of educated workers. This follows as taxation is more harmful to high income earners, and consequently a tax reduction will improve the income relatively more for high income earners.

However, let us first consider the impact of a more zealous enforcement policy on education, for a given tax rate. We have the following results:

Proposition 4 *An increase in the audit rate, p, or the punishment fee, α, will increase (reduce) the number of educated workers if the relative productivity of education is in the following range $\frac{y_h}{y_m} \in \left[\frac{o_m}{o_h}, g(\kappa_h, \kappa_m)\frac{o_m}{o_h}\right]$ ($\frac{y_h}{y_m} \in \left[g(\kappa_h, \kappa_m)\frac{o_m}{o_h}, \infty\right]$) where $g(\kappa_h, \kappa_m) > 1$ if $\kappa_h > \kappa_m$.*

Proof. We know from above that the existence of an interior solution of $\hat{e}$ requires that $y_h/y_m > o_m/o_h$. Differentiating the educational equation with respect to expected punishment reveals that the impact on education is determined by the sign of $y_m |do_m/d(p\alpha)| - y_h |do_h/d(p\alpha)|$ which is equal to the sign of $y_h/y_m - g(\kappa_h, \kappa_m)(o_m/o_h)$, where the term $g(\kappa_h, \kappa_m)$ is independent of y_h/y_m and larger than one for $\kappa_h > \kappa_m$:

$$g(\kappa_h, \kappa_m) \equiv \frac{A_h \left(\frac{\theta_h^F}{\theta_h^I}\right)^{\frac{1}{1-\gamma}-\frac{1}{2}} \psi_h^{\frac{1}{1-\gamma}} + \psi_h}{A_m \left(\frac{\theta_m^F}{\theta_m^I}\right)^{\frac{1}{1-\gamma}-\frac{1}{2}} \psi_m^{\frac{1}{1-\gamma}} + \psi_m} > 1, \ for\ \kappa_h > \kappa_m$$

where $A_l = (1 + o_l) \,/\, \left(\frac{1}{\psi_l} + o_l\right)$. See the appendix for a full proof. ■

The impact of a more zealous enforcement policy on educational attainment depends on how attractive underground work is to manual and educated workers, respectively. When concealment costs are higher for highly educated workers, more zealous enforcement policies tend to induce more workers to educate themselves. This follows as $\kappa_h > \kappa_m$ implies that manual workers to a larger extent face informal labour market opportunities. Therefore, more zealous enforcement policies, which makes it less attractive to work in the informal sector, will be more harmful to manual workers.

This effect may, however, be counteracted by the fact that highly educated workers have higher productivity and therefore earn higher wages. As also informal activities are highly productive for these workers, this implies that more harsh punishment, in this perspective, are more harmful for highly educated worker. Thus, even if highly educated workers face less informal employment opportunities, these opportunities are more profitable. This reduces educational incentives.

Which of the two effects dominate will thus depend on how sizable the differences in informal employment opportunities and productivity are. If underground employment opportunities in an economy foremost are available to manual workers, more harsh punishment of underground activities will push more workers into education. Thus increasing the stock of educated workers in the economy. However, if these employment opportunities to a large extent also are available for highly educated workers, harder punishment will harm highly educated workers more as these opportunities are more profitable to productive workers. This leads to that less workers educate themselves.

Note that proposition 4 only provides the sufficient conditions for when the educational stock increases and when it falls with more harsh punishment of the informal sector without considering the financing of the reform. Provided that we are located on the positively sloped side of the Laffer curve, we can conclude the following:

Proposition 5 *If an increase in the audit rate, p, or in the punishment rate, α, increases the number of educated workers as given by proposition 4, the financing of the reform will further increase the stock of educated workers.*

This simply follows as taxation as a direct effect is more harmful for high income earners, and consequently a tax reduction, in order to maintain a balanced government budget, will be more beneficial for high income earners, thus encouraging educational attainments.

3.4 Unemployment

From proposition 3, 4 and 5, it follows that more severe punishment of the informal sector potentially increases the total number of unemployed workers. If the formal sector is larger than the informal sector, the unemployment rates for both manual and highly educated workers are augmented. Moreover, if informal employment opportunities to a significantly larger extent are available for manual workers, more workers will attain higher education when informal activities is punished more severely. This tends to increase total unemployment as the actual unemployment rate, including informal work, is higher for highly educated workers. Thus, in this case, total unemployment increases both because the unemployment rates for all workers increase, and because workers are reallocated towards the sector where the unemployment rate is highest. More generally, the proposition summarizes the result:

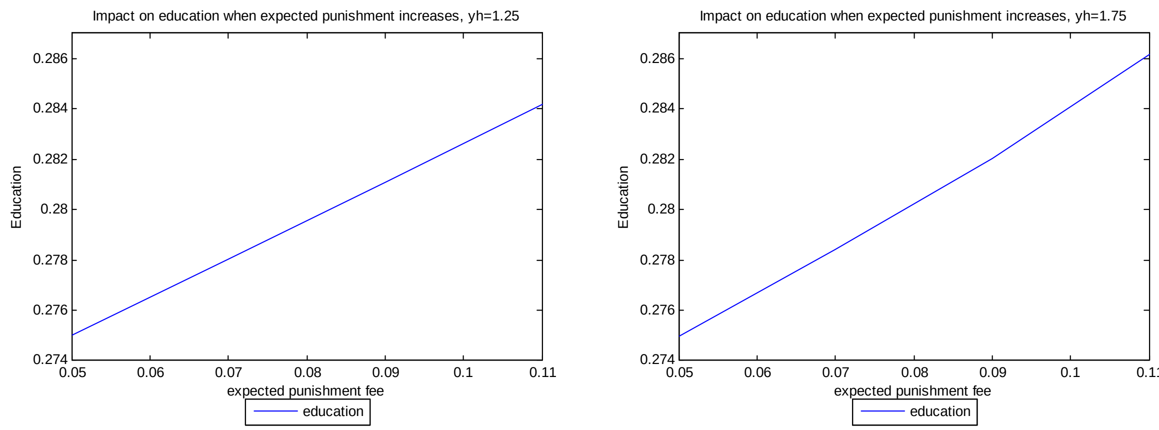
Proposition 6 *A fully financed increase in the audit rate, p, or in the punishment fee, α, increases(decreases) the number of unemployed workers if the relative productivity of education is in the following range* $\frac{y_h}{y_m} \in \left[\frac{o_m}{o_h}, g\left(\kappa_h, \kappa_m\right)\frac{o_m}{o_h}\right]$ *where* $g\left(\kappa_h, \kappa_m\right) > 1$ *if* $\kappa_h > \kappa_m$*, and* $\psi_l > 1 (< \psi_l)$.

4 Numerical Simulations

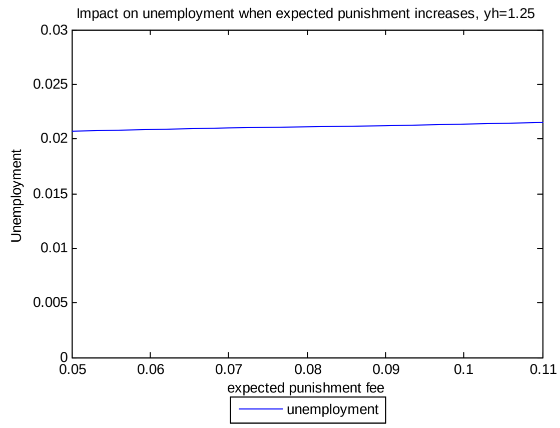
This section considers numerical simulations of the model in order to quantify the effect of more zealous enforcement policy on educational attainment, unemployment, and the size of the informal sector. For the numerical simulations, the year is the basic time unit, and the productivity of manual workers are normalized to unity, $y_m = 1$. The discount rate is set to $r = 0.08$ as is the separation rate, $s = 0.08$ (see Millard and Mortensen 1997). The productivity of highly educated workers y_h is assumed to be 1.25. The educational costs are captured by the following cost function: $c\left(e\right) = g\left(1-e\right)^t$, where the parameters g and t, the hiring cost parameter, k, the search efficiency parameter, γ, the concealment costs, κ_h and κ_m, are set to match the OECD average fraction of 25 to 64 years old with tertiary education of 27.5 percent (OECD 2009), an average observable unemployment rate of 10%, with an observable unemployment rate for manual workers of slightly less than 12%, and for highly educated workers of about 6%, the employment rates for the informal sector are chosen to be between 5% and 15% and informal employment relatively to formal employment to be between 3% and 10%. The official unemployment rates are slightly higher than the pre-financial crises average in the OECD countries. The reason for this is partly that the model overestimates the official unemployment rates as all informally employed workers are assumed to be unemployed job searchers.[10] The endogenous payroll tax rate, z, which always balances the government budget constraint is initially equal to 20%. The parameter values we use are:

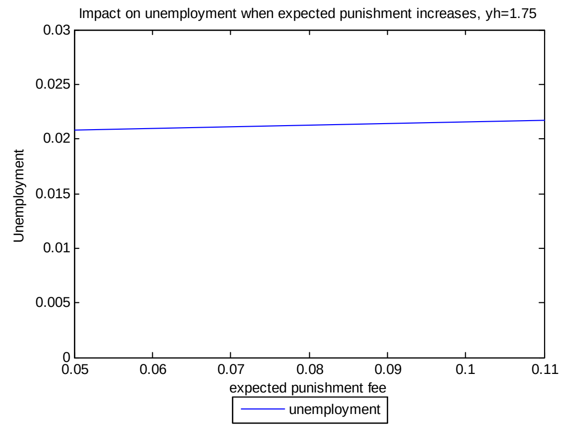
r	s	κ_h	κ_m	k	γ	y_h	y_m	$p\alpha$	M	g	t	R
0.08	0.08	0.9	0.75	0.4	0.35	1.25	1	0.05	3	0.5	1.1	0.1045

