ASSORTATIVE MATCHING, SEARCH AND DISCRIMINATION

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Abstract

We show that a model of employer taste-based discrimination in a labor market characterized by search frictions and production complementarities in worker and employer productivity types can replicate the main empirical regularities pertaining to racial discrimination in the U.S. We build on Shimer and Smith (2000) partnership model and assume that a positive share of employers are prejudiced and a positive share of workers are discriminated against. The key distinctive features of our model with respect to other search discrimination models is the introduction of two-sided productivity heterogeneity and production complementarities. These features allow us to replicate the main regularities of observed wage and employment gaps between black and white male workers with different levels of skill. They also allow us to study the extent of worker segregation by skill and race in the labor market. The final goal of the paper is to quantify the importance of discrimination and skill in differences in labor market outcomes across workers by estimating the model with U.S. data.

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1. Introduction

Discrimination in the labor market is still perceived by many as a first-order cause to explain differences in labor market outcomes between workers belonging to minority and nonminority groups (race, gender etc.), e.g. Altonji and Blank (1999).[1] The direct evidence used to support this view comes from experimental studies, e.g. Bertrand and Mullainathan (2004), and regression-based methods, e.g. Carneiro (2005).[2] We are motivated to study discrimination in the labor market given the persistence of differences in labor market outcomes of blacks and whites, see Lang and Lehmann (2011). Although these differences are less striking than thirty years ago, e.g. Hsieh and Klenow (2011), the role of discrimination in explaining those differences is still not fully understood.

In this paper we develop a model of discrimination that aims at explaining the main stylized facts pertaining to racial discrimination in the U.S. In a recent survey Lang and Lehmann (2011) highlighted negative black vs. white wage and employment differentials as the two main empirical regularities that a model of discrimination should replicate. Critically, they stressed that these differentials vary substantially by skill. In particular, wage gaps '*are smaller or nonexistent for very high-skill workers and possibly for very low-skill workers*' and the employment gaps are '*somewhat smaller among high-skill than among low-skill workers.*' The second set of empirical regularities we wish to replicate concern segregation by skill and race in the labor market. Occupational segregation by race is well established, although less well than gender occupational segregation, e.g. Altonji and Blank (1999). Hsieh and Klenow (2011) describe the evolution of women and black occupational segregation in the U.S. over the past fifty years. Hellerstein and Neumark (2008) studied workplace segregation in the U.S. and have found evidence of segregation by race

[1] Although initially encompassing both gender and racial discrimination, the discrimination literature has evolved to focus on racial discrimination.

[2] There are well-known problems with these approaches that may render their findings invalid, e.g. Charles and Guryan (2011). While we acknowledge that discrimination might be mismeasured, we still think it is a valuable theoretical exercise to study how discrimination comes about and what are its implications for labor market outcomes of workers belonging to different groups.

and education and that '*education plays very little role in generating workplace segregation by race*'.

The strategy we pursue to construct a unified framework capable of accounting for all these facts is to introduce two-sided productivity heterogeneity and production complementarities in a model of taste-based discrimination with search frictions. We follow the long-standing approach initiated in Becker (1971) and model discrimination as a result of the presence of prejudiced employers, where prejudiced is modeled as a taste that enters directly into the employers' utility function. In our model economy there are two types of workers (minority and nonminority workers) and two types of employers (prejudiced and nonprejudiced). Prejudiced employers incur a psychic cost of employing a minority worker and this utility cost is transferable to the worker. Employers and workers are further distinct with respect to their productivity levels. Lastly, our environment is characterized by search frictions (agents meet potential partners randomly) and production complementarities in workers' and employers' productivity levels.

To operationalize this characterization of the labor market we build on Shimer and Smith (2000) partnership model. In this setting employers and workers decide which matches are acceptable, where potential matches are characterized by the partner's type (minority/nonminority, prejudiced/nonprejudiced) and its productivity level. These set of decisions constitute an agent's matching set and fully described her strategy. Workers and employers meet potential partners at endogenous rates, mediated by an aggregate matching function. Finally, once a worker and a firm meet the wage is set according to the Nash-bargaining solution.

The richness of our setting provides our model with a number of interesting features and implications, some of which are novel to the literature. Similar to previous models of taste-based discrimination in a frictional environment, e.g. Bowlus and Eckstein (2002), Rosén (2003) and Flabbi (2010), our model generate wage discrimination and hiring discrimination. We follow the literature, e.g. Cain (1986), and define economic discrimination as the unequal treatment of equals on account of noneconomic factors, where equal workers are those with the same productivity level. When a minority worker is paid a wage lower than an equally productive nonminority worker we say there is wage discrimination. When an employer accepts to form a match with a nonminority worker but refuses to match with an equally productive minority worker, we say there is hiring discrimination.

The distinctive feature of our model is the presence of two-sided productivity heterogeneity.[3] There are several reasons why this a desirable feature. As Lang and Lehmann (2011) emphasize, differences in labor market outcomes are heterogeneous across skill. Furthermore, it is a well-known critique to regression-based methods used to measure discrimination to say that unobserved heterogeneity (typically, both worker and firm productivity) bias estimates of discrimination. Modeling productivity differences explicitly allows us to gain insight on how the different sources of heterogeneity (race, prejudice and skill) interact to produce different labor market outcomes for minority and nonminority workers, and how the latter effects change with worker productivity. To analyze these features of the equilibrium we recourse to calibration methods.

Another, admittedly small, contribution of this paper is to bring together two different strands of the literature, the discrimination literature and the recent literature on sorting. In Becker (1971) pioneering contribution the existence of prejudice employers generates segregation — the market mechanism allocates minority workers to nonprejudiced firms, so that minority workers are overrepresented in nonprejudiced firms. The concept of segregation captures the degree of inequality in the distribution of workers of different types across groups, where these groups can be firms, areas of town, occupations etc. Worker segregation by race across firms in the labor market occurs when workers of similar race are overly concentrated in particular groups of firms, e.g. blacks might be overrepresented in firms with black managers. A recent and very active literature seeks to measure a specific type of segregation,

[3]Although the model developed by Bartolucci (2009) also features two-sided productivity heterogeneity it is not, strictly speaking, a discrimination model. In his model discrimination is a black-box, defined as the difference between bargaining parameters of minority and nonminority workers.

known as sorting, using matched employer-employee data.[4] Sorting in the labor market occurs when high-skill workers are disproportionately employed in high-productivity firms.[5] Our model generates sorting along two dimensions: skill and race. That is to say, high-productivity workers are overly concentrated in high-productivity firms and minority workers are overrepresented in non-prejudiced firms. On top of this, our model generates a third type of segregation: minority workers are overly represented in low-productivity firms. We argue that, under a particular intepretation of our model, this last prediction can speak to the patterns of occupational segregation by race that we observe in the North-American labor market.

The final step of this paper will involve carrying out a quantitative assessment of the sources of differences in labor market outcomes according to the structure present in the model. The fact that two-sided productivity heterogeneity is a distinctive feature of the model calls for an estimation strategy that makes use of matched employer-employee data. Unfortunately, for the U.S. labor market there exists no such database readily and publicly available for researchers based outside the U.S. On the other hand, the model finds its motivation in U.S. labor markets, so ideally we want to use North-American data. Our current estimation strategy seeks to address both these concerns. We match worker data from the Current Population Survey (CPS) with data on value-added per worker collected by the U.S. Census of Manufactures. We estimate the parameters of the model using indirect inference, matching moments obtained from the data to the theoretical moments generated by the model.

The paper is structured in the following way. In Sections 2. and 3. we present the theoretical model, derive the equilibrium and establish some theoretical results. In Sections 4. we analyze the quantitative properties of the model, namely its ability to reproduce the main empirical regularities of interest, given a reasonable calibrated vector of parameter values. In Section 5. we

[4] See e.g. Abowd et al. (1999), Postel-Vinay and Robin (2002), Lopes De Melo (2008) and Bagger and Lentz (2008).

[5] Shimer and Smith (2000) have shown that in a frictional environment there will be sorting if the production function is log-supermodular. Lopes De Melo (2008) proposes using a measure of segregation to measure sorting.

set out our estimation strategy and provide a preliminary parameter estimates. In section 6. we conclude.

2. The Model

The model we develop in this section builds on Shimer and Smith (2000) partnership model extending it to the case where there exist two types of workers and two types of firms, and where one type of firm is prejudiced against one type of worker.

2.1. The Environment

We consider a labor market with L workers and G jobs. There exist two job types and two worker types. A share m of workers are of type-1 and a share $1-m$ are of type-2. Similarly, a share π of jobs are within prejudiced (P) firms and a share $(1-\pi)$ are not (N). Agents also differ in their productivity, $h \in [\underline{h}, \overline{h}]$ is the index of workers productivity and $x \in [\underline{x}, \overline{x}]$ that of jobs, which can also be interpreted as the productivity of firms. In what follows we use the terms *jobs* and *firms* interchangibly.

Let $\ell_i(h)$ and $g^j(x)$ denote the density functions of population measures of type-i workers of productivity h and type-j firms of productivity x.[6] Let $u_i(h)$ and $v^j(x)$ denote the density functions of measures of unemployed type-i workers of productivity h and vacant type-j firms of productivity x.[7] Thus, the total number of unemployed type-i workers is $u_i = \int u_i(h)\,\mathrm{d}h$ and the total number of vacant type-j firms $v^j = \int v^j(x)\,\mathrm{d}x$.

Time is continuous and both workers and firms are risk neutral. Like in Becker (1971), prejudiced firms incur a psychic cost d of employing a type-2 worker. When a worker and firm meet, their flow output depends on their productivity types via a production function $f(h,x)$. We assume the production function $f(h,x)$ satisfies certain regularity conditions (see appendix B1.). We

[6] So that $mL = \int \ell_1(h)\,\mathrm{d}h$ and $\pi G = \int g^P(x)\,\mathrm{d}x$.

