The cyclical behavior of the unemployment, job finding and separation rates

Camille Abeille-Becker∗
Pierrick Clerc †

February 15, 2012

Abstract

In this paper, we follow a calibration strategy close to Hagedorn and Manovskii (2008) to make the search and matching model with endogenous separations capable of replicating the unemployment standard deviations with the right contributions of the job finding and separation rates. This framework calls for a value of the worker’s opportunity cost of employment less extreme and does not require a real wage stickiness, as compared to the constant separation rate model. While determining this particular value for the opportunity cost, we stress two new and significant points. First, the separation rate volatility is as sensitive as the job finding volatility to the opportunity cost. Secondly, for somewhat high values of the opportunity cost, there is a mechanism making the separation rate standard deviation amplifies that of the job finding, the amplification being increasing and convex in the value of this cost. This amplification mechanism explains the lower value for the opportunity cost.

JEL Codes : E24, J63, J64.

Keywords: Endogenous separation, Unemployment volatility, Matching.

∗Paris School of Economics and Paris 1 University. Email : camille.abeille-becker@univ-paris1.fr
†Paris School of Economics and Paris 1 University. Email : pierrick.clerc@malix.univ-paris1.fr
1 Introduction

The “unemployment volatility puzzle”, following the well-known contribution of Shimer (2005), embodies the inability of the Mortensen-Pissarides (henceforth MP) class of models to replicate the volatility of the unemployment rate in the US. Empirically, the standard deviation of the unemployment rate is explained for an half by the standard deviation of the job finding rate, and for the other by that of the separation rate (Fujita and Ramey (2009), Elsby and al.(2010)), even though this proportion is still debated. However, the first attempts to solve this puzzle narrowed on the canonical version of the matching model, in which the separation rate is purely exogenous and constant. In his paper, Shimer (2005) targeted the volatility of the job finding rate but this solution reproduces only half of the unemployment standard deviation. Hagedorn and Manovskii (2008) targeted the unemployment variability in their calibration of the worker’s opportunity cost of employment. In this case, all the unemployment volatility stems from the job finding, which clearly overstates the job finding standard deviation.

Since the seminal work of Mortensen and Pissarides (1994), the cyclical behavior of the separation rate has received an increasing attention. Pissarides (2007), Mortensen and Nagypal (2007) and Fujita and Ramey (2011) notably introduce an additional margin on which the firms adjust the number of their jobs. In all these papers, the volatility of the separation rate is restituted but the job finding variability (and thus the unemployment volatility) is always far below its empirical counterpart.

The aim of this paper is to replicate the unemployment standard deviation and the relative contributions of the job finding and separation rates. The strategy followed is close to the one used by Hagedorn and Manovskii for the constant separation rate model: the worker’s opportunity cost of employment is calibrated endogenously to make the search and matching framework with cyclical separations capable of reproducing the right volatilities of the job finding and separation rates.

We notably find that this value for the worker’s opportunity cost is much lower than in the constant separation rate model. Furthermore, the job finding and separation rate standard deviations are reproduced for a traditional symmetric Nash bargaining. Hence, the MP framework with cyclical separations is able to solve the unemployment volatility puzzle without resting on any real wage stickiness.

While determining the required value for the opportunity cost, we point out two interesting results. First, the opportunity cost has a critical impact on the separation rate volatility. More precisely, the separation rate standard deviation is an increasing and convex function of this cost. This is an important point since previous papers lead to the misleading conclusion that the separation rate standard deviation is always attainable by simply adjusting the standard deviation of the distribution of the idiosyncratic shocks. Secondly, for
somewhat high values of the worker’s opportunity cost, introducing a cyclical separation rate amplifies the job finding standard deviation. Intuitively, since the firms have now a second margin to adjust the number of jobs, we could expect that a higher separation rate volatility would reduce the job finding response and induce a trade-off between the two variabilities. Though, we stress the existence of an amplification mechanism, operating through the profit of the firm and making the separations volatility raises the job finding response. The amplification is also an increasing and convex function of the opportunity cost and explains the lower value of the opportunity cost required for the endogenous separations framework.

The rest of the paper is organized as follows. In the next section, we present the model with endogenous separations. We also examine how the opportunity cost affects the separation rate standard deviation and describe the amplification mechanism between separations and job finding volatilities. In the third section, we find a calibration for the worker’s opportunity cost of employment that replicates the right standard deviations and elasticities for the conventional symmetric wage bargaining and illustrate the critical role of the opportunity cost on the separation rate volatility and on the amplification. Section 4 concludes.

2 The model with endogenous separations and the job finding - job separations relations

The framework is derived from the paper of Mortensen and Pissarides (1994). Here we use the presentation of Ramey (2008).

2.1 The framework

2.1.1 Assumptions

The model with endogenous separations sets the same assumptions than the canonical one, the only element that differs is the modelisation of the separation rate, which is no longer an idiosyncratic exogenous probability, but now depends on the productivity of the match.