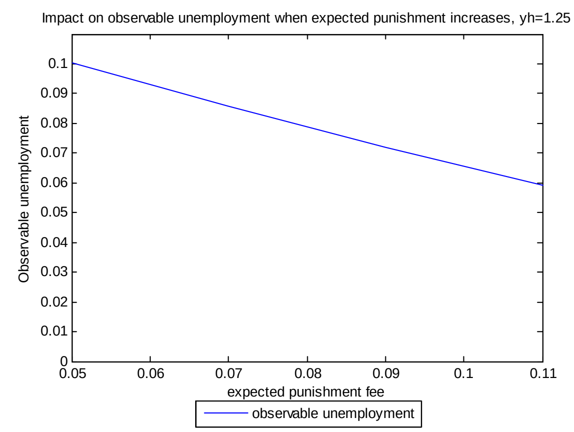
The impact of stricter punishment policies on education is positive (Figure 2, panel 1):

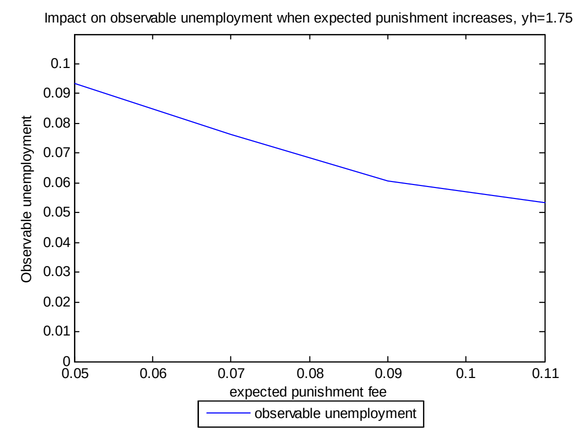


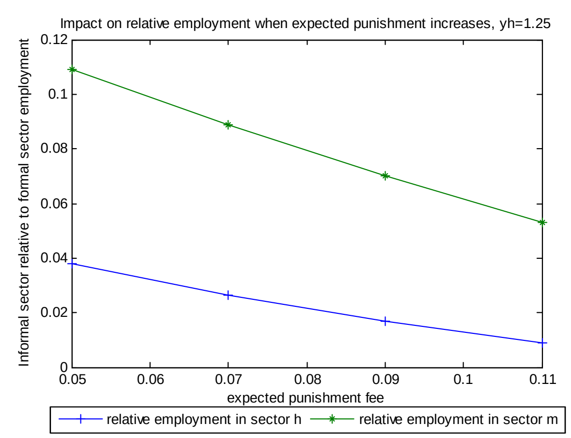
[10]Based on labour market surveys, the official unemployment rate is measured by $u^o = \frac{U^o}{LF^o}$, where the U^o is the total number of unemployed job searchers and LF^o is the official labour force. In the model, everyone who is employed in the informal sector belongs to the labour force which implies that $u^o = \frac{U^o+E^I}{U+E^F+E^I}$. However, if some workers employed in the informal sector is counted as out of the labour force, the observed unemployment rate in the labour statistics meassure $u^o = \frac{U^o+\alpha E^I}{U+E^F+\alpha E^I}$, where α is the share of the informal workers being out of the labour force. Thus an $\alpha < 1$ implies that the model value of the official unemployment rate should match to a higher value of unemployment than suggested by the labour statistics. This is of no importance for the analytical results, but biases the values slightly in the numerical analysis.

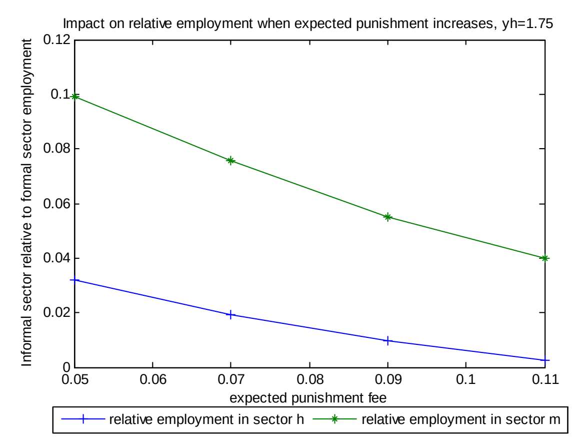












From the analytical part we know that tighter enforcement policy have an ambiguous effect on educational attainments. Harsher punishment tends to increase educational attainments as manual workers face more informal employment opportunities when $\kappa_h > \kappa_m$ i.e., the "employment opportunity effect". On the other hand, more severe punishment tends to reduce educational attainments because highly educated workers are more productive, i.e., the "productivity effect". Moreover, in equilibrium (when z adjusts so to balance the government revenue), a reduction in z reinforces the employment opportunity effect.

From our numerical exercise the impact of stricter punishment policies on education turns out to be positive (see figure 2, panel 1). Thus the employment opportunity effect dominates the productivity effect. This is explained by the relatively sharp reduction in informal to formal sector employment for manual workers in relation to highly educated workers as seen in panel 4. We further notice that unemployment is hardly affected (panel 2), whereas observable unemployment is significantly reduced (panel 3) corresponding to the shift of workers from formal to informal employment (panel 4). We showed analytically that the relative importance for the two counteracting effects on educational attainments hinged on differences in κ_l and y_l. But even for very high productivity levels, $y_h = 1.75$, facing the highly educated workers, education increases with $p\alpha$ (right hand graphs). We increase the parameter g (to $g = 1.3836$) simultaneously so to keep the initial educational level at $1 - \hat{e} = 0.275$. We furthermore notice that the impact on education is slightly higher for $y_h = 1.75$ than for $y_h = 1.25$, which may be due to that z falls to $z = 0.189$ when $y_h = 1.75$. We therefore keep z fixed, and observe in our benchmark case, see Figure 3, that when z is exogenously fixed, educational attainments increase with stricter punishment of informal work, although the effect is smaller.

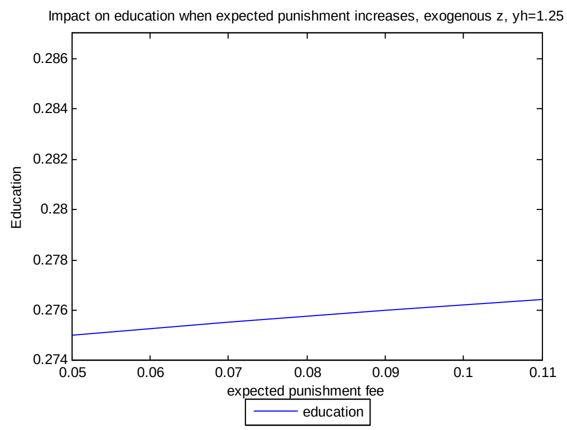


Figure 3

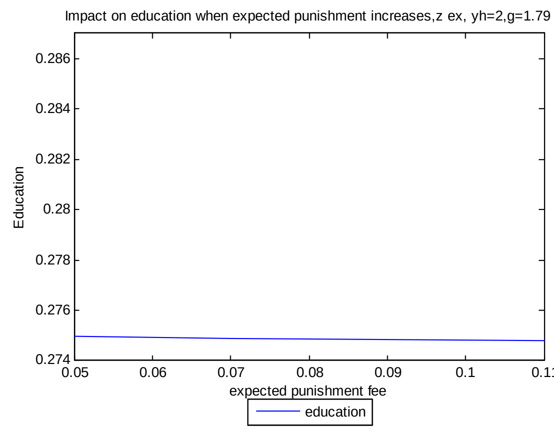


Figure 4

As shown analytically in proposition 4, letting κ_m increase to the same size as κ_h, causes the productivity effect to dominate, leading to a reduction in the educational attainments for a given z. This is, however, a less plausible outcome in terms of the relative size of informal-formal employment.

Sensitivity analysis with respect to the other parameters of the model, reveals that the results are sensitive to changes in γ. When γ is relative small, i.e. $\gamma = 0.25$, and κ_m increases to $\kappa_m = 0.85$ education falls for fixed z. See Figure 4. However allowing for z to adjust to maintain government budget balance reestablishes the positive impact on education.

5 Welfare

This section is concerned with welfare analysis and the optimal design of punishment policies. As shown above, increasing the punishment fees or the audit rates affect the number of educated workers as well as the allocation of search and jobs across the formal and informal sectors. This is essential when considering the impact on welfare.

We make use of a utilitarian welfare function, which is obtained by adding all individuals' steady state flow values of welfare. This accounts for that both the formal and the informal economy generate welfare in the economy. The social welfare function is written as:

$$W = \hat{e}\tilde{W}_m + \int_{\hat{e}}^{1} \tilde{W}_h de,$$

where

$$\begin{aligned}
\tilde{W}_m &= u_m r U_m + \sum_{j=F,I} n_m^j r E_m^j + \sum_{j=F,I} n_m^j r J_m^j + \sum_{j=F,I} v_m^j r V_m^j + n_m^I r J_m^{law}, \\
\tilde{W}_h &= u_h r U_h + \sum_{j=F,I} n_h^j r E_h^j + \sum_{j=F,I} n_h^j r J_h^j + \sum_{j=F,I} v_h^j r V_h^j + n_h^I r J_h^{law}.
\end{aligned}$$

We assume that firms are owned by "renters" who do not work. This explains the presence of $\sum_{j=F,I} n_m^j r J_m^j + \sum_{j=F,I} v_m^j r V_m^j$ and $\sum_{j=F,I} n_h^j r J_h^j + \sum_{j=F,I} v_h^j r V_h^j$ in the welfare function. Moreover, we assume that the concealment costs for tax evasion facing firms are payments to "lawyers" who only engage in concealing taxable income for other firms. The welfare function therefore includes $n_m^I r J_m^{law} = n_m^I w_m^I \kappa_m$ and $n_h^I r J_h^{law} = n_h^I w_h^I \kappa_h$. This assumption enables us to disregard from the waste associated with tax evasion if firms only pay these expenses to nobody.