[7] To simplify the notation we assume workers and firms of different types draw their productivity from a common distribution. In sections 4. and 5. we will relax these assumptions in line with the available evidence.

take complementarities in skill as a descriptive feature of modern labor markets and so assume the production function is supermodular (see appendix B1.).[8] There is an emerging and very active literature that aims to identify the sign or the strength of assortative matching, or sorting, in the labor market using matched employer-employee data, see e.g. Abowd et al. (1999), Lopes De Melo (2008), Bagger and Lentz (2008) and Eeckhout and Kircher (2011). We abstract from such endeavour in this paper. Rather we assess whether the predictions of our model are consistent with our observations of labor market outcomes.

We assume that only unemployed workers and vacant firms search for a partner, ruling out on-the-job search. The number of meetings per period is given by the aggregate matching function $M(u_1 + u_2, v^p + v^n)$. Workers face an instantaneous probability to meet a firm $\lambda^W = \frac{M(u_1+u_2,v^P+v^N)}{u_1+u_2}$ and firms sample the pool of unemployed workers with probability $\lambda^F = \frac{M(u_1+u_2,v^P+v^N)}{v^P+v^N}$.

Once a firm and worker meet they decide whether or not to form a match. We define a match indicator function $\alpha_i^j(h,x)$ which is equal to 1 if a type-i worker and a type-j firm decide to match upon meeting. Matches are randomly destroyed with a flow probability $\delta > 0$, in which case, both worker and firm re-enter the pool of searchers.[9]

2.2. Value Functions

Workers can be in one of two different states: employed or unemployed. While unemployed a worker receives a flow utility b. The value function for an employed type-i worker of productivity h in a type-j firm of productivity x is given below, where $w_i^j(h,x)$ is the wage resulting from the bargaining game, and $U_i(h)$

[8] Shimer and Smith (2000) show that if the production function is log-supermodular then there is positive assortative matching. We have not proven that, in our environment, a supermodular production function implies positive assortative matching. However, in the countless simulations of the model we have done, positive assortative matching always holds.

[9] We are aware that in many applications, one should expect the job destruction shock to be heterogenous across worker types. In sections 4.4. we examine the effect of allowing for that extra source of heterogeneity.

is the value of being unemployed:

$$\rho W_i^j(h,x) = w_i^j(h,x) + \delta\left[U_i(h) - W_i^j(h,x)\right]. \tag{1}$$

Here $U_i(h)$ is given by:

$$\begin{aligned}\rho U_i(h) = & b + \lambda^W\Bigg[\int \alpha_i^P(h,x)\left[W_i^P(h,x) - U_i(h)\right]\frac{v^P(x)}{v^P+v^N}\,\mathrm{d}x \\ & + \int \alpha_i^N(h,x)\left[W_i^N(h,x) - U_i(h)\right]\frac{v^N(x)}{v^P+v^N}\,\mathrm{d}x\Bigg].\end{aligned} \tag{2}$$

Since $v^j(x)$ are density measures, $\frac{v^j(x)}{v^P+v^N}$ is the probability of sampling a vacant type-j job of productivity x from the pool of unmatched firms.

Firms can be in three states: vacant, filled or inactive. $J_i^j(h,x)$ is the value of a filled vacancy of a type-j firm of productivity x, filled with a type-i worker of productivity h:

$$\rho J_i^j(h,x) = f(h,x) - d\mathbf{1}_{\{i=2,j=P\}} - w_i^j(h,x) + \delta\left[V^j(x) - J_i^j(h,x)\right] \tag{3}$$

Here, d is a psychic non-pecuniary cost that a prejudiced employer incurs upon matching with a type-2 worker. One way to interpret this condition is that a prejudiced firm is acting *as if* the value of production were lower than it *actually* is. Note that the prejudice enters additively in the firms value equation. This assumption is standard in search models of taste-based discrimination with match-specific heterogeneity, e.g. Rosén (2003) and Flabbi (2010).

We assume that posting a vacancy has a positive flow cost $\kappa > 0$. Firms only enter the market if the present discounted value of opening a vacancy is positive. The value of posting a vacancy, $V^j(x)$, is dependent upon the probability of the vacancy being filled by each of the types of worker. It is given by:

$$\begin{aligned}\rho V^j(x) = & -\kappa + \lambda^F\Bigg[\int \alpha_1^j(h,x)\left[J_1^j(h,x) - V^j(x)\right]\frac{u_1(h)}{u_1+u_2}\,\mathrm{d}h \\ & + \int \alpha_2^j(h,x)\left[J_2^j(h,x) - V^j(x)\right]\frac{u_2(h)}{u_1+u_2}\,\mathrm{d}h\Bigg]\end{aligned} \tag{4}$$

Again, since $u_i(h)$ is a density measure, $\frac{u_i(h)}{u_1+u_2}$ is the probability of sampling an unemployed type-i worker of productivity h from the pool of unmatched workers.

2.3. Match Surplus

From the four value functions written above, we can determine the total surplus generated by any match. This surplus $S_i^j(h,x)$ is bargained over and split according to Nash's bargaining rule, with the worker taking a share β and the firm a share $(1-\beta)$.

$$S_i^j(h,x) = \frac{J_i^j(h,x) - V^j(x)}{1-\beta} = \frac{W_i^j(h,x) - U_i(h)}{\beta} \tag{5}$$

The wage equation that solves this bargaining problem is given by:

$$w_i^j(h,x) = \beta\left[f(h,x) - d\mathbf{1}_{\{i=2,j=P\}} - \rho V^j(x)\right] + (1-\beta)\rho U_i(h) \tag{6}$$

Finally, a match is formed whenever it generates nonnegative surplus:

$$\alpha_i^j(h,x) = \mathbf{1}\left[S_i^j(h,x) \geq 0\right]. \tag{7}$$

An important remark concerning the wage equation is that our model assumes the psychic cost d is transferable among the parties. In case of a match between a prejudiced firm and a minority worker, it implies the worker and the firm share this cost according to their rent-sharing parameters.[10]

For a type-i worker of productivity h his strategy is given by two sets $\mathcal{M}_i^j(h)$ for $j \in \{P,N\}$ that contain all the acceptable firms with whom he is willing to match and who are willing to match with him. Similarly, a firm's strategy is defined by two sets $\mathcal{M}_i^j(x)$ for $i \in \{1,2\}$. By construction, matching sets are symmetric, so all matches in an agent's matching set are mutually acceptable

[10]One way to ascertain the impact of assuming nontransferable psychic cost is to simulate a model where d is removed from the prejudiced firm's value function and the condition for match feasibility between type-2 workers and type-P firms is changed to $\alpha_2^P(h,x) = \mathbf{1}\left[(1-\beta)S_2^P(h,x) \geq d\right]$, i.e. the surplus of the match for the prejudiced firm has to be at least equal to d. The results show that this model delivers similar patters to the one with transferable utility. This leads us to conclude that this assumption is not crucial.

and therefore feasible. This property of matching sets follows directly from the assumption of Nash-bargaining, as the implied wage-setting rule is joint-rent maximizing, see e.g. Pissarides (2000). Using the indicator function $\alpha_i^j(h,x)$ we can express each worker matching set as:

$$\mathcal{M}_i^j(h) = \{x \,|\, \alpha_i^j(h,x) = 1\} \tag{8}$$

and each firm matching set:

$$\mathcal{M}_i^j(x) = \{h \,|\, \alpha_i^j(h,x) = 1\} \tag{9}$$

3. Equilibrium

A steady-state equilibrium of this model is described by three conditions: (i) everyone maximizes their expected payoff, taking all other strategies as given; (ii) if matching increases both agents' payoff, then they accept to match; (iii) all unmatched rates are in steady-state. Conditions (i) and (ii) are given respectively by firms and workers' value functions and their matching sets. We now state the assumptions necessary to ensure (iii).

3.1. Flow Balance Equations

To fix all unmatched rates in steady-state, flow creation and flow destruction of matches for every type of agents must exactly balance. This is given by:

$$\lambda^W \alpha_i^j(h,x)\, u_i(h) \frac{v^j(x)}{v^P + v^N} = \delta \gamma_i^j(h,x) \tag{10}$$

where $\gamma_i^j(h,x)$ is a joint measure of matched type-i workers of productivity h and type-j firms of productivity x.

Then the steady-state stock of type-i workers of productivity h is:

$$\ell_i(h) - u_i(h) = \int \gamma_i^N(h,x)\, \mathrm{d}x + \int \gamma_i^P(h,x)\, \mathrm{d}x. \tag{11}$$

The total population of type-i workers of productivity h is equal to the sum of the unemployed and employed populations.

Similarly, for the population of type-j firms of productivity x:

$$g^j(x) - v^j(x) = \int \gamma_1^j(h,x)\,\mathrm{d}h + \int \gamma_2^j(h,x)\,\mathrm{d}h,\ \forall\, x\ \textit{s.t.}\ V^j(x) > V^j(x^\star). \tag{12}$$

Conditional on entering the market, the total number of firms must equal the number of vacant and matched firms.

3.2. Equilibrium Description

A steady-state equilibrium can be summarized by three conditions: (i) who is matching with whom, $\alpha_i^j(h,x)$, (ii) measures of types searching, $u_i(h)$ and $v^j(x)$, and (iii) how much everyone's time is worth, $rU_i(h)$ and $rV^j(x)$. Shimer and Smith (2000) have a proof of equilibrium existence when $d = 0$ and Assumptions 1 and 2 hold (see appendix B1..[11] Formally, the equilibrium is defined as:

DEFINITION 1 (EQUILIBRIUM): *Given exogenous parameters L, G, m, d, π, ρ, β, b, κ, δ, a production function $f(h,x)$, a matching function $M(\lambda,\eta,\mu)$, with exogenous parameter values (λ,η,μ), and density functions of population measures of firms' and workers' $\ell_i(h)$, $g^j(x)$, an equilibrium is a fixed point*

$$(\alpha(h,x)_i^j, u_i(h), v^j(x), U_i(h), V^j(x))$$

that solves the system of equations given in appendix A4..

3.3. Equilibrium Characterization

We now explore some implications of equilibrium for workers and firms of different types and productivity levels. Proofs of stated results can be found in the

[11]We have not formally proven that an equilibrium exists when $d > 0$ and $\pi \in (0,1)$. Shimer and Smith (2000) have shown that an equilibrium exists when there is sorting along one dimension, productivity. In our model there are two sources of heterogeneity, productivity and race/prejudice, and so sorting can occur along these two dimensions. However, for all the simulations of the model we have made we have always found an equilibrium.

appendix A4.. Before proceeding with those implications, however, we establish some definitions. In the economics literature discrimination exists when equally productive workers are treated differently based on noneconomic factors, such as race or gender, e.g. Cain (1986). The first instance of economic discrimination we are interested in characterizing pertains to wages.