By making use of the asset equations, imposing the flow equilibrium conditions as well as the government budget restriction in (20), and considering the case of no discounting, i.e., $r \to 0$, we can write the welfare function as:

$$
\begin{aligned}
W &= \hat{e}W_m + \int_{\hat{e}}^{1} W_h de, && (21)\\
&W_m = (1-u_m)\, y_m - u_m k y_m \Theta_m, && (22)\\
&W_h = (1-u_h)\, y_h - u_h k y_h \Theta_h - c\,(e)\,, && (23)
\end{aligned}
$$

where $\Theta_l = \left(1-\sigma_l^I\right)^{\gamma}\theta_l^F + \left(\sigma_l^I\right)^{\gamma}\theta_l^I,\ l = m, h.$ This welfare measure is analogous to the welfare measure described in, for example, Pissarides (2000) as it includes aggregate production minus total vacancy costs, i.e. note that $u_l\Theta_l k = \left(v_l^F + v_l^I\right)k,\ l = m, h.$ With the assumption of risk neutral individuals, we ignore distributional issues and hence wages will not feature in the welfare function.

Let us first derive the socially optimal choice of tightness, search and stock of educated workers by maximizing the welfare function in (21)-(23) with respect to $\theta_m^F, \theta_m^I, \theta_h^F, \theta_h^I, \sigma_m^I, \sigma_h^I$ and $\hat{e}$. The socially optimal solution are solved from the following seven conditions[11]:

$$
\begin{aligned}
\left(\sigma_l^{I*}\right)^{(\gamma-1)} - \left(1-\sigma_l^{I*}\right)^{\gamma-1} &= 0, \ \rightarrow\ \sigma_l^{I*} = \frac{1}{2} \qquad l = m, h && (24)\\
-sk\left(\theta_l^{*I}\right)^{\frac{1}{2}} + \frac{1}{2}\left[1 - \frac{k\theta_l^{*I}}{\left(\frac{1}{2}\right)^{1-\gamma}}\right] &= 0, \qquad l = m, h, j = F, I && (25)\\
\left(y_h - y_m\right)\frac{k\theta_l^{*I}}{\left(\frac{1}{2}\right)^{1-\gamma}} - c\left(\hat{e}^*\right) &= 0. && (26)
\end{aligned}
$$

We can now compare the socially optimal solution with the market outcome. From (11), (14), and (15) it follows that the market solution for search and tightness coincides with the socially optimal allocation if the imposed

[11] See the appendix for the second order conditions.

tax and punishment policy are such that $\psi_m = \psi_h = 1$.[12]

This conclusion is intuitive as any policy that induces a deviation of the $\psi^j, j = m, h$, from unity, implies a favourable treatment of the formal or the informal sector which, in turn, induces a distortion in the sectorial allocation of search and tightness between the formal and informal sectors. For example, if search to a larger extent is allocated to the formal or informal sector instead of the other, search is inefficiently used as less efficient search methods in that sector needs to be used. Moreover, as discussed in relation to proposition 3, a favourable treatment of either the formal or the informal sector induce too many firms to open vacancies in that sector without accounting for the externality they impose on others. In fact, unemployment is minimized when the allocation of search and tightness across the formal and informal sector is equal, and so is vacancy costs. Thus, welfare is maximized when search and tightness are allocated equally across the formal and the informal sector.

Now let us compare the socially optimal stock of educated workers with the educational outcome induced by the market. As the market outcome in terms of sectorial allocation of search and tightness coincided with the socially optimal one when the government lets the market face $\psi_m = \psi_h = 1$, we evaluate also the private outcome of education under these conditions. This yields the following market outcome of the stock of educated workers:

$$(y_h - y_m) \frac{k\theta_l^I}{(1+z)\left(\frac{1}{2}\right)^{1-\gamma}} - c(\hat{e}) = 0. \tag{27}$$

[12] When $\psi_m = \psi_h = 1$ is imposed on the private solution, it follows from (14), (15) that tightness in the formal and the informal sector is equal, and that search must be split equally between the formal and the informal sector, i.e., $\sigma = \frac{1}{2}$ from (11). Imposing $\sigma = \frac{1}{2}$ and $\theta_l^F = \theta_l^I$, $l = m, h$, and assuming $r = 0$ in (14) and (15), yields the same expression as (25).

It immediately follows that a tax and punishment policy which implies that $\psi_m = \psi_h = 1$, will not provide incentives to the market to generate a socially optimal stock of educated workers. Comparing (26) and (27), in fact, reveals that the market outcome induces too few workers to educate themselves if formal and informal sector jobs face uniform treatment in terms of $\psi_m = \psi_h = 1$. This follows as taxes, captured by $(1 + z)$ in (27), hits highly educated workers more severely than manual workers, which reduces the incentives of education. From this we can conclude that welfare would increase if more workers chose to educate themselves when $\psi_m = \psi_h = 1$.

This discussion brings us to the government's explicit choice of tax and punishment policy. How should the government punish informal work in order to maximize welfare?

5.1 Optimal punishment policy

The welfare analysis above indicates that it may be optimal to punish tax evading activities carried out by manual workers more severely than those carried out by highly educated workers. For example, if concealment costs are higher for highly educated workers, a punishment policy with $\psi_m = \psi_h = 1$ is only possible if the manual workers to a larger extent than highly educated workers face punishment of informal activities. That is, $p\alpha$ have to be set relatively higher for manual workers if $\kappa_m < \kappa_h$ in order to induce $\psi_m = \psi_h = 1$.

This raises the question of whether it is possible or not to target the punishment fees and audit rates towards the sector employing manual vs highly educated workers. While governments potentially could, and in fact do,[13] target their audits to specific sectors, i.e. allowing for p_m to differ from p_h, this is less likely the case for the fee rates.

To find the socially optimal choice of audit rates for the sector employing manual workers and the sector employing highly educated workers, the welfare function in (21)-(23) is maximized by choice of p_m and p_h subject to the market reactions given by (11), (14), (15), (17) and (19) and the government budget restriction in (20). This yields the following first order conditions:

$$\frac{dW}{dp_m} = \hat{e}\frac{dW_m}{d\psi_m}\frac{d\psi_m}{dp_m} + \frac{dW}{d(1-e)}\frac{d(1-e)}{dp_m} = 0, \tag{28}$$

$$\frac{dW}{dp_h} = (1-\hat{e})\frac{dW_h}{d\psi_h}\frac{d\psi_h}{dp_h} + \frac{dW}{d(1-e)}\frac{d(1-e)}{dp_h} = 0, \tag{29}$$

where $\frac{dW_l}{d\psi_l} = \left[\sum_{j=F,I}\frac{dW_l}{d\theta_l^j}\frac{d\theta_l^j}{d\psi_l} + \frac{dW_l}{d\sigma_l^I}\frac{d\sigma_l^I}{d\psi_l}\right], j = m, h$. Evaluating the first order conditions at the levels of p_m and p_h ensuring that $\psi_m = \psi_h = 1$ turns out to be very convenient and gives:

$$\frac{dW}{dp_m} \mid_{\psi_m=1} = \frac{dW}{d(1-\hat{e})}\frac{d(1-\hat{e})}{dp_m} > 0, \tag{30}$$

$$\frac{dW}{dp_h} \mid_{\psi_h=1} = \frac{dW}{d(1-\hat{e})}\frac{d(1-\hat{e})}{dp_h} < 0. \tag{31}$$

Provided that we are located on the positively sloped side of the Laffer curves, we can conclude that:

Proposition 7 *Welfare is maximized when the sector employing manual workers are audited to a larger extent than the sector employing highly educated workers, i.e.,* $p_m > p_h$ *so to get* $\psi_h^* < 1 < \psi_m^*$ *if* $\kappa_h \geq \kappa_m$.

[13] See, for example, Kleven et al 2011.

Proof. Evaluate the first order conditions (28) and (29) at $\psi_m = \psi_h = 1$. From the socially optimal allocation of search and tightness, $\psi_l = 1$ implies that $\frac{dW_l}{d\theta_l^F} = \frac{dW_l}{d\theta_l^I} = \frac{dW_l}{d\sigma_l^I} = 0,\ l = m, h$. Then $\frac{dW}{dp_m} \mid_{\psi_m=1} = \frac{dW}{d(1-e)} \frac{d(1-e)}{dp_m} > 0$, and $\frac{dW}{dp_h} \mid_{\psi_h=1} = \frac{dW}{d(1-e)} \frac{d(1-e)}{dp_h} < 0$ as $\frac{dW}{d(1-e)} > 0$ from (26) and (27) and $\frac{d(1-\hat{e})}{dp_m} > 0$, $\frac{d(1-\hat{e})}{dp_h} < 0$ from (17). Thus welfare improves by reallocation of audits towards the manual sector. If $\kappa_h = \kappa_m$, $p_m = p_h$ at $\psi_m = \psi_h = 1$, welfare improves by setting $p_m > p_h$. If $\kappa_h > \kappa_m$, the results are reinforces as $p_m > p_h$ already when $\psi_m = \psi_h = 1$, and welfare improves by further increasing p_m and reducing p_h. ■

The result in proposition 7 follows straightforwardly from the first order conditions when evaluated at the p_m and p_h which induces $\psi_m = \psi_h = 1$. The first term on the right hand side of equations (28) and (29) then disappears as the distortions in search and allocation of tightness across the formal and the informal sectors are fully eliminated. In this case there are no other distortions present except for that too few workers have chosen to educate themselves. Recall that this is a consequence of that taxation harms high income earners relatively more. This distortion can, however, be corrected for by increasing the audits in the manual sector and reducing them in the sector for highly educated workers, which is captured by the right hand side in (30) and (31). As informal sector work for manual workers becomes less attractive when the government increases the number of audits, manual workers are encouraged to acquire higher education. Similarly, less audits in the highly educated sector further encourages workers to acquire higher education.

If concealment costs are higher in the sector employing highly educated workers, i.e., $\kappa_h > \kappa_m$, there are even further incentives for the government to focus their audits on the manual sector. This follows as high concealment costs work as a self-regulating punishment of informal sector activities.

Thus, if concealment costs are higher in the sector employing highly educated workers, this sector will be in less need of audits as concealment costs will do part of the job limiting the size of the informal sector.

Moreover it follows that:

Corollary 8 *The stock of educated workers is below its socially optimal value when the audit rates are chosen so to maximize welfare.*

Proof. See appendix. ■

When deciding on the optimal audit rates the government face a trade-off between two distortions, and it is never optimal to fully eliminate one of them. When the stock of educated workers is at its socially optimal level, there is an inefficient allocation of search and jobs across the formal and informal sectors. Welfare then improves as the stock of educated workers is reduced below its socially optimal level as this will only be a second order effect in comparison to the improved welfare following a more efficient sectorial allocation.

5.2 Optimal punishment policy when concealment costs are high

In deriving the optimal audit rates in the previous section, it was implicitly assumed that audit rates could be chosen freely without restrictions. For example, according to proposition 7, the audit rates should be chosen such that $p_m^* > p_h^*$ so to get $\psi_h^* < 1 < \psi_m^*$. However, this is only possible if concealment costs are not too high. If, for example, $\kappa_h > z$ then $\psi_h > 1$ even when p_h is very small. Replacing the first order condition in (29) with the appropriate Kuhn-Tucker conditions, $\frac{dW}{dp_h} + \mu = 0$, $p_h \geq 0$, and $\mu p_h = 0$, where μ is the Lagrange multiplier for the constraint $p_h \geq 0$, then suggest that the audit rate in the sector should be set as low as possible when $\kappa_h > z$. Concealment costs are simply high enough to self-regulate the size of the informal sector facing highly educated workers, and there is no need for additional audits of this sector.[14]

Taking off in real world observations from western economies, this may not be an unrealistic scenario. Evidence indicates that manual workers, or workers with a lower level of formal education, to a substantially larger degree face informal employment opportunities compared to highly educated workers. Pedersen and Smith (1998) using comprehensive survey data, find that almost half of the informal sector activities in Denmark are carried out within the construction sector. They also find that around 70 percent of the total hours performed in the informal sector is carried out within the service sector or construction sector.

Potential explanations for why manual, in contrast to highly educated, workers engage in informal activities are that manual workers to a larger extent work in industries which handles cash-payments or are to a lesser extent subject to third-party reporting. Firms and workers in industries dealing with cash-payments, or which to a lesser extent are subject to third-party reporting, will find it easier, and thus less costly, to conceal their tax evasion. Taking this at face value implies that concealment costs for highly educated workers, κ_h, could be very large. If κ_h is assumed to approach infinity, informal employment opportunities facing highly educated workers will become infinitely small, leading to that basically no firms will post informal sector vacancies to highly educated workers and none of the highly educated workers will allocate search effort into the informal sector. All the results derived

[14]This clearly holds also for the manual sector if concealment cost are higher than the tax rate.

in propositions 1 to 6 accounts for this special case, including the now clear cut result that higher punishment fees, or a general increase in the audit rate, encourage more workers to educate themselves. This follows as less workers will remain as manual workers as the forgone informal employment opportunities when attaining education has become less attractive. Moreover, the socially optimal audit rate is again being determined by an audit rate which implies that p_m^* is set large enough so to get $\psi_m^* > 1$, although not high enough to induce an efficient stock of educated workers.

6 Conclusion

There has recently been an intensified focus on issues related to tax evasion and informal activities from both a policy and research perspective.[15] The study by Kleven et al (2011), which conducted a large field experiment in Denmark, made it possible to address, and convincingly answer, a number of questions related to tax compliance behaviour, that had not been answered before.

This paper uses this knowledge to investigate the general equilibrium implications of informal sector activities on economic performance. A number of questions can be asked. How will informal employment opportunities affect labour market performance and educational attainments? Can informal jobs really be turned into formal jobs by more zealous punishment policies? And if so, to what extent will formal sector jobs replace jobs in the informal sector?

[15] The OECD recently initiated the "Global forum of transparency and exchange of information for tax purposes" (OECD, 2010), whereas the European commission conducted the first EU wide comparable questionnaire in order to increase the knowledge about tax evasion in Europe (see EC, 2007).

In order to address these questions, we develop a four sector equilibrium search and matching model with informal sector employment opportunities and educational choice. We find that informal activities reduces the incentives to acquire higher education if informal employment opportunities mainly are available to low educated workers. More zealous enforcement policies will in this case improve educational incentives as it reduces the attractiveness of remaining a low educated worker. Moreover, we find that stricter enforcement policies will create new jobs in the formal sector, although most likely to a lesser extent than the number of jobs destructed in the informal sector. This will lead to an increase in the actual unemployment rates although the official unemployment rates fall. Numerically it turns out the destroyed jobs in the informal sector following harsher punishment of informal sector work is to a very large extent replaced by newly created formal sector jobs, inducing only a very marginal reduction in the actually unemployment rate. Finally, characterizing the optimal enforcement policies, we find that relatively more audits should be targeted towards the sector employing low educated workers, elsewise a too low stock of educated workers could materialize.

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7 Appendix

For more details see an on-line Appendix.

7.1 Tightness relatively to search intensity

We show that $\frac{\theta_t^F}{\left(1-\sigma_t^I\right)^{1-\gamma}} < \frac{\theta_x^F}{\left(1-\sigma_x^I\right)^{1-\gamma}}$ when $\kappa_t > \kappa_x$ in the following way. Differentiating equations (14),(15) and (11) with respect to κ_l gives around the equilibrium:

$$\frac{d\theta_l^F}{d\kappa_l} = \frac{\frac{(1-\gamma)}{1-\sigma_l^I}\frac{k\theta_l^F}{\left(1-\sigma_l^I\right)^{1-\gamma}}\frac{1}{2}\left(1+\frac{k\theta^I}{\left(\sigma_l^I\right)^{1-\gamma}}\right)\frac{1}{\theta_l^I}}{D_l}(1+z)>0, \tag{32}$$

$$\frac{d\theta_l^I}{d\kappa_l} = -\frac{\frac{1}{2}\left(1+\frac{k\theta_l^F}{\left(1-\sigma_l^I\right)^{1-\gamma}}\right)\frac{1}{\theta_l^F}(1-\gamma)\,k\theta_l^I\left(\sigma_l^I\right)^{\gamma-2}}{D_l}(1+z)<0, \tag{33}$$

$$\frac{d\sigma_l^I}{d\kappa_l} = -\frac{\frac{1}{2}\left(1+\frac{k\theta_l^F}{\left(1-\sigma_l^I\right)^{1-\gamma}}\right)\frac{1}{\theta_l^F}\frac{1}{2}\left(1+\frac{k\theta_l^I}{\left(\sigma_l^I\right)^{1-\gamma}}\right)\frac{1}{\theta_l^I}}{D_l}(1+z)<0, \tag{34}$$

where

$$D_l=\frac{(1-\gamma)\frac{1}{\sigma_l^I}}{\theta_l^I\theta_l^F4\left(1-\sigma_l^I\right)}\left(\left(\left(\frac{\frac{1}{\psi_l}k\theta_l^I}{\left(\sigma_l^I\right)^{1-\gamma}}+1\right)\left(1-\frac{k\theta_l^I}{\left(\sigma_l^I\right)^{1-\gamma}}\right)\left(1-\sigma_l^I\right)+\left(\frac{k\theta_l^I}{\left(\sigma_l^I\right)^{1-\gamma}}+1\right)\sigma_l^I\left(1-\frac{\frac{1}{\psi_l}k\theta_l^I}{\left(\sigma_l^I\right)^{1-\gamma}}\right)\right)\right)>$$

0. Now, differentiating $\frac{\theta_l^F}{\left(1-\sigma_l^I\right)^{1-\gamma}}$ with respect to κ_l gives:

$$\begin{aligned}
\frac{d\frac{\theta_l^F}{\left(1-\sigma_l^I\right)^{1-\gamma}}}{d\kappa_l} &= \frac{d\left(1-\sigma_l^I\right)^{\gamma-1}\theta_l^F}{d\kappa_l}=\left(1-\sigma_l^I\right)^{\gamma-1}\theta_l^F\left((1-\gamma)\left(1-\sigma_l^I\right)^{-1}\frac{d\sigma_l^I}{d\kappa_l}+\frac{1}{\theta_l^F}\frac{d\theta_l^F}{d\kappa_l}\right)\\
&= \frac{\left(1-\sigma_l^I\right)^{\gamma-1}\theta_l^F}{2}\left(\begin{array}{c}-(1-\gamma)\left(1-\sigma_l^I\right)^{-1}\frac{\left(1+\frac{k\theta_l^F}{\left(1-\sigma_l^I\right)^{1-\gamma}}\right)\frac{1}{\theta_l^F}\frac{1}{2}\left(1+\frac{k\theta_l^I}{\left(\sigma_l^I\right)^{1-\gamma}}\right)\frac{1}{\theta_l^I}}{D_l}\\+\frac{\frac{(1-\gamma)}{1-\sigma_l^I}\frac{k\theta_l^F}{\left(1-\sigma_l^I\right)^{1-\gamma}}\left(1+\frac{k\theta_l^I}{\left(\sigma_l^I\right)^{1-\gamma}}\right)\frac{1}{\theta_l^I}}{\theta_l^F D_l}\end{array}\right)(1+z)\\
&= -\frac{\left(1-\sigma_l^I\right)^{\gamma-1}\theta_l^F}{4D_l}\frac{(1-\gamma)}{1-\sigma_l^I}\left(1+\frac{k\theta_l^I}{\left(\sigma_l^I\right)^{1-\gamma}}\right)\frac{1}{\theta_l^I}\frac{1}{\theta_l^F}\left(1-\frac{k\theta_l^F}{\left(1-\sigma_l^I\right)^{1-\gamma}}\right)(1+z)<0.
\end{aligned}$$

Hence, if $\kappa_t>\kappa_x$ then $\frac{\theta_t^F}{\left(1-\sigma_t^I\right)^{1-\gamma}}<\frac{\theta_x^F}{\left(1-\sigma_x^I\right)^{1-\gamma}}$.