DEFINITION 2 (WAGE DISCRIMINATION): *A type-i worker of productivity $h \in [\underline{h}, \overline{h}]$ experiences wage discrimination if she is paid a lower wage than an equally productive type-$k \neq i$ worker matched with an equally productive firm of productivity $x \in [\underline{x}, \overline{x}]$ of type $j = N, P$, that is*

$$w_i^j(h, x) < w_{k \neq i}^j(h, x) \text{ for } j = N, P.$$

The second instance of economic discrimination relates to agents decision of whom to match with. In an economy with no match surplus losses due to prejudice, type-1 and type-2 workers with the same productivity level match with firms within the same range of productivity levels. When there is prejudice, in general, this will no longer be the case and the matching sets of two equally productive workers of different types will differ. One reason why these matching sets differ is due to hiring discrimination. Formally, we have that:

DEFINITION 3 (HIRING DISCRIMINATION): *A type-i worker of productivity $h \in [\underline{h}, \overline{h}]$ experiences hiring discrimination if, upon meeting a firm of productivity $x \in [\underline{x}, \overline{x}]$ of type $j = N, P$, he is not hired, but an equally productive type-$k \neq i$ worker is; that is,*

$$\exists (h, x), \alpha_i^j(h, x) = 0 \text{ and } \alpha_{k \neq i}^j(h, x) = 1 \text{ for } j = N, P.$$

Hiring discrimination describes discriminatory behavior by employers that is materialized in the decision to hire a worker — a decision that is different from that of how much to pay him (wage discrimination), but that stems from same cause, viz. prejudice.

The first implication of our model is that, for a positive value of d, type-P firms (those who are prejudiced) and type-2 workers (those who are prejudiced against) face worse perspectives in the labor market compared to type-N firms and type-1 workers, respectively. This result is stated in the following proposition.

PROPOSITION 1 (OUTSIDE OPTION EFFECTS): *For any equilibrium such that* $\pi \in (0,1)$ *and* $d > 0$ *and for any productivity levels* $x \in [\underline{x}, \overline{x}]$ *and* $h \in [\underline{h}, \overline{h}]$*:*

(i) for any two workers with productivity h *and of different types, the value of unemployment of a type-1 worker is higher than that of a type-2 worker, that is,* $U_1(h) > U_2(h)$*; and*

(ii) for two firms with productivity x *and of different types, the value of vacancy of a type-N firm is higher than that of type-P firm, i.e.* $V^N(x) > V^P(x)$.

A corollary of Proposition 1 is that, if there are any prejudiced employers in this model economy, there will be wage discrimination.

COROLLARY 1 (TYPE-2 WAGE DISCRIMINATION): *For any equilibrium such that* $\pi \in (0,1)$ *and* $d > 0$*, type-2 workers experience wage discrimination in both types of firms.*

One can also show that, given certain mild assumptions about the economy (see assumptiom 3), the presence of prejudice employers implies that type-2 workers will hire-discriminated in equilibrium. This result is stated below:

COROLLARY 2 (TYPE-2 HIRING DISCRIMINATION BY PREJUDICED FIRMS): *If the economy satisifies certain regularity conditons, stated in assumption 3, for any equilibrium such that* $\pi \in (0,1)$ *and* $d > 0$*, some type-2 workers experience hiring discrimination by prejudiced firms.*

Differences in unemployed workers' outside options will generate type-1 hiring discrimination by non-prejudiced firms. This result is stated below.

COROLLARY 3 (TYPE-1 HIRING DISCRIMINATION BY NONPREJUDICED FIRMS): *If the economy satisifies certain regularity conditons, stated in assumption 3, for* *any equilibrium such that $\pi \in (0,1)$ and $d > 0$, some type-1 workers experience hiring discrimination by nonprejudiced firms.*

The latter effect is weaker since it is due to differences in outside options (see Proposition 1) and not a direct psychic cost.

We now turn to the model implications regarding the productivity distribution of prejudiced and nonprejudiced firms in equilibrium. We first state a result that will be instrumental in showing the main result of interest. It states that, for each type of firm, the value of posting a vacancy is a monotonically increasing function of the firm's productivity.

PROPOSITION 2 (MONOTONICALLY INCREASING OUTSIDE OPTIONS): *Given Assumption 2:*

(i) $V^j(x)$ is monotonically increasing in x, for $j = N, P$; and

(ii) $U_i(h)$ is monotonically increasing in h, for $i = 1, 2$.

Note that, for all $\kappa > 0$, $\frac{\partial V^j(x)}{\partial \kappa} < 0$ and it is constant. Thus, if κ is large enough, there exists a threshold productivity required for a type-j firm to enter the market, denoted $x^{j\star}$, such that $V^j(x^{j\star}) = 0$. All type-j firms of productivity below $x^{j\star}$ will choose not to enter the market and so $v^j(x) = 0, \forall x < x^{j\star}$. Proposition 1 and Proposition 2 give us the following result:

COROLLARY 4 (THRESHOLD PRODUCTIVITY DIFFERENCES): *For all $d > 0$ and $\pi \in (0,1)$, $x^{P\star} > x^{N\star}$.*

Hence, in equilibrium, conditional on the primitive productivity distribution across firm types being the same, prejudiced firms will be on average unambiguously more productive than nonprejudiced firms. This follows from the fact that prejudiced firms have to *pay* the cost of prejudice d.

Our model contains two sources of frictions which drive inefficiency - labor market and discriminatory frictions. See the appendix for detailed decomposition of discriminatory inefficiencies.

4. Simulations

The model developed in the previous section does not have a closed formed solution. To investigate the main features of the equilibrium outcomes and describe the mechanisms that deliver those features, we first analyze an illustrative numerical example. The second simulation exercise involves investigating whether our model of discrimination can replicate the main features of differences in labor market outcomes among male and black white workers in the U.S. labor market. In both simulation exercises we follow the methodology and model specification described in sections 4.1. and 4.2..

4.1. Methodology

We assign values to all the exogenous parameters ($\ell_i(h)$, $g^j(x)$, L, G, m, π, b, κ, δ, λ, μ, η, d) and solve for the endogenous variables of the model, using the following multi-stepped algorithm:

1. We set initial parameter values for all the endogenous objects of the model and discretize the supports of the productivity measures of firms and workers in the economy as a whole, $\ell_i(h)$ and $g^j(x)$.

2. Using the initial values we iterate over equations (2) and (4) to determine $U_i(h)$ and $V^j(x)$, at each stage updating the region of feasible matches determined by equation (7).

3. $V^j\left(x^{j\star}\right) = 0$ determines $x^{j\star}$.

4. $u_i(h)$ and $g^j(x)$ are obtained from equations (11) and (12).

5. Given the values determined in the previous step, new values of u_i and v^j as well as λ^W are determined.

6. Steps 2 through 5 are updated until the endogenous distributions $u_i(h)$ and $g^j(x)$ converge.

4.2. Model Specification

We specify a production function that is supermodular in h and x, namely $f(h,x) = hx$. In this respect we depart from Shimer and Smith (2000), who assume a log-supermodular production function. Contact rates between firms and workers are assumed to be governed by the aggregate number of vacancies and unemployed workers via a matching function, which we specify to be Cobb-Douglas

$$M(u_1 + u_2, v^N + v^P) = \lambda(u_1 + u_2)^{\eta}(v^N + v^P)^{\mu} \tag{13}$$

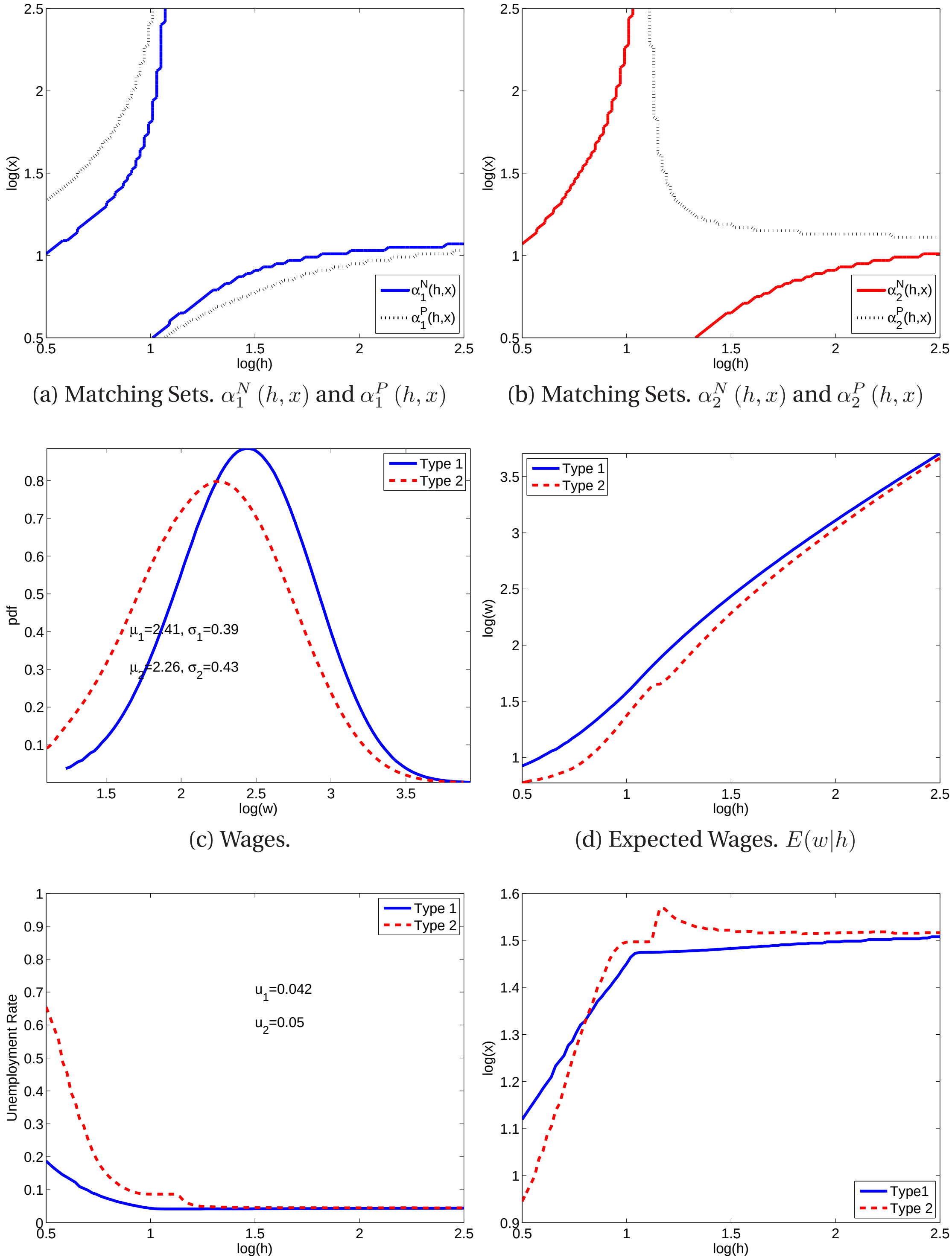
where λ is the matching efficiency and η and μ are the matching elasticities of unemployed workers and vacancies. In all simulations η and μ are set to equal 1, so that our matching function is quadratic. A quadratic matching function is perhaps not common in the literature, but has been used by Shimer and Smith (2000) and Lopes De Melo (2008). Finally, we assume both worker and firm productivity distributions are log-normal with a mean 1.5 and a standard error 0.25. The supports of these distributions are discretized, truncated and defined over the support $[0.5, 2.5]$.