7.2 Existence of $\hat{e}\in(0,1)$.

Consider the educational equation (17). For a non-trivial solution, there need to be a net gain in expected income of higher education. Thus, $y_h/y_m > o_m/o_h$. Moreover, to guarantee a non-trivial interior solution where at least some, but not all, individuals choose to acquire education, the individual with highest ability face a very low costs of education, more specifically $c(1) = 0$, and the individual with the lowest ability face very high cost of education, i.e., $\lim_{e\to 0} c(e) = \infty$.

In case $\kappa_h \leq \kappa_m$ then $o_m/o_h < 1$ and hence $y_h/y_m > o_m/o_h$ holds as $y_h > y_m$. If educated workers face higher concealment costs than manual workers $\kappa_h > \kappa_m$, then we need to assume that the productivity gain of education is large enough to assure that $y_h/y_m > o_m/o_h$ holds, which is possible as the right hand side is independent of y_l.

7.3 Relative unemployment rates (Proposition 1)

Unemployment is increasing in concealment costs if $\psi_l > 1$. Hence if $\kappa_t > \kappa_x$ then $u_t > u_x$ if $\psi_l > 1$. We show that in the following way, $u_t > u_x$ if and only if $s/\left(s + \lambda_x^F + \lambda_x^I\right) < s/\left(s + \lambda_t^F + \lambda_t^I\right)$ if an only if $\lambda_t^F + \lambda_t^I < \lambda_x^F + \lambda_x^I$. Hence, the condition holds if

$$\begin{aligned}
\frac{d\left(\lambda_l^F + \lambda_l^I\right)}{d\kappa_l} &= \frac{d\left[\left(1-\sigma_h^I\right)^\gamma \left(\theta_h^F\right)^{\frac{1}{2}} + \left(\sigma_h^I\right)^\gamma \left(\theta_h^I\right)^{\frac{1}{2}}\right]}{d\kappa_l} \\
&= \gamma\left(-\left(1-\sigma_l^I\right)^{\gamma-1}\left(\theta_l^F\right)^{\frac{1}{2}} + \left(\sigma_l^I\right)^{\gamma-1}\left(\theta_l^I\right)^{\frac{1}{2}}\right)\frac{d\sigma_l^I}{d\kappa_l} \\
&\quad + \frac{1}{2}\left(\frac{\left(1-\sigma_h^I\right)^\gamma}{\left(\theta_h^F\right)^{\frac{1}{2}}}\frac{d\theta_l^F}{d\kappa_l} + \frac{\left(\sigma_h^I\right)^\gamma}{\left(\theta_h^I\right)^{\frac{1}{2}}}\frac{d\theta_l^I}{d\kappa_l}\right) \\
&< 0
\end{aligned}$$

We substitute for the derivatives and the first order condition for search intensity to obtain the condition

$$\gamma\left(\frac{1}{\psi_l}\frac{1}{\left(\theta_l^F\right)^{\frac{1}{2}}}-\frac{1}{\left(\theta_l^I\right)^{\frac{1}{2}}}\right)\left(1+\frac{1}{\psi_l}\frac{k\theta_l^I}{\left(\sigma_l^I\right)^{1-\gamma}}\right)\left(1+\frac{k\theta^I}{\left(\sigma_l^I\right)^{1-\gamma}}\right)$$
$$+\left(1-\gamma\right)\frac{k\theta_l^I}{\left(\sigma_l^I\right)^{1-\gamma}}\left(\frac{1}{\left(\theta_l^F\right)^{\frac{1}{2}}}\frac{1}{\psi_l}\left(\frac{1}{\psi_l}+\frac{1}{\psi_l}\frac{k\theta^I}{\left(\sigma_l^I\right)^{1-\gamma}}\right)-\frac{1}{\left(\theta_h^I\right)^{\frac{1}{2}}}\left(1+\frac{1}{\psi_l}\frac{k\theta_l^I}{\left(\sigma_l^I\right)^{1-\gamma}}\right)\right)$$

which is negative when $\psi_l > 1$, as then $\theta_l^F > \theta_l^I$ giving $\frac{1}{\psi_l}\frac{1}{\left(\theta_l^F\right)^{\frac{1}{2}}} < \frac{1}{\left(\theta_l^I\right)^{\frac{1}{2}}}$ and $\left(\frac{1}{\psi_l}+\frac{1}{\psi_l}\frac{k\theta^I}{\left(\sigma_l^I\right)^{1-\gamma}}\right) < \left(1+\frac{1}{\psi_l}\frac{k\theta_l^I}{\left(\sigma_l^I\right)^{1-\gamma}}\right)$. Hence unemployment increases with ψ_l and hence $u_t > u_x$ when $\kappa_t > \kappa_x$.

The official unemployment rate facing t workers is higher than the official unemployment rate facing x workers, $u_t^o > u_x^o$ if and only if $\left(s+\lambda_t^I\right)/\left(s+\lambda_t^F+\lambda_t^I\right) > \left(s+\lambda_x^I\right)/\left(s+\lambda_x^F+\lambda_x^I\right)$. This holds if an only if $\lambda_x^F\left(s+\lambda_t^I\right) > \lambda_t^F\left(s+\lambda_x^I\right)$,which is true when $\lambda_x^F > \lambda_t^F$ and $\lambda_x^I > \lambda_t^I$, that is when $\kappa_t > \kappa_x$.

7.4 Impact of higher punishment on sector allocation (Proposition 2)

Raising the audit rate p_l or the punisment fee α, increases the wedge, $\psi_l = \left(1+p\alpha+\kappa_l\right)/\left(1+z\right)$. Differentiating equations (14),(15) and (11) with respect to ψ_l gives around the equilibrium:

$$\frac{d\theta_l^F}{d\psi_l} = \frac{\frac{(1-\gamma)}{1-\sigma_l^I}\frac{k\theta_l^F}{\left(1-\sigma_l^I\right)^{1-\gamma}}\frac{1}{2}\left(1+\frac{k\theta^I}{\left(\sigma_l^I\right)^{1-\gamma}}\right)\frac{1}{\theta_l^I}}{D_l}\frac{1}{\psi_l} > 0, l=m,h,$$
$$\frac{d\theta_l^I}{d\psi_l} = -\frac{\frac{1}{2}\left(1+\frac{k\theta_l^F}{\left(1-\sigma_l^I\right)^{1-\gamma}}\right)\frac{1}{\theta_l^F}\left(1-\gamma\right)k\theta_l^I\left(\sigma_l^I\right)^{\gamma-2}}{D_l}\frac{1}{\psi_l} < 0, l=m,h,$$
$$\frac{d\sigma_l^I}{d\psi_l} = -\frac{\frac{1}{2}\left(1+\frac{k\theta_l^F}{\left(1-\sigma_l^I\right)^{1-\gamma}}\right)\frac{1}{\theta_l^F}\frac{1}{2}\left(1+\frac{k\theta^I}{\left(\sigma_l^I\right)^{1-\gamma}}\right)\frac{1}{\theta_l^I}}{D_l}\frac{1}{\psi_l} < 0, l=m,h,$$

$$D_l = \frac{(1-\gamma)}{\theta_l^I \theta_l^F 4\left(1-\sigma_l^I\right)\sigma_l^I}\left(\left(\frac{1}{\psi_l}\frac{k\theta_l^I}{\left(\sigma_l^I\right)^{1-\gamma}}+1\right)\left(1-\frac{k\theta_l^I}{\left(\sigma_l^I\right)^{1-\gamma}}\right)(1-\sigma_l)+\left(\frac{k\theta_l^I}{\left(\sigma_l^I\right)^{1-\gamma}}+1\right)\sigma_l\left(1-\frac{1}{\psi_l}\frac{k\theta_l^I}{\left(\sigma_l^I\right)^{1-\gamma}}\right)\right)$$

0. Hence, as $\lambda_{li}^I = \left(\sigma_{li}^I\right)\gamma\left(\theta_l^I\right)^{\frac{1}{2}}$ and $\lambda_{li}^F = \left(1-\sigma_{li}^I\right)^{\gamma}\left(\theta_l^F\right)^{\frac{1}{2}}$, by inspection of equation (18) it follows that $dn_l^F/d\psi_l > 0, dn_l^I/d\psi_l < 0, l = m, h$. The impact on wages is then

$$\begin{aligned} \frac{d\omega_l^F}{d\psi_l} &= \frac{1}{2}y_l k \frac{d\frac{\theta_l^F}{\left(1-\sigma_l^I\right)^{1-\gamma}}}{d\psi_l} < 0, l = m, h, \\ \frac{d\omega_l^I}{d\psi_l} &= \frac{1}{2}y_l k \frac{d\frac{\theta_l^I}{\left(\sigma_l^I\right)^{1-\gamma}}}{d\psi_l} > 0, l = m, h. \end{aligned}$$

as

$$\begin{aligned} \frac{d\frac{\theta_l^F}{\left(1-\sigma_l^I\right)^{1-\gamma}}}{d\psi_l} &= \frac{d\left(\left(1-\sigma_l^I\right)^{\gamma-1}\theta_l^F\right)}{d\psi_l} = \left(1-\sigma_l^I\right)^{\gamma-1}\theta_l^F\left((1-\gamma)\left(1-\sigma_l^I\right)^{-1}\frac{d\sigma_l^I}{d\psi_l}+\frac{1}{\theta_l^F}\frac{d\theta_l^F}{d\psi_l}\right) \\ &= -\frac{\left(1-\sigma_l^I\right)^{\gamma-1}}{4D_l}\frac{(1-\gamma)}{1-\sigma_l^I}\left(1+\frac{k\theta_l^I}{\left(\sigma_l^I\right)^{1-\gamma}}\right)\frac{1}{\theta_l^I}\left(1-\frac{k\theta_l^F}{\left(1-\sigma_l^I\right)^{1-\gamma}}\right)\frac{1}{\psi_l} < 0. \quad (35) \end{aligned}$$