4.3. Illustrative Numerical Example

In the numerical example we set the populations share of workers of both types equal to a half. We also set the size of the prejudiced firms to half the total number of firms. These firms incur a psychic cost d equal to 5 when employing a type-2 worker. This value corresponds to about 20% of average productivity of type-2 workers. We note that we calibrated these parameters for illustrative purposes; the main objective of this exercise being the description of the model. The remaining parameter values can be found in Table 1.

4.3.1. Results

Figure 1a plots the contour lines of the matching sets of type-1 workers with prejudiced and nonprejudiced employers. Workers productivity levels are displayed on the horizontal axis and firms productivity levels on the vertical axis.



(a) Matching Sets. $\alpha_1^N(h,x)$ and $\alpha_1^P(h,x)$

(b) Matching Sets. $\alpha_2^N(h,x)$ and $\alpha_2^P(h,x)$

(c) Wages.

(d) Expected Wages. $E(w|h)$

(e) Unemployment Rates

(f) Expected firm types. $E(x|h)$

Figure 1: Numerical Example

Table 1: Parameters used in the Simulation

Description	Parameter	Value
Total measure of workers	L	1
Total measure of firms	G	1
Job destruction rate	δ	0.01
Matching efficiency	λ	5
Matching elasticity	η	1
Matching elasticity	μ	1
Flow value of unemployment	b	2
Flow value of vacancy	κ	0
Discount rate	ρ	0.0043
Share of type-1 workers	m	0.5
Share of prejudiced firms	π	0.5
Psychic cost of discrimination	d	5
Worker's bargaining power	β	0.5

The solid lines indicate the bounds of the union of matching sets between type-1 workers and type-N firms, while the dashed lines indicate the bounds of the union of matching sets between type-1 workers and type-P firms. The region in the interior of each of the two pairs of lines contains all matches that produce a positive surplus. A quick inspection of the plots of the two unions of matching sets indicates that there is positive assortative matching in productivity types: low (high) productivity firms are matching with low (high) productivity workers. Matches in the northwestern corner generate negative surpluses: high productivity firms have a high outside option that can only be compensated when they meet high productivity workers and production is high. Conversely, in the southeastern corner, the outside option of high productivity workers can only be outweigthed when they meet high productivity firms.

Turning our attention to differences in type-1's matching sets across prejudiced and nonprejudiced firms, we note that if $d = 0$ the two unions of matching sets would coincide. However, when d and π are positive, workers' and firms' equilibrium strategies differ. The union of matching sets of type-1 workers with prejudiced firms expand on the northwestern corner. This means that some prejudiced firms are willing to match with certain productivity type-1 workers that nonprejudiced firms are unwilling to match with. This is due to the outside option effect on prejudiced firms. Their outside option is lower than an equally productive nonprejudiced firm, since there is a positive probability of meeting a minority worker, with whom their matches are always less productive than an equally productive nonprejudiced firm. Note that this effect benefits lower productivity type-1 workers, who will tend to match, on average, with higher productivity prejudiced firms, compared to equally productive minority workers.

Figure 1b plots the matching sets of type-2 workers with the two types of employers. The solid lines indicate the bounds of the union of matching sets of type-2 workers with nonprejudiced firms and the dashed line the bound of the union of matching sets with type-P employers. Clearly, the equilibrium strategies of prejudiced and nonprejudiced firms are very different. While the shape of the union of matching sets with nonprejudiced firms is similar to that depicted in Figure 1a, the union of matching sets with prejudiced firms is not only considerably smaller, but also it is fully concentrated on the northeastern region of the plot. This shape translates the direct negative effect of a positive d on the match surplus between minority workers and prejudiced employers. Positive assortative matching in productivity is even stronger: for both firms and workers there is a threshold productivity level below which no matches are formed, whatever the productivity of the partner on the other side of the market. Finally, comparing figures 1a and 1b one can see that nonprejudiced firms have larger matching sets with type-2 workers, particularly in the the southeastern region. Again, this results from the outside option effect of a positive d on the minority workers outside option, which makes matches with some nonprejudiced firms feasible.

The remaining four figures describe the differences in labor market outcomes for minority and nonminority workers of different productivity types. Since in this simulation exercise the productivity distributions of worker and firm types are the same, these patterns follow from the impact of the discrimination parameters π and d on workers and firms equilibrium strategies (their matching sets). In all graphs the solid line indicates monminority workers outcomes, while the dashed line indicates the outcomes of minority workers. Figure 1e shows the unemployment rates of both types of workers for different levels of productivity. For both minority and nonminority workers the unemployment rates of low productivity workers are higher than those of more productive types. This is due to positive assortative matching in productivity, as high productivity firms only match with high productivity workers. On the other hand, for a considerable range of low productivity types, the unemployment rates of minority workers are higher than those of nonminority workers. This happens because, for a wide range of productivity levels, minority workers are not hired at all by nonprejudiced firms. This effect dominates the countervailing effect of larger matching sets of prejudiced employers with minority workers.

Figure 1f plots the expected firm productivity for different levels of worker productivity. The upward-sloping curves provide yet another another visual illustration of positive assortative matching in productivity. The differences in these curves highlight the distinct job assignment patterns of minority and nonminority workers. Low productivity minority workers are expected to match with firms of lower productivity compared to type-1 workers of low productivity. These patterns illustrate the second source of assortative matching; due to prejudice minority workers will match disproportionately more with nonprejudiced firms. We will treat these patterns in more detail when analyzing black segregation in simulations based on realistic parameters values for the population shares.

Lastly, figures 1c and 1d illustrate the differences in wage distributions across worker types. Figure 1c plots the probability density functions of equilibrium wages. The wage distribution of type-1 workers stochastically dominates the wage distribution of type-2 workers. In Figure 1d one can see a more detailed illustration of wage differences. The expected wage differential between non-minority and minority workers is high for low levels of workers productivity but it converges to zero at higher levels of worker productivity.

4.4. Calibration

The racial discrimination literature highlights two causes for differences in labor market outcomes between black and white workers: (i) various forms of discrimination against blacks and (ii) lower average skill endowments of the black populations vs their white counterparts, e.g. see Neal (2006). In the context of our model we interpret (ii) as generating differences in productivity. In this section and the next we apply the model outlined thus far to racial discrimination in the U.S. labor market, treating white males as the majority workers (type-1) and black males as the minority workers (type-2). We now allow a further source of heterogeneity between the two worker types, no longer imposing that their productivities are drawn from a common distribution. This further source of heterogeneity is important as we want to be able to distinguish the contribution of productivity differences and discrimination for the observed differences in labour market outcomes between black and white male workers.

The objective of the calibration exercise is to assess how well the model developed in section 2. performs in replicating the main empirical regularities pertaining to differences in labor market outcomes between black and white workers in the U.S. We use CPS data to construct a sample of black and white male individuals and to quantify differences in labor market outcomes across these two groups.[12] The statistics in the first panel of Table 2 summarize those differences: blacks represent around 8% of the active population; they have a higher unemployment rate w.r.t. whites (more than the double of whites unemployment rate) and a job finding rate that is 10 p.p. lower than of whites; their average wage is also smaller but with a lower standard deviation. These differences provide empirical counterparts to the endogenous variables in our

[12]This sample is less restrictive than the one used in the estimation section, in that we include workers that are were not employed in the manufacturing sector at same stage during the period covered in the sample. The sample is described in more detail in the next section.

model and form the benchmark to which the quantitative predictions of our model must be tested.

The second panel of Table 2 contains estimates of exogenous parameters of the model. The average job destruction rate of the sample of black and white workers is an estimate of δ. The estimates of π and d and the difference in average productivity of blacks and whites are those obtained in Bowlus and Eckstein (2002). The last panel of Table 2 contains parameter values calibrated in line with the practice in the literature.