and

$$\begin{aligned} \frac{d\frac{\theta_l^I}{\left(\sigma_l^I\right)^{1-\gamma}}}{d\psi_l} &= \frac{d\left(\left(\sigma_l^I\right)^{1-\gamma}\theta_l^I\right)}{d\psi_l} = \left(\sigma_l^I\right)^{\gamma-1}\theta_l^I\left(-(1-\gamma)\left(\sigma_l^I\right)^{-1}\frac{d\sigma_l^I}{d\psi_l}+\frac{1}{\theta_l^I}\frac{d\theta_l^I}{d\psi_l}\right) \\ &= \frac{\left(\sigma_l^I\right)^{\gamma-1}}{\psi_l 4D_l}\frac{(1-\gamma)}{\sigma_l^I}\left(1+\frac{k\theta_l^F}{\left(1-\sigma_l^I\right)^{1-\gamma}}\right)\frac{1}{\theta_l^F}\left(1-\frac{k\theta^I}{\left(\sigma_l^I\right)^{1-\gamma}}\right) > 0 \end{aligned}$$

7.5 Impact of higher punishment on unemployment rates (Proposition 3)

Raising the audit rate p or the punisment fee α, increases the wedge, $\psi_l = (1 + p\alpha + \kappa_l)/(1+z)$. Differentiating equation (19) with respect to ψ_l gives:

$$\frac{du_l}{d\psi_l} = -\frac{s}{\left(s+\lambda_l^I+\lambda_l^F\right)^2}\left(\frac{d\lambda_l^F}{d\psi_l}+\frac{d\lambda_l^I}{d\psi_l}\right)$$

where

$$\begin{aligned}\frac{d\left(\lambda_l^F+\lambda_l^I\right)}{d\psi_l} &= \frac{\left(1-\sigma_h^I\right)^\gamma\left(\theta_h^F\right)^{\frac{1}{2}}+\left(\sigma_h^I\right)^\gamma\left(\theta_h^I\right)^{\frac{1}{2}}}{d\psi_l}= \\ &= \gamma\left(-\left(1-\sigma_l^I\right)^{\gamma-1}\left(\theta_l^F\right)^{\frac{1}{2}}+\left(\sigma_l^I\right)^{\gamma-1}\left(\theta_l^I\right)^{\frac{1}{2}}\right)\frac{d\sigma_l^I}{d\psi_l} \\ &\quad +\frac{1}{2}\left(\frac{\left(1-\sigma_h^I\right)^\gamma}{\left(\theta_h^F\right)^{\frac{1}{2}}}\frac{d\theta_l^F}{d\psi_l}+\frac{\left(\sigma_h^I\right)^\gamma}{\left(\theta_h^I\right)^{\frac{1}{2}}}\frac{d\theta_l^I}{d\psi_l}\right)\end{aligned}$$

Substitute for the derivatives and the first order condition for search intensity we obtain that $du_l/d\psi_l$ has the same sign as

$$\begin{aligned}&-\gamma\left(\frac{1}{\psi_l}\frac{1}{\left(\theta_l^F\right)^{\frac{1}{2}}}-\frac{1}{\left(\theta_l^I\right)^{\frac{1}{2}}}\right)\left(1+\frac{1}{\psi_l}\frac{k\theta_l^I}{\left(\sigma_l^I\right)^{1-\gamma}}\right)\left(1+\frac{k\theta^I}{\left(\sigma_l^I\right)^{1-\gamma}}\right) \\ &-(1-\gamma)\frac{k\theta_l^I}{\left(\sigma_l^I\right)^{1-\gamma}}\left(\frac{1}{\left(\theta_l^F\right)^{\frac{1}{2}}}\frac{1}{\psi_l}\left(\frac{1}{\psi_l}+\frac{1}{\psi_l}\frac{k\theta^I}{\left(\sigma_l^I\right)^{1-\gamma}}\right)-\frac{1}{\left(\theta_h^I\right)^{\frac{1}{2}}}\left(1+\frac{1}{\psi_l}\frac{k\theta_l^I}{\left(\sigma_l^I\right)^{1-\gamma}}\right)\right).\end{aligned}$$

Hence

$$\frac{du_l}{d\psi_l}\lesseqgtr 0 \text{ if and only if } \psi_l\lesseqgtr 1.$$

The impact on the official unemployment rate resulting from an increase in the audit rate or the punishment fee, corresponds to

$$\frac{du_l^o}{d\psi_l}=\frac{\left(s+\lambda_l^I+\lambda_l^F\right)\frac{d\lambda_l^I}{d\psi_l}-\left(s+\lambda_l^I\right)\left(\frac{d\lambda_l^F}{d\psi_l}+\frac{d\lambda_l^I}{d\psi_l}\right)}{\left(s+\lambda_l^I+\lambda_l^F\right)^2}=\frac{\lambda_l^F\frac{d\lambda_l^I}{d\psi_l}-\left(s+\lambda_l^I\right)\left(\frac{d\lambda_l^F}{d\psi_l}\right)}{\left(s+\lambda_l^I+\lambda_l^F\right)^2}<0\,\forall\psi_l, l=m,h.$$

7.6 Impact of higher punishment on education (Proposition 4 and 5)

A closer examination of (17) reveals that changes in the audit rates or punishment rates affect the share of educated workers, $1-\hat{e}$, through ψ_l only, whereas changes in the tax rate, z, have a direct effect on $1-\hat{e}$ in addition to the effects working through ψ_l. Therefore, in order to consider the effects of a fully financed change in the punishment rates on the number of educated workers, we have to account for repercussions on $1-\hat{e}$ following adjustments in the tax rate. However, let us first consider the impact on $1-\hat{e}$ of a change in the tax and expected punishment separately:

$$\frac{\partial(1-\hat{e})}{\partial(p\alpha)}|_z = -\frac{k}{c'(e)(1+z)}\left(y_h\frac{d\frac{\theta_h^F}{\left(1-\sigma_h^I\right)^{1-\gamma}}}{d(p\alpha)} - y_m\frac{d\frac{\theta_m^F}{(1-\sigma_m^I)^{1-\gamma}}}{d(p\alpha)}\right)$$

$$\frac{\partial(1-\hat{e})}{\partial z}|_{p_l\alpha} = \psi_l\frac{\partial(1-\hat{e})}{\partial(p\alpha)}|_z - \frac{c(\hat{e})}{c'(\hat{e})(1+z)}$$

Using equation (35) we obtain

$$\frac{d\frac{\theta_l^F}{\left(1-\sigma_l^I\right)^{1-\gamma}}}{d(p\alpha)} = -\frac{\frac{1}{\psi_l}\frac{k\theta_l^I}{\left(\sigma_l^I\right)^{1-\gamma}}}{\frac{\left(\frac{1}{\psi_l}\frac{k\theta_l^I}{\left(\sigma_l^I\right)^{1-\gamma}}+1\right)\left(1-\frac{k\theta_l^I}{\left(\sigma_l^I\right)^{1-\gamma}}\right)}{\left(1+\frac{k\theta_l^I}{\left(\sigma_l^I\right)^{1-\gamma}}\right)\left(1-\frac{1}{\psi_l}\frac{k\theta_l^I}{\left(\sigma_l^I\right)^{1-\gamma}}\right)}\frac{1-\sigma_l^I}{\sigma_l^I}+1}\frac{1}{\psi_l}\frac{1}{1+z}, l = h, m,$$

whereby the educational impacts become

$$\frac{\partial(1-\hat{e})}{\partial(p\alpha)}|_z = -\frac{1}{c'(e)(1+z)^2}\left(y_h\frac{do_h}{d\psi_h} - y_m\frac{do_m}{d\psi_m}\right),$$

$$\frac{\partial(1-\hat{e})}{\partial z}|_{p_l\alpha} = -\psi_l\frac{\partial(1-\hat{e})}{\partial(p\alpha)}|_z + \frac{c(\hat{e})}{c'(\hat{e})(1+z)},$$

where $o_l = \frac{1}{\psi_l}\frac{k\theta_l^I}{\left(\sigma_l^I\right)^{1-\gamma}} = \frac{k\theta_l^F}{\left(1-\sigma_l^I\right)^{1-\gamma}}, l = h, m$ and

$$\frac{do_l}{d\psi_l} = -\frac{\frac{1}{\psi_l}o_l}{\frac{(o_l+1)(1-\psi_l o_l)}{(\psi_l o_l+1)(1-o_l)}\frac{\left(1-\sigma_l^I\right)}{\sigma_l^I}+1} < 0, l = h, m \tag{36}$$

For existence of an interior solution for education we need $y_h o_h - y_m o_m > 0$. Hence, education increases if $y_h\frac{do_h}{d\psi_h} - y_m\frac{do_m}{d\psi_m} > 0$. As $\frac{do_l}{d\psi_l}, l = h, m$ is negative, and $y_h/y_m > o_{m/}o_h$, then for existence of an interior solution for $\hat{e}$, if

$$\left|\frac{do_m}{d\psi_m}\right| / \left|\frac{do_h}{d\psi_h}\right| > y_h/y_m > o_{m/}o_h \tag{37}$$

then education increases with $p\alpha$. Consider the case where $\kappa_h > \kappa_m$. As ψ_l increases with κ_l, then for such a solution to exist, we need that $\left|\frac{do_l}{d\psi_l}\right|, l = h, m$ is decreasing in concealment costs whereby $\left|\frac{do_m}{d\psi_m}\right| > \left|\frac{do_h}{d\psi_h}\right|$. We first show that that is the case. Multiply the numerator and denominator by ψ_l to obtain

$$\left|\frac{do_l}{d\psi_l}\right| = \frac{o_l}{\frac{(1+o_l)(1-\psi_l o_l)}{\left(\frac{1}{\psi_l}+o_l\right)(1-o_m)}\frac{1-\sigma_l^I}{\sigma_l^I} + \psi_l}.$$

Substituting for the tightness equations, $1-\frac{k\theta_l^F}{\left(1-\sigma_l^I\right)^{1-\gamma}} = 1-o_l = k\left(r+s\right)\left(\theta_l^F\right)^{\frac{1}{2}}2$ and $1-\frac{k\theta_l^I}{\left(\sigma_l^I\right)^{1-\gamma}} = 1-\psi_l o_l = k\left(r+s\right)\left(\theta_l^I\right)^{\frac{1}{2}}2$ and use the fact that $\frac{1-\sigma_l^I}{\sigma_l^I} = \left(\frac{\theta_l^F}{\theta_l^I}\right)^{\frac{1}{1-\gamma}}\psi_l^{\frac{1}{1-\gamma}}$ according to the search equation to obtain

$$\left|\frac{do_l}{d\psi_l}\right| = \frac{o_l}{A_l\left(\frac{\theta_l^F}{\theta_l^I}\right)^{\frac{1}{1-\gamma}-\frac{1}{2}}\psi_l^{\frac{1}{1-\gamma}} + \psi_l},$$

where $A_l = \frac{(1+o_l)}{\left(\frac{1}{\psi_l}+o_l\right)}$. Differentiating this expression with respect to ψ_l we obtain

$$\frac{d\left|\frac{do_l}{d\psi_l}\right|}{d\psi_l} = \frac{\frac{do_l}{d\psi}\left(A_l\left(\frac{\theta_l^F}{\theta_l^I}\right)^{\frac{1}{1-\gamma}-\frac{1}{2}}\psi_l^{\frac{1}{1-\gamma}} + \psi_l\right) - o_l\psi_l^{\frac{1}{1-\gamma}}\left(\frac{\theta_l^F}{\theta_l^I}^{\frac{1}{1-\gamma}-\frac{1}{2}}\left(\frac{dA_l}{d\psi_l} + \frac{A_l}{\psi_l}\left(\frac{1}{1-\gamma}\right)\right) + A_l\frac{d\frac{\theta_l^F}{\theta_l^I}^{\frac{1}{1-\gamma}-\frac{1}{2}}}{d\psi} + 1\right)}{\left(A_l\left(\frac{\theta_l^F}{\theta_l^I}\right)^{\frac{1}{1-\gamma}-\frac{1}{2}}\psi_l^{\frac{1}{1-\gamma}} + \psi_l\right)^2}$$

as substituting for $\frac{do_l}{d\psi_l}$ using the expression from equation (36) gives

$$\frac{dA_l}{d\psi_l} = \frac{\frac{do_l}{d\psi_l}\left(\frac{1}{\psi}-1\right)+\frac{1+o_l}{\psi^2}}{\left(\frac{1}{\psi_l}+o_l\right)^2} = \frac{\frac{1}{\psi_l}o_l+\frac{1+o_l}{\psi^2}\frac{(o_l+1)(1-\psi_l o_l)}{(\psi_l o_l+1)(1-o_l)}\frac{\left(1-\sigma_l^I\right)}{\sigma_l^I}+\frac{1}{\psi^2}}{\left(\frac{1}{\psi_l}+o_l\right)^2\left(\frac{(o_l+1)(1-\psi_l o_l)}{(\psi_l o_l+1)(1-o_l)}\frac{\left(1-\sigma_l^I\right)}{\sigma_l^I}+1\right)} > 0,$$

and from the equilibrium equations we have $d\left(\theta_h^F/\theta_h^I\right)/d\psi_l > 0$ and $do_l/d\psi_l < 0$.

Hence as $\frac{d\left|\frac{do_l}{d\psi_l}\right|}{d\psi_l} < 0$ then $\frac{d\left|\frac{do_l}{d\psi_l}\right|}{d\kappa_l} < 0$ so when $\kappa_h > \kappa_m$ then $\left|\frac{do_m}{d\psi_m}\right| > \left|\frac{do_h}{d\psi_h}\right|$. We observe $\frac{d\left|\frac{do_l}{d\psi_l}\right|}{d\psi_l} < 0$ both because the numerator decreases with ψ_l and the denominator increases with ψ_l. Rewriting the expression determining the sign of $\frac{\partial(1-\hat{e})}{\partial(p\alpha)}|_z$, equation (37) as

$$\frac{\frac{o_m}{A_m\left(\frac{\theta_m^F}{\theta_m^I}\right)^{\frac{1}{1-\gamma}-\frac{1}{2}}\psi_m^{\frac{1}{1-\gamma}}+\psi_m}}{\frac{o_h}{A_h\left(\frac{\theta_h^F}{\theta_h^I}\right)^{\frac{1}{1-\gamma}-\frac{1}{2}}\psi_h^{\frac{1}{1-\gamma}}+\psi_h}} = g\left(\kappa_h,\kappa_m\right)o_m/o_h > y_h/y_m > o_m/o_h,$$

where

$$g\left(\kappa_h,\kappa_m\right) \equiv \frac{D_{\frac{do_h}{d\psi_h}}}{D_{\frac{do_m}{d\psi_m}}} = \frac{A_h\left(\frac{\theta_h^F}{\theta_h^I}\right)^{\frac{1}{1-\gamma}-\frac{1}{2}}\psi_h^{\frac{1}{1-\gamma}}+\psi_h}{A_m\left(\frac{\theta_m^F}{\theta_m^I}\right)^{\frac{1}{1-\gamma}-\frac{1}{2}}\psi_m^{\frac{1}{1-\gamma}}+\psi_m} > 1,$$

when $\kappa_h > \kappa_m$ as the denominator of $\left|\frac{do_l}{d\psi_l}\right|$ increases with ψ_l. We conclude that if $\frac{y_h}{y_m} \in \left[\frac{o_m}{o_h}, g\left(\kappa_h,\kappa_m\right)\frac{o_m}{o_h}\right]$ education increases with $p\alpha$ and when $\frac{y_h}{y_m} \in \left[g\left(\kappa_h,\kappa_m\right)\frac{o_m}{o_h}, \infty\right]$ education falls with $p\alpha$.

7.7 Impact of higher punishment on unemployment (Proposition 6)

Raising the audit rate p or the punisment fee α, increases the wedge, $\psi_l = (1+p_l\alpha+\kappa_l)/(1+z)$. Differentiating total unemployment with respect to ψ_l gives

$$\frac{dU_{TOT}}{d\psi_l} = \frac{d\hat{e}}{d\psi_l}(u_m - u_h) + \hat{e}\frac{du_m}{d\psi_l} + (1-\hat{e})\frac{du_h}{d\psi_l}.$$

The last two terms are positive (≤ 0) when ψ_l is larger than one (≤ 1). The first term is positive if $\frac{y_h}{y_m} \in \left[\frac{o_m}{o_h}, g(\kappa_h, \kappa_m)\frac{o_m}{o_h}\right]$ where $g(\kappa_h, \kappa_m) > 1$ if $\kappa_h > \kappa_m$, and $\psi_l > (=)1$. as then $(u_m - u_h) < (=)0$ and $\frac{d\hat{e}}{d\psi_l} < 0$. However, when $\psi_l < 1$ and $\kappa_h > \kappa_m$ then $(u_m - u_h) > 0$ and in case $\frac{d\hat{e}}{d\psi_l} < 0$ then unemployment falls, $\frac{dU_{TOT}}{d\psi_l} < 0$. If, $\frac{y_h}{y_m} \in \left[\frac{o_m}{o_h}, g(\kappa_h, \kappa_m)\frac{o_m}{o_h}\right]$ then $\frac{d\hat{e}}{d\psi_l} > 0$ and $\frac{dU_{TOT}}{d\psi_l}$ has an ambiguous sign.

Total official unemployment changes according too

$$\frac{dU^o_{TOT}}{d\psi_l} = \frac{d\hat{e}}{d\psi_l}(u^o_m - u^o_h) + \hat{e}\frac{du^o_m}{d\psi_l} + (1-\hat{e})\frac{du^o_h}{d\psi_l} < 0,$$

where the last two terms are negative and therefore when $\kappa_h > \kappa_m$ we obtain $\frac{dU^o_{TOT}}{d\psi_l} < 0$ when $\frac{d\hat{e}}{d\psi_l} \leq 0$ as $(u^o_m - u^o_h) > 0$. When $\frac{d\hat{e}}{d\psi_l} > 0$ the sign of $\frac{dU^o_{TOT}}{d\psi_l}$ is ambiguous.