Table 2: Statistics and Parameters used in Calibrations

Statistic/Parameter	Value
Share of whites	0.92
Whites unemployment rate	0.044
Blacks unemployment rate	0.1
Whites job finding rate	0.39
Blacks job finding rate	0.29
Whites average log wage	2.6
Blacks average log wage	2.5
Whites wage s.d.	0.5
Blacks wage s.d.	0.45
Job destruction rate	0.13
Share of prejudiced firms	0.56
Psychic cost of discrimination	$0.31 \times E(ln(h)\|white)$
$E(ln(h)\|black)/E(ln(h)\|white)$	0.96
Flow value of unemployment	0
Flow value of vacancy	0
Discount rate	0.0043
Worker's bargaining power	0.5
Matching efficiency	10

The first exercise we perform aims at answering the following question:

conditional on the structure of our model, what can productivity differences alone account for in terms of differences in labor market outcomes between black and white workers? Calibrating the job destruction rate to equal the average between the two races (0.12) we conclude that productivity differences account for some but not all of those facts. The plots corresponding to this calibration exercise are displayed in Figure 2. One can see that there is no significant unemployment differential (both unemployment rates are equal to 0.036), the average wage differential is 1 p.p. higher than the average productivity differential, respectively 0.95 and 0.96, and the difference in job finding rates is minimal (both rounded job finding rates are equal to 0.36). Obviously, due to complementarities there is positive assortative matching and the employment share of blacks by firm productivity level is monotonically decreasing, with blacks overrepresented (i.e. their employment share is above their population share) in low productivity firms and underrepresented in high productivity firms.

Our attention now shifts to the impact of prejudice on labor market incomes. The plots corresponding to this calibration are displayed in Figure 3. Setting the productivity distributions of blacks and whites to equal each other and using Bowlus and Eckstein (2002) estimates of the discrimination parameters (π and d), the following effects are observed. The unemployment differential is sizable though about half the observed differential. Interestingly, the unemployment differential clearly decreases by skill level, starting at about 2% and converging to zero for the highest levels of skill. The average wage differential is higher than in the model with productivity differences only, at around 0.92. The pattern of expected wage differentials by skill is decreasing, starting at about 4% and converging to a value very close to zero for the highest levels of skill. The differences in average job finding rates of blacks and whites are also in line with what we see in the data, respectively 0.36 and 0.19. Unsurprisingly, there is positive assortative matching. A curious effect is that the distribution of blacks employment share by firm productivity is v-shaped, meaning that blacks are overrepresented in low and high productivity jobs and underrepresented in average productivity jobs. This v-shaped curve is the result of sorting on skill and race: blacks are overrepresented in nonprejudiced firms, and more so in low productivity firms, and are underrepresented in low productivity prejudiced firms — in fact, they do not match at all with these firms. Interestingly, blacks are overrepresented in high productivity prejudiced firms: for very high productivity firms the production loss due to prejudice is very small and so the outside option effect (blacks outside option is smaller) dominates the direct effect of d.

In conclusion, our calibration exercises suggest that prejudice plays a prominent role in explaining the main regularities regarding differences in labor market outcomes between blacks and whites in the U.S.[13] Obviously, due to the nature of this exercise, these results are merely suggestive. In order to to obtain a more robust response to this question, we need to study how the parameters of the model can be identified and estimated using U.S. labor market data. These issues are addressed in the following section.

5. Data and Identification

We use worker data from the Current Population Survey (CPS), merging the Monthly Outgoing Rotation Groups (MORG) extracts with the Basic Monthly (BM) extracts, thus gathering information on individual wages and transition rates across different labor market states. Our sample runs from January 2005 to December 2006. We limit the sample to include exclusively men who declare themselves to be either black or white[14]. We limit the sample to men to avoid further complications of modeling labor supply decisions and to be as precise as possible about the type of prejudice we are estimating. We also restrict our sample to individuals between the ages of 20 and 60 (inclusively) who remain active in the labor market throughout their spell in the sample. Finally, we restrict our sample to individuals who at some point during the sample were

[13]One aspect of the calibration exercise we have omitted from the discussion relates to the second moment of blacks and whites wage distributions. In almost all simulations, the predictions are contrary to what we see in the data; the variance of blacks wages being higher than that of whites.

[14]Note, there is some ambiguity about whether Hispanics declare themselves as white or white other. We've run both specifications and find our results robust to either definition of white.

employed at a manufacturing firm.

The last restriction we imposed follows from the fact that the only publicly available data containing information on firm value-added is the U.S. Census of Manufacturing, which only surveys this sector of the economy.[15] We match *firm* level data to worker level data by four-digit industry code. This is to say, we assume productivity differences across firms are only explained by the four-digit industry at which they operate. We compute the average value-added per worker per hour for each four-digit industry and interpret that value as the production generated by the match between that particular pair of worker and firm, i.e. $f(h, x)$. We are aware that this estimation strategy implies a particularly restrictive interpretation of the model. For the time being, our main objective is to show that the model can be identified with currently available data.

We take information about wages from the MORG, which we observe a maximum of twice per individual. The wages are weekly and measured at the time of the interview. There is some literature about the misreporting of wages in the CPS - the main source of measurement error is believed to be over reporting at low levels and under reporting at high levels, see Bollinger (1998). Further there is a problem of top-coding, where a maximum observed wage is imposed. One solution to account for measurement error would be to assume that wages are measured with error and generated from an estimable parametric distribution. Instead, like similar models we simply trim the top and bottom 5 % of the wage distributions for black and white males, replacing observed wages as missing. Using information on working hours we also convert the wages to be hourly, this further reduces the problem of top-coding.

Thus an observation in our rolling panel is given by $\psi_{it} = \{e_{it}, w_{it}, r_{it}, v_{j(i,t)}\}$, where i indexes the worker, t the month and j the *firm*. The variables e, w, r and v are the employment state, wage and race of the worker and the value added of the match, respectively.

[15] See http://www.census.gov/econ/overview/ma0100.html.

5.1. Identification

Without a closed form solution to the model, proofs of identification are seemingly impossible; instead, we justify identification with simulations. We therefore err on the side of caution and calibrate some of the parameters of the model, the values of which are given in Table 3. Remaining consistent with Shimer and Smith (2000), and for simplicity, we assume a quadratic matching function. We set ρ equal to a monthly rate of 0.0043, meaning agents discount the future at an annual rate of approximately 5%.

Table 3: Calibrated Parameters

Description	Parameter	Value
Matching elasticity w.r.t. to unemploymet	η	1
Matching elasticity w.r.t. to vacancies	μ	1
Flow value of unemployment	b	0
Flow cost of a vacancy	κ	0
Discount rate	ρ	0.0043

It is convenient at this stage to declare exactly which parameters we intend to estimate. After calibrating the parameters in Table 3 we are left with a vector, Ω, of parameters to be estimated:

$$\Omega = \{\beta, \lambda, \delta, d, \pi, \ell(x), g_1(h), g_2(h)\} \tag{14}$$

The focus of the rest of this section is to show that we can differentiate between three sources of heterogeneity. Worker productivity differences and employer prejudice determine wage and employment differentials between races. Simultaneously, we need to identify the relative contributions of worker and firm productivities to output, and by extension, to wages.

We implement a multi-stepped minimum distance estimator, identifying our parameters from three sources: worker transition rates, firm (sector) productivity distributions, cross racial surplus shares and the distribution of wages. For the three productivity distributions in Ω we make a parametric assumption, that they each follow a log-normal distribution. Simplifying, the estimation of three distribution into six parameters, $\{\mu_1, \mu_2, \mu_x, \sigma_1, \sigma_2, \sigma_x\}$.

5.2. Auxiliary Models

The choice of auxiliary models is a key step in indirect inference, our choices are based upon what we deem to be the crucial aspects of our theoretical model. Specifically, they are the rate at which workers exit and enter employment, the distribution of productivity across occupation, the different sorting patterns and the distribution of wages for whites and blacks.

5.2.1. Transition Rates

Our model predicts three different rates of transition which will perfectly determine employment rates. Empirically we can also observe different job destruction rates between blacks and whites as well as job-to-job transition rates on top of these. Although imposing different job destruction rates are perfectly identified we choose not to make this extension. Firstly, because this is another form of discrimination which we impose without modeling it theoretically, and secondly, because it will impose further complementarities between the race of a worker and a firms output. Thus it will have an effect on sorting patterns hindering our identification strategy of π and d.

Thus the three moments we match are the rate of job destruction and the rate at which whites and blacks exit unemployment, their theoretical counterparts are given below.

$$jtu = 1 - e^{\delta \times 1} \tag{15}$$

$$utj_i = \int \left(1 - exp\left(\frac{\lambda^W}{v^P + v^N} \sum_{j=\{P,N\}} \int \alpha_1^j(h,x) v^j(x)\ dx \times 1\right)\right) \left(\frac{\ell_i(h) - u_i(h)}{\int \ell_i(h) - u_i(h)\, dh}\right) dh \tag{16}$$

δ is perfectly identified by Equation 15 and λ can be obtained from equation 16 conditional on all the other elements of Ω. We use a weigthed average of the transition rate from unemployent to employment, for blacks and whites, respectively, as our moment condition to estimate λ.

5.2.2. Distribution of Wages and Value Added

Worker and firm productivity heterogeneity are identified using the distribution of matched wages and value added productivity conditional on both worker types. Heterogeneity in these four distributions are driven by worker and firm productivity differences and the presence of prejudice employers. In this discussion we abstract away from discrimination, returning to it in the following section. Thus if firms are homogeneous, wage dispersion is entirely explained by worker productivity. If all workers were homogenous, the wage distributions would be degenerate. While firm heterogeneity and discrimination also effect the wage distribution, all else equal it is identified by worker productivity. The equivalent argument is made for the distribution of value-added productivity, $f_i(h, x)$, for identifying the parameters of the primitive distribution of firm productivity.

So as we map the entire distribution we fit 19 virgintiles of the four distributions in question; minimizing the Euclidean distance between the empirical distribution and our theoretical counterpart.

5.2.3. Rent Sharing

The final auxiliary model to be estimated is of a similar ilk to the literature on rent-sharing. Blanchflower and Sanfey (1996) determine the share of profits workers in U.S. manufacturing firms get, matching the CPS with information on profits of 2 digit codes for sector. Instead, we follow the structure of our model and look at how the value added is divided between the worker and firm, see equation (6).

Conditional on h and x the surplus is divided up according to the worker and firm types, the discrimination parameter π and d and the bargaining parameter β. The final moments we match are the relationship between expected wage and value added. We mean smooth the relationship of wage on value added and match in the same non-parametric way as above (quantile fit).

To understand how this identifies β, π and d, first imagine there are no prejudice employers ($\pi = 0$) then white and black workers will get an equal share of value added, thus β would identify the relationship between expected wage and value added. Any differences in the two worker types surplus share can only be due to the discrimination parameters, thus π and d are identified.