7.8 Socially optimal solution for $\theta^F_m, \theta^I_m, \theta^F_h, \theta^I_h, \sigma^I_m, \sigma^I_h, \hat{e}$.

From the first order conditions for tightness in the formal and informal sector for manual and highly educated workers, i.e.,$\frac{\partial W}{\partial \theta^I_l} = 0$, $\frac{\partial W}{\partial \theta^F_l} = 0, l = m, h$, we get the following conditions: $2sk\left(\theta^I_l\right)^{\frac{1}{2}} = u_l\left[1 + k\Theta_l\right]$ and $2sk\left(\theta^F_l\right)^{\frac{1}{2}} = u_l\left[1 + k\Theta_l\right]$, $l = m, h$, which gives $\theta^F_l = \theta^I_l$. Substitute $\theta^F_l = \theta^I_l$ into the first order condition for search effort, $\frac{\partial W}{\partial \sigma^I_l} = 0$, and the following condition determins the social optimal level of search: $\left(\sigma^I_m\right)^{\gamma-1} - \left(1 - \sigma^I_m\right)^{\gamma-1} = 0$. This yields $\sigma^I_l = \frac{1}{2}, l = m, h$. Substitute $\sigma^I_l = \frac{1}{2}, l = m, h$ into $2sk\left(\theta^I_l\right)^{\frac{1}{2}} = u_l\left[1 + k\Theta_l\right]$ and $2sk\left(\theta^F_l\right)^{\frac{1}{2}} = u_l\left[1 + k\Theta_l\right]$, $l = m, h$, which yields the four equations in (25) determining $\theta^F_m, \theta^I_m, \theta^F_h$, and θ^I_h in equilibrium. The socially optimal educational stock is determined by: $\frac{\partial W}{\partial(1-\hat{e})} = W_h(\hat{e}) - W_m =$

$y_h\left[1-u_h\left[1+k\Theta_h\right]\right]-y_m\left[1-u_m y_m\left[1+k\Theta_m\right]\right]-c\left(\hat{e}\right)=0$. Now use the equations determining the optimal levels of tightness, $2sk\left(\theta_l^I\right)^{\frac{1}{2}}=u_l\left[1+k\Theta_l\right]$ and $2sk\left(\theta_l^F\right)^{\frac{1}{2}}=u_l\left[1+k\Theta_l\right]$, $l=m,h,$, and the equation for the optimal educational level given by (26). To show that we have a global maximum we differentiate W with respect to $\sigma_l^I,\theta_l^I,\theta_l^F,l=m,h$ and $1-\hat{e}$ to obtain

$$\begin{aligned}
\left(\sigma_l^{I*}\right)^{\gamma-1}-\left(1-\sigma_l^{I*}\right)^{\gamma-1} &= 0, l=m,h,\\
-sk\left(\theta_l^{*I}\right)^{\frac{1}{2}}+\frac{1}{2}\left[1-\frac{k\theta_l^{*I}}{\left(\sigma_l^I\right)^{1-\gamma}}\right] &= 0, l=m,h,\\
-sk\left(\theta_l^{*F}\right)^{\frac{1}{2}}+\frac{1}{2}\left[1-\frac{k\theta_l^{*F}}{\left(1-\sigma_l^I\right)^{1-\gamma}}\right] &= 0, l=m,h,\\
\left(y_h\frac{k\theta_h^{*I}}{\left(\sigma_h^I\right)^{1-\gamma}}-y_m\frac{k\theta_m^{*I}}{\left(\sigma_m^I\right)^{1-\gamma}}\right)-c\left(\hat{e}^*\right) &= 0.
\end{aligned}$$

The associated Hessian matrix is then

$$\begin{vmatrix}
\left(\gamma-1\right)S_m & 0 & 0 & 0 & 0 & 0 & 0\\
-\frac{k\theta_m^I}{2\left(\sigma_m^I\right)^{2-\gamma}}\left(\gamma-1\right) & \Delta_m^I & 0 & 0 & 0 & 0 & 0\\
\frac{k\theta_m}{2\left(1-\sigma_m^I\right)^{2-\gamma}}\left(\gamma-1\right) & 0 & \Delta_m^F & 0 & 0 & 0 & 0\\
0 & 0 & 0 & \left(\gamma-1\right)S_h & 0 & 0 & 0\\
0 & 0 & 0 & -\frac{k\theta_h^I}{2\left(\sigma_h^I\right)^{2-\gamma}}\left(\gamma-1\right) & \Delta_h^I & 0 & 0\\
0 & 0 & 0 & \frac{k\theta_h}{2\left(1-\sigma_h^I\right)^{2-\gamma}}\left(\gamma-1\right) & 0 & \Delta_h^F & 0\\
-y_m\left(\gamma-1\right)\frac{k\theta_m^{*I}}{\left(\sigma_m^I\right)^{2-\gamma}} & -y_m k\left(\sigma_m^I\right)^{\gamma-1} & 0 & \left(\gamma-1\right)y_h\frac{k\theta_h^{*I}}{\left(\sigma_h^I\right)^{2-\gamma}} & y_h k\left(\sigma_h^I\right)^{\gamma-1} & 0 & c'\left(\hat{e}^*\right)
\end{vmatrix}$$

where $S_l=\left(\left(\sigma_l^I\right)^{\gamma-2}+\left(1-\sigma_l^I\right)^{\gamma-2}\right), l=m,h, \Delta_l^I=-\frac{1}{2}\left(sk\left(\theta_l^I\right)^{-\frac{1}{2}}+k\left(\sigma_l^I\right)^{\gamma-1}\right), l=m,h$ and $\Delta_l^F=-\frac{1}{2}\left(sk\left(\theta_l^F\right)^{-\frac{1}{2}}+k\left(1-\sigma_l^I\right)^{\gamma-1}\right), l=m,h$. Therefore, $H_1=\left(\gamma-1\right)\left(\left(\sigma_m^I\right)^{\gamma-2}+\left(1-\sigma_m^I\right)^{\gamma-2}\right)<0$ and the principal minors alternate in sign, for all variable values, i.e., $H_2=-\left(\gamma-1\right)\left(\left(\sigma_m^I\right)^{\gamma-2}+\left(1-\sigma_m^I\right)^{\gamma-2}\right)\Delta_m^I>0,...,H_7=\left(\gamma-1\right)\left(\left(\sigma_m^I\right)^{\gamma-2}+\left(1-\sigma_m^I\right)^{\gamma-2}\right)\Delta_m^I\Delta_m^F\left(\gamma-1\right)\left(\left(\sigma_h^I\right)^{\gamma-2}+\left(1-\sigma_h^I\right)^{\gamma-2}\right)\Delta_h^I\Delta_h^F c'\left(\hat{e}^*\right)$ 0, by which we have a global maximum.

7.9 Optimal not to induce the socially efficient stock of education. (Corollary 8)

Evaluating (28) and (29) at p_m^e and p_h^e such that the socially optimal level of education is reached, i.e., $\frac{dW}{d(1-e)} = 0$. From 7 this requires that $\psi_m^e > 1 > \psi_h^e$. This yields: $\frac{dW}{dp_m} \mid_{\psi_m^e > 1} = \hat{e} \left[\frac{dW}{d\theta_m^F} \frac{d\theta_m^F}{d\psi_m} + \frac{dW}{d\theta_m^I} \frac{d\theta_m^I}{d\psi_m} + \frac{dW}{d\sigma_m^I} \frac{d\sigma_m^I}{d\psi_m} \right] \frac{d\psi_m}{dp_m}$ and $\frac{dW}{dp_h} \mid_{\psi_h^e < 1} = (1-\hat{e}) \left[\frac{dW}{d\theta_h^F} \frac{d\theta_h^F}{d\psi_h} + \frac{dW}{d\theta_h^I} \frac{d\theta_h^I}{d\psi_h} + \frac{dW}{d\sigma_h^I} \frac{d\sigma_h^I}{d\psi_h} \right] \frac{d\psi_h}{dp_h}$. From the derivations of the socially optimal solution for $\theta_m^F, \theta_m^I, \theta_h^F, \theta_h^I, \sigma_m^I, \sigma_h^I$ it follows that $\frac{dW}{d\theta_l^F} \mid_{\psi_l > 1} < 0$, $\frac{dW}{d\theta_l^I} \mid_{\psi_l > 1} > 0$, $\frac{dW}{d\sigma_l^I} \mid_{\psi_l > 1} > 0$ and $\frac{dW}{d\theta_l^F} \mid_{\psi_l < 1} > 0$, $\frac{dW}{d\theta_l^I} \mid_{\psi_l < 1} < 0$, $\frac{dW_l}{d\sigma_l^I} \mid_{\psi_l < 1} < 0$ as the welfare function is maximized at $\psi_l = 1$, i.e., $\frac{dW}{d\theta_l^F} \mid_{\psi_l = 1} = \frac{dW_l}{d\theta_l^I} \mid_{\psi_l = 1} = \frac{dW_l}{d\sigma l} \mid_{\psi_l = 1} = 0$. It then follows that $\frac{dW}{dp_m} \mid_{\psi_m^e > 1} < 0$ and $\frac{dW}{dp_h} \mid_{\psi_h^e < 1} > 0$.

7.10 Optimal punishment policy including auditing costs.

The government budget constraint with auditing costs, $\varphi(p)$, is $\hat{e} \sum_{j=F,I} n_m^j w_m^j z + (1-\hat{e}) \sum_{j=F,I} n_h^j w_h^j p\alpha - \varphi(p) = R$, where p is the total intensity of audits, $p = p_m + p_h$. Adding costs of auditing has no impact on the positive analyses. The welfare function, however, is equal to: $W = \hat{e} W_m + \int_{\hat{e}}^1 W_h de - \varphi(p)$, with first order conditions for optimal audit rates:

$$\frac{dW}{dp_m} = \hat{e} \frac{dW_m}{d\psi_m} \frac{d\psi_m}{dp_m} + \frac{dW}{d(1-e)} \frac{d(1-e)}{dp_m} - \varphi'(p) = 0, \tag{38}$$

$$\frac{dW}{dp_h} = (1-\hat{e}) \frac{dW_h}{d\psi_h} \frac{d\psi_h}{dp_h} + \frac{dW}{d(1-e)} \frac{d(1-e)}{dp_h} - \varphi'(p) = 0, \tag{39}$$

where, $\frac{dW_l}{d\psi_l} = \left[\sum_{j=F,I} \frac{dW_l}{d\theta_l^j} \frac{d\theta_l^j}{d\psi_l} + \frac{dW_l}{d\sigma_l^I} \frac{d\sigma_l^I}{d\psi_l} \right], j = m, h$. The optimal level of audits are reduced in both sectors. The result from proposition 7, that welfare is maximized when the government to a larger extent targets its audits to the manual sector, i.e., $p_m > p_h$ if $\kappa_h \geq \kappa_m$, will however still hold.