5.3. Results

Preliminary results are given in Table 4. A current avenue of research is the assessment of the goodness of fit of these parameters.

Table 4: Parameter Estimates

Parameter	Estimate
β	0.235
λ	43.5
δ	0.0125
d	2.34
π	0.677
μ_1	2.10
μ_2	2.08
μ_x	2.20
σ_1	0.114
σ_2	0.0939
σ_x	0.198

[To be continued].

6. Conclusion

[To be written].

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A Mathematical appendix

A1. Match Surplus

Match surplus is the sum of worker and firm surpluses from matching:

$$S_i^j(h,x) = J_i^j(h,x) - V^j(x) + W_i^j(h,x) - U_i(h). \tag{17}$$

Given (1) an (3)

$$S_i^j(h,x) = \frac{f(h,x) - d\mathbf{1}_{\{i=2,j=P\}} - \rho U_i(h) - \rho V^j(x)}{\rho+\delta} \tag{18}$$

Hence

$$\alpha_i^j(h,x) = \mathbf{1}\left[\frac{f(h,x) - d\mathbf{1}_{\{i=2,j=P\}}}{\rho} - U_i(h) - V^j(x) > 0\right] \tag{19}$$

A2. Values of Unemployment and Vacant Job

Given (1) and (5)

$$\rho U_i(h) = \frac{b + \frac{\lambda^W \beta}{\rho+\delta}\sum_{j=\{N,P\}}\int \alpha_i^j(h,x)\left[f(h,x) - d\mathbf{1}_{\{i=2,j=P\}} - \rho V^j(x)\right]\frac{v^j(x)}{v^P+v^N}\,\mathrm{d}x}{1 + \frac{\lambda^W \beta}{\rho+\delta}\sum_{j=\{N,P\}}\int \alpha_i^j(h,x)\frac{v^j(x)}{v^P+v^N}\,\mathrm{d}x} \tag{20}$$

Given (3) and (5)

$$\rho V^j(x) = \frac{-\kappa + \frac{\lambda^F(1-\beta)}{\rho+\delta}\sum_{i=\{1,2\}}\int \alpha_i^j(h,x)\left[f(h,x) - d\mathbf{1}_{\{i=2,j=P\}} - \rho U^i(h)\right]\frac{u_i(h)}{u_1+u_2}\,\mathrm{d}h}{1 + \frac{\lambda^F(1-\beta)}{\rho+\delta}\sum_{i=\{1,2\}}\int \alpha_i^j(h,x)\frac{u_i(h)}{u_1+u_2}\,\mathrm{d}h} \tag{21}$$

A3. Measures of Unmatched Agents

Given (10) and (11)

$$u_i(h) = \frac{l_i(h)}{1 + \frac{\lambda^W}{\delta} \sum_{j=\{N,P\}} \int \alpha_i^j(h,x) \frac{v^j(x)}{v^P + v^N} \, \mathrm{d}x} \tag{22}$$

Given (10) and (12)

$$v^j(x) = \begin{cases} \frac{g^j(x)}{1 + \frac{\lambda^W}{\delta} \sum_{i=\{1,2\}} \int \alpha_i^j(h,x) \frac{u_i(h)}{u_1+u_2} \, \mathrm{d}h}, & \forall x \geq x^{j\star} \\ 0, & \forall x < x^{j\star} \end{cases} \tag{23}$$

A4. Equilibrium

Equilibrium solves a system of equations (20), (21), (22), (23) where $u_i = \int u_i(h) \, \mathrm{d}h$, $v^j = \int v^j(x) \, \mathrm{d}x$, $\lambda^W = \frac{M(u_1+u_2, v^P+v^N)}{u_1+u_2}$ and $\alpha_i^j(h,x)$ is given by (19).

B Assumptions

B1. Production Function Assumptions

ASSUMPTION 1 (REGULARITY CONDITIONS): *$f(h,x)$ is nonnegative, symmetric, continuous, and twice differentiable, with uniformly bounded first partial derivatives on $[0,1] \times [0,1]$.*

ASSUMPTION 2 (STRICT SUPERMODULARITY): *f is strictly supermodular. If $x > x'$ and $y > y'$, then $f(x,y) + f(x',y') > f(x',y) + f(x,y')$.*

B2. Model Economy Assumptions

ASSUMPTION 3 (ECONOMY REGULARITY CONDITIONS): *The flow value of unemployment b, the psychic cost bore by prejudiced employers d are sufficiently small and the entry cost κ is sufficiently large with respect to the value of production $f(h,x)$, so that, $\exists (h,x) \in [\underline{h}, \overline{h}] \times [\underline{x}, \overline{x}]$, such that matches are feasible, i.e. $\alpha_i^j(h,x) = 1$.*

C Omitted proofs

PROOF OF PROPOSITION 1. OUTSIDE OPTION EFFECTS

Part 1 of Lemma 1 in Shimer and Smith (2000) states that, for any worker of any productivity level, her unemployment value can only be smaller if evaluated at some alternative (non-optimal) matching set. In particular, it implies that, for any type-1 worker with productivity $h \in [0,1]$:

$$\rho U_1(h) \geq b + \lambda^W \left[\int \alpha_2^P(h,x) \left[W_1^P(h,x) - U_1(h) \right] \frac{v^P(x)}{v^P + v^N} \mathrm{d}x \right. \\ \left. + \int \alpha_2^N(h,x) \left[W_1^N(h,x) - U_1(h) \right] \frac{v^N(x)}{v^P + v^N} \mathrm{d}x \right]. \tag{24}$$

Subtracting $\rho U_2(h)$ to both sides of this inequality, substituting by (18) and rearranging one obtains the following inequality:

$$U_1(h) - U_2(h) \geq \frac{d}{\rho} \times \frac{\frac{\lambda^W \beta}{\rho+\delta} \int \alpha_2^P(h,x) \frac{v^P(x)}{v^P+v^N} \mathrm{d}x}{1 + \frac{\lambda^W \beta}{\rho+\delta} \sum_{j=P,N} \int \alpha_2^j(h,x) \frac{v^j(x)}{v^P+v^N} \mathrm{d}x} \tag{25}$$

If $\pi \in (0,1)$ then all workers face a positive probability of meeting a prejudiced firm, i.e. $\frac{v^P(x)}{v^P+v^N} > 0$ for $x > x^{P\star}$. By assumption 3, for each $d > 0$, there $\exists (h,x)$ such that $\alpha_2^P(h,x) = 1$. Then, the integral in the numerator is always positive and $U_1(h) > U_2(h)$.

By an analogous argument one can prove that $V^N(x) > V^P(x)$ for $\forall x \in [0,1]$.

PROOF OF COROLLARY 1. WAGE DISCRIMINATION

Take an arbitrary h and x. Then, if $\pi \in (0,1)$ and $d > 0$:

$$w_2^P(h,x) = \beta \left[f(h,x) - d - \rho V^P(x) \right] + (1-\beta) \rho U_2(h) < \\ \beta \left[f(h,x) - \rho V^P(x) \right] + (1-\beta) \rho U_2(h) = w_1^P(h,x);$$

By Proposition 1, $U_2(h) < U_1(h)$, so that:

$$\begin{aligned} w_2^N(h,x) =&\beta\left[f(h,x) - \rho V^N(x)\right] + (1-\beta)\rho U_2(h) < \\ &\beta\left[f(h,x) - \rho V^N(x)\right] + (1-\beta)\rho U_1(h) = w_1^N(h,x). \end{aligned}$$

Proof of Corollary 2. Type-2 Hiring Discrimination by Prejudiced Firms

Using equation (18) one can write $S_1^P(h,x)$ as a function of $S_2^P(h,x)$, namely

$$S_1^P(h,x) = S_2^P(h,x) + \frac{d + \rho(U_2(h) - U_1(h))}{\rho + \delta}$$

A type-2 worker will not be hired if $S_2^P(h,x) < 0$ which will be the case if $S_1^P(h,x) < \frac{d+\rho(U_2(h)-U_1(h))}{\rho+\delta}$. From (25) it can be shown that $\frac{d+\rho(U_2(h)-U_1(h))}{\rho+\delta} > 0$.

Hence, all unmatched type-2 workers of productivity h such that type-1 workers of the same productivity are matched with a prejudiced firm of productivity x generating a surplus $0 \leq S_1^P(h,x) < \frac{d+\rho(U_2(h)-U_1(h))}{\rho+\delta}$ will be hiring discriminated by prejudiced firms.

If some matches are not feasible, then $\exists(h,x)$ such that $0 \leq S_1^P(h,x) < \frac{d+\rho(U_2(h)-U_1(h))}{\rho+\delta}$.

Proof of Corollary 3. Type-1 Hiring Discrimination by Nonprejudiced Firms

Using equation (18) one can write $S_2^N(h,x)$ as a function of $S_1^N(h,x)$, namely

$$S_1^N(h,x) = S_2^N(h,x) - \frac{\rho(U_1(h) - U_2(h))}{\rho + \delta}$$

A type-1 worker will not be hired if $S_1^N(h,x) < 0$ which will be the case if $S_2^N(h,x) < \frac{\rho(U_1(h)-U_2(h))}{\rho+\delta}$. From Proposition 1 we know that $U_1(h) - U_2(h) > 0$, $\forall h$.

Hence, all type-1 workers of productivity h that are unmatched with non-prejudiced firm such that type-2 workers of the same productivity are matched with a prejudiced firm of productivity x generating a surplus $0 \leq S_2^N(h,x) <$

$\frac{\rho(U_1(h)-U_2(h))}{\rho+\delta}$ will be hiring discriminated by nonprejudiced firms.

Again, if some matches are not feasible, then $\exists(h,x)$ such that $0 \leq S_2^N(h,x) < \frac{\rho(U_1(h)-U_2(h))}{\rho+\delta}$.

Proof of Proposition 2. Monotonically Increasing Outside Options

The proof is, again, based on Part 1 of Lemma 1 in Shimer and Smith (2000). Given Assumption 1 about $f(x,y)$, the value inequality lemma states that:

$$\rho V^j(x) \geq -\kappa + \lambda^F \left[\int \alpha_1^j(h,x') \left[J_1^j(h,x) - V^j(x) \right] \frac{u_1(h)}{u_1+u_2} \mathrm{d}h + \int \alpha_2^j(h,x') \left[J_2^j(h,x) - V^j(x) \right] \frac{u_2(h)}{u_1+u_2} \mathrm{d}h \right] \tag{26}$$

for an arbitrary $x' \in [\underline{x}, \overline{x}]$.

Hence, for all $x_1 < x_2$

$$\rho V^j(x_2) - \rho V^j(x_1) \geq \lambda^F \left[\int \alpha_1^j(h,x_1) \left[J_1^j(h,x_2) - J_1^j(h,x_1) - V^j(x_2) + V^j(x_1) \right] \frac{u_1(h)}{u_1+u_2} \mathrm{d}h + \int \alpha_2^j(h,x_1) \left[J_2^j(h,x_2) - J_2^j(h,x_1) - V^j(x_2) + V^j(x_1) \right] \frac{u_1(h)}{u_1+u_2} \mathrm{d}h \right] \tag{27}$$

and using (18)

$$V^j(x_2) - V^j(x_1) \geq \frac{\frac{\lambda^F(1-\beta)}{\rho+\delta} \sum_{i=\{1,2\}} \int \alpha_i^j(h,x_1) \left[f(h,x_2) - f(h,x_1) \right] \frac{u_i(h)}{u_1+u_2} \mathrm{d}h}{\rho + \rho \frac{\lambda^F(1-\beta)}{\rho+\delta} \sum_{i=\{1,2\}} \int \alpha_i^j(h,x_1) \frac{u_i(h)}{u_1+u_2} \mathrm{d}h} \tag{28}$$

Since Assumption 1 implies $f(h,x_2) - f(h,x_1) \geq 0$ for all $x_1 < x_2$, we also have $V^j(x_2) - V^j(x_1) \geq 0$. An analogous argument also proves that the value function $U(h)$ is monotonically increasing in worker productivity, h.

Proof of Corollary 4. Threshold Productivity Differences

Given Proposition 1, $V^N(x^{N\star}) > V^P(x^{N\star})$. Given Proposition 2, $\exists x^{P\star} > x^{N\star}$ such that $V^N(x^{N\star}) = V^P(x^{P\star})$.

Proof of Proposition ??. Mean Wages are Monotonically increasing in Firm Productivity

$$\frac{\partial w_i^j(h,x)}{\partial x} < 0, \textit{for some}\{(h,x,i,j)|\alpha_i^j(h,x)=1\}$$

Differentiating equation (6) with respect to x:

$$\frac{\partial w_i^j(h,x)}{\partial x} = \beta f_x(h,x) - \beta\rho\frac{\partial V^j(x)}{\partial x}$$

The second term can be obtained by differentiating equation (21):

$$\rho\frac{\partial V^j(x)}{\partial x} = \frac{\lambda^F(1-\beta)}{(\rho+\delta)(u_1+u_2)}\sum_{i=\{1,2\}}\int \alpha_i^j(h,x) f_x(h,x)\, u_i(h)\; dh \tag{29}$$

As h tends to its minimum, $f_x(h,x)$ tends to 0 and assuming non-emptiness of matching sets $\rho\frac{\partial V^j(x)}{\partial x} > 0$, thus for sufficiently low h, $\frac{\partial w_i^j(h,x)}{\partial x} < 0$.

However, the mean wage is increasing in firm productivity:

$$\frac{\partial \overline{w}_i^j(h)}{\partial h} > 0, \forall\{(h,i,j)|\alpha_i^j(h,x)=1\}$$

$$\begin{aligned}\frac{\partial \overline{w}_i^j(x)}{\partial x} &= \frac{\int\left(\alpha_1^j(h,x)\gamma_1^j(h,x)+\alpha_2^j(h,x)\gamma_2^j(h,x)\right)\left(\beta f_x(h,x)-\rho\beta\frac{\partial V^j(x)}{\partial x}\right)dh}{\int \alpha_1^j(h,x)\gamma_1^j(h,x)+\alpha_2^j(h,x)\gamma_2^j(h,x)\,dh}\\ &= \beta\frac{\int\left(\alpha_1^j(h,x)\gamma_1^j(h,x)+\alpha_2^j(h,x)\gamma_2^j(h,x)\right)f_x(h,x)}{\int \alpha_1^j(h,x)\gamma_1^j(h,x)+\alpha_2^j(h,x)\gamma_2^j(h,x)\,dh} - \rho\beta\frac{\partial V^j(x)}{\partial x}\end{aligned}$$

Note, the joint measure of workers and firms is given by:

$$\gamma_i^j(h,x) = \frac{\lambda^W}{\delta(v^P+v^N)}\alpha_i^j(h,x)\,u_i(h)\,v^j(x)$$

Thus from equation (29) we know:

$$\rho\frac{\partial V^j(x)}{\partial x} < \int\sum_{i=\{1,2\}}\alpha_i^j(h,x)\gamma_i^j(h,x)f_x(h,x)\,dh$$

Thus,

$$\frac{\partial \overline{w}_i^j(x)}{\partial x} > \beta \frac{\int \left(\alpha_1^j(h,x)\gamma_1^j(h,x) + \alpha_2^j(h,x)\gamma_2^j(h,x)\right)\left(f_x(h,x) - f_x(h,x)\right)}{\int \alpha_1^j(h,x)\gamma_1^j(h,x) + \alpha_2^j(h,x)\gamma_2^j(h,x)\, dh} = 0$$

D Inefficiency Decomposition

Our model contains two sources of frictions which drive inefficiency - labor market and discriminatory frictions. The degree of labor market frictions on either side of the market are determined by the ratios of $\frac{\lambda^W}{\delta}$ for workers and $\frac{\lambda^F}{\delta}$ for firms. We abstract from any discussion of efficiency losses that arise from labor market friction, as the frictions associated with prejudice are the novel feature of our model. For a discussion of efficiency associated with labor market frictions in a similar model, see Teulings and Gautier (2004).
Frictions that arise because of the existence of prejudiced firms create four sources of inefficiency. Firstly, the psychic cost is by definition a deadweight loss and benefits no-one. Secondly, the frictions increase both unemployment and vacancy rates, thus changing total asset value of the unmatched agents. Thirdly, the frictions change the shape of the matching sets and result in mismatch, and finally, the existence of prejudice firms allows for the entry of less productive firms into the market. In this section we attempt to quantify the loss of output associated with each of the three sources of inefficiency.

Let us first define the steady state welfare (O). It is equal to the discounted sum of utilities of all economic agents.

$$\rho O = \sum_{i=\{1,2\}} \int \rho U_i(h)\mathrm{d}h + \sum_{j=\{N,P\}} \int_{x^{*j}} \rho V^j(x)\mathrm{d}x + \sum_{i=\{1,2\}} \sum_{j=\{N,P\}} \int \int_{x^{*j}} \left(\rho W_i^j(h,x) + \rho J_i^j(h,x)\right) \gamma_i^j(h,x)\mathrm{d}x\mathrm{d}h \tag{30}$$

As in Hosios (1990), the steady state welfare can be rewritten as the total asset value of the economy, i.e. the discounted sum of the benefit received by unemployed workers, minus the cost incurred by vacant firms, plus the total ouput net of the psychic cost of prejudice.

$$\rho O = b \sum_{i=\{1,2\}} \int u_i(h)\mathrm{d}h - \kappa \sum_{j=\{N,P\}} \int_{x^{*j}} v^j(x)\mathrm{d}x + \sum_{i=\{1,2\}} \sum_{j=\{N,P\}} \int \int_{x^{*j}} \gamma_i^j(h,x) \left(f(h,x) - d\mathbf{1}_{\{i=2,j=P\}} \right) \mathrm{d}x\mathrm{d}h \tag{31}$$

To evaluate the loss of welfare from discrimination, we compare the asset value for positive π and d with it evaluated at $d = 0$ (no discrimination), which we call $\rho O|_{d=0}$. The total output loss is hence given by $\rho O|_{d=0} - \rho O$, which we can decompose into each of the three sources stated. The discounted level of total psychic cost incurred by the economy is given by equation (32).

$$\rho P = d \int \int_{x^{*P}} \gamma_2^P(h,x)\mathrm{d}x\mathrm{d}h \tag{32}$$

The discounted change in total asset value of the unmatched agents is given by:

$$\rho UV = b \sum_{i=\{1,2\}} \int \left[u_i(h)|_{d=0} - u_i(h) \right] \mathrm{d}h - \kappa \left[\sum_{j=\{N,P\}} \int_{x^*} v^j(x)|_{d=0}\mathrm{d}x - \sum_{j=\{N,P\}} \int_{x^{*j}} v^j(x)\mathrm{d}x \right] \tag{33}$$

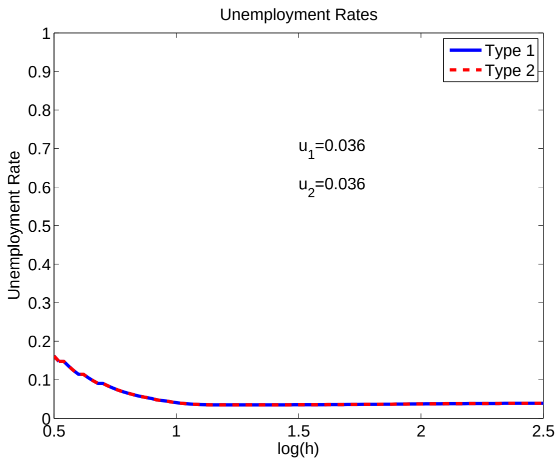
In the absence of prejudice, there is a single type of firm, let us define the threshold productivity that satisfies entry (equalising equation (4) to zero) as x^*. Thus the discounted efficiency loss is associated with the exit of the least productive prejudiced firms and the entry of less productive non-prejudiced firms and it is given by equation (34).

$$\rho X = \sum_{i=\{1,2\}} \int \int_{x^*}^{x^{*P}} \left[\gamma_i^P(h,x) \right]_{d=0} f(h,x)\mathrm{d}x\mathrm{d}h - \sum_{i=\{1,2\}} \int \int_{x^{*N}}^{x^*} \gamma_i^N(h,x) f(h,x)\mathrm{d}x\mathrm{d}h \tag{34}$$

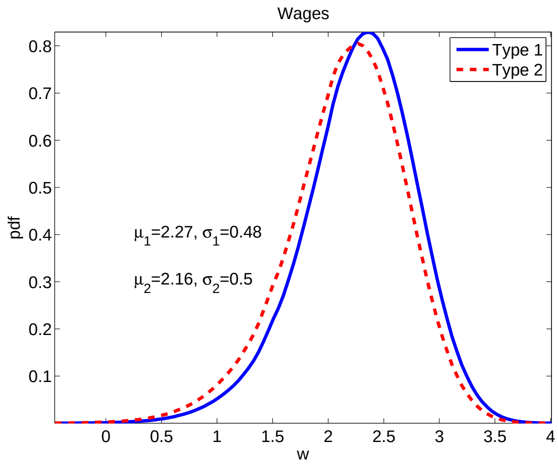
Finally, the loss in mismatch, ρM, a less efficient allocation of matches between workers and firms is thus the remaining difference.

$$\rho O|_{d=0} - \rho O = \rho P + \rho UV + \rho X + \rho M \tag{35}$$

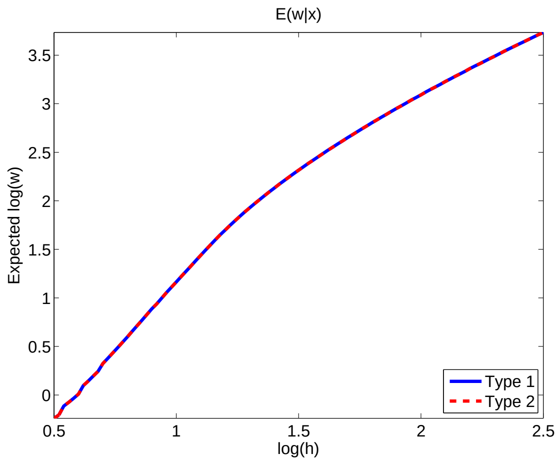
E Calibration Figures



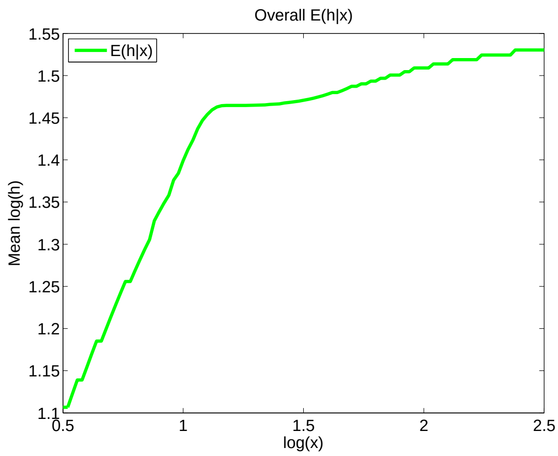
(a) Unemployment Rates



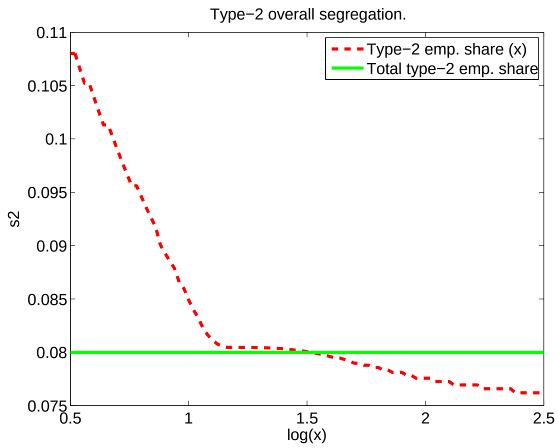
(b) Wages



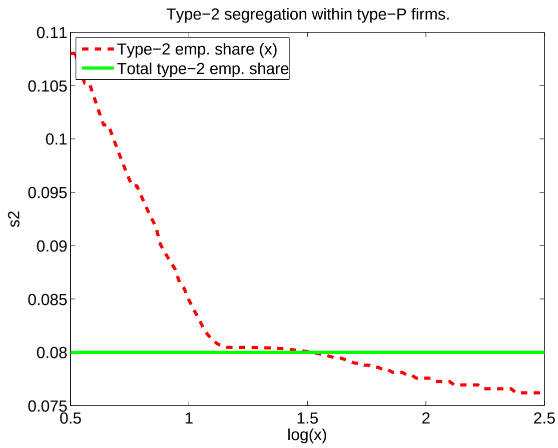
(c) Expected Wages $E(w|h)$



(d) Overall $E(h|x)$

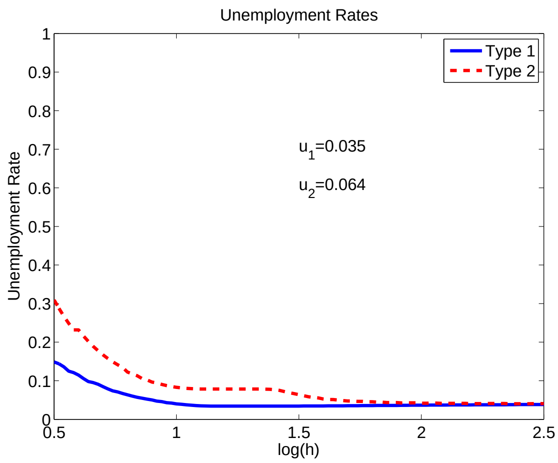


(e) Blacks employment share

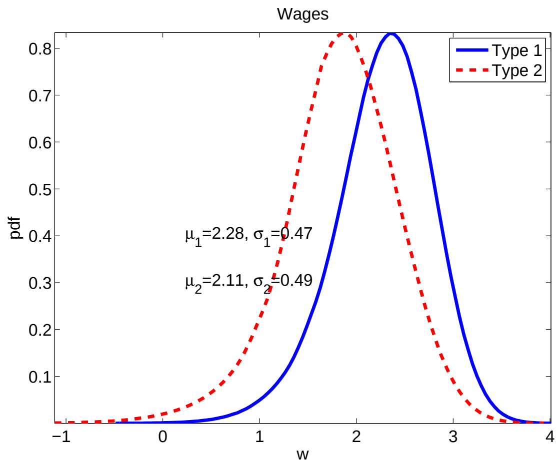


(f) Blacks employment share in type-P firms

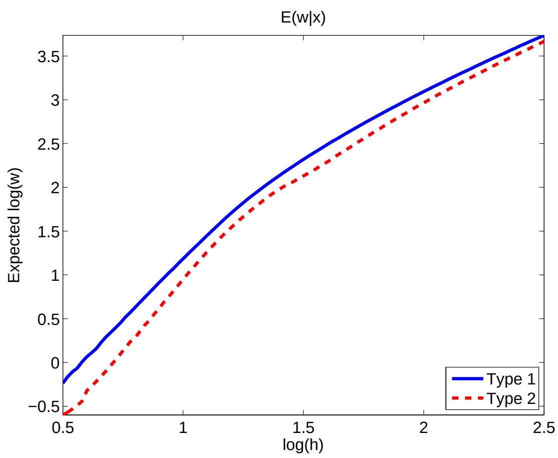
Figure 2: Calibration — Model with productivity differences only



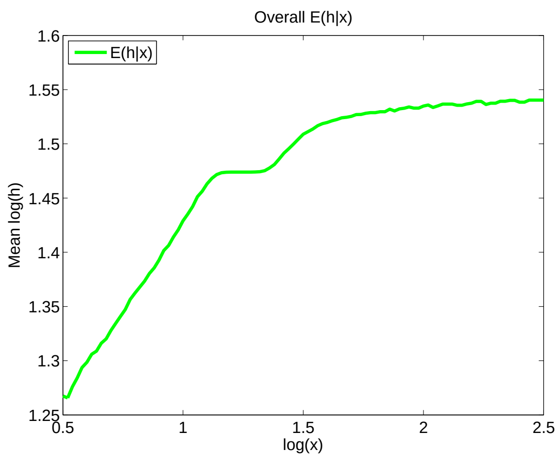
(a) Unemployment Rates



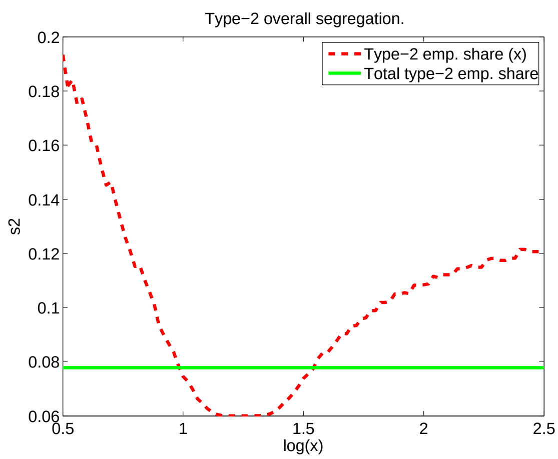
(b) Wages



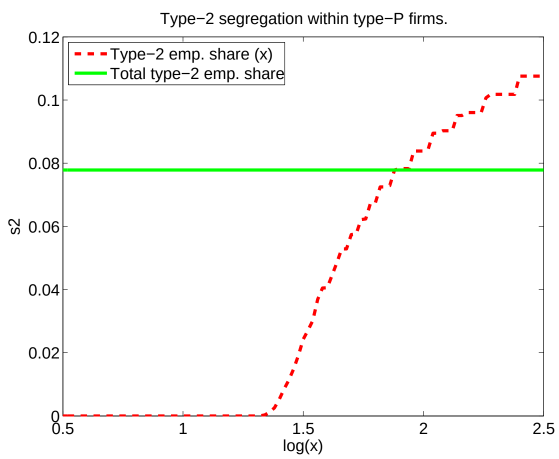
(c) Expected Wages $E(w|h)$



(d) Overall $E(h|x)$



(e) Blacks employment share



(f) Blacks employment share in type-P firms

Figure 3: Calibration — Model with prejudice only