There are two types of agents in the economy, the workers and the firms. At every period $t$, each worker is either working, and receives a wage $w$, or unemployed, and receives a flow benefit $z$ (which represents the value of leisure, home production and unemployment benefits). $z$ is also the opportunity cost of employment for the worker. Each firm is either matched with a worker or posting a vacancy, with a cost $c$.

The situation in the labor market is characterized by the tightness $\theta_t = \frac{v_t}{u_t}$, with $v_t$ the number of vacant jobs and $u_t$ the number of unemployed. The higher $\theta_t$ is, the more
vacant jobs there are with respect to the number of unemployed and so the better it is for the unemployed workers. At every period \( t \), some vacant jobs can be filled and some unemployed workers can find a job: the number of matches that are created every period is given by the following matching function \( m(u_t, v_t) = A u_t^\alpha v_t^{1-\alpha} \), which is a Cobb-Douglas function with constant return to scale. We can now define the probability \( q(\theta_t) \) of filling a vacancy \( q(\theta_t) = \frac{m(u_t, v_t)}{v_t} = A \theta_t^\alpha \) and the job finding rate \( \theta q(\theta_t) = \frac{m(u_t, v_t)}{u_t} = A \theta_t^{1-\alpha} \).

When a match is filled, the worker receives a wage \( w_t \), which is the result of a Nash bargaining and a function the bargaining power of the worker \( \gamma \), and the firm receives \( p_t x_t - w_t \), with \( p_t x_t \) the productivity of the match at time \( t \).

Indeed, there is two sources of productivity in the economy: \( p_t \) the aggregate productivity (henceforth macro productivity) and \( x_t \) a match specific productivity factor (henceforth micro productivity). \( x_t \) is a random variable, which evolves according to the c.d.f \( G(x) \).

Every match begins at the maximum micro productivity \( x_h \) and has a probability \( \lambda \) of being hit by a shock at every period. If the micro productivity falls under a certain level \( R \), called the reservation productivity and which corresponds to the situation where the total surplus of the match becomes negative, the match is destroyed. Besides, the match has still an exogenous probability \( s \) of being destroyed at every period. So in this model, the separation rate is equal to \( s + (1-s)\lambda G(R) \). It is endogenous, because the higher the productivity of the economy, the lower \( R \) and then the lower the separation rate.

### 2.1.2 Creation and destruction

With a vacancy posting cost \( c \) and a match beginning at the higher productivity, we can write the following value function \( V \) of a vacant job:

\[
V = -c + \beta [q(\theta_t) J(p_t x_h) + (1 - q(\theta_t)) V]
\]

with \( \beta \) the discount factor and \( J(p_t x_h) \) the value of a new filled job which begins at the maximum micro productivity \( x_h \).

The free entry condition imposes that vacancies are opened as long as the expected profit is higher than the cost, so at equilibrium, \( V = 0 \), which gives the job creation condition:

\[
\beta J(p_t x_h) = \frac{c}{q(\theta)} \iff \beta(1 - \gamma) S(p_t x_h) = \frac{c}{A} \theta_t^\alpha
\]

This condition imposes that the discounted cost of posting a vacancy should be equal to the discounted profit of filling this vacancy. We can rewrite this equation:

\[
\beta(1 - \gamma) \left( \frac{p_t(x_h - R)}{1 - \beta(1-s)(1-\lambda)} \right) = \frac{c}{A} \theta_t^\alpha
\]

(1) with \( \left( \frac{p_t(x_h - R)}{1 - \beta(1-s)(1-\lambda)} \right) \) the joint surplus of a new vacancy.

Symmetrically, we can write the value function of unemployment \( U \):

\[
U = z + \beta [\theta_t q(\theta_t) W(p_t x_h) + (1 - \theta_t q(\theta_t)) U]
\]
with \( W(p_t|x_h) \) the value function of a worker.

We can also write the value function of an employed worker \( W(p_t|x_t) \) and of a filled job \( J(p_t|x_t) \):

\[
W(p_t|x_t) = w(x_t) + \beta(1-s)[\lambda E_t \int_R^{x_h} W(p_{t+1}|y)dG(y) + (1-\lambda)W(p_{t+1}|x_t)] + \beta[s+(1-s)\lambda G(R)]U
\]

\[
J(p_t|x_t) = p_t x_t - w(x_t) + \beta(1-s)[\lambda E_t \int_R^{x_h} J(p_{t+1}|y)dG(y) + (1-\lambda)J(p_{t+1}|x_t)] + \beta[s+(1-s)\lambda G(R)]V
\]

From these four value functions, we can deduce the value of the total surplus of a match, which is

\[
S(p_t|x_t) = (J(p_t|x_t) - V) + (W(p_t|x_t) - U).
\]

As \( V = 0 \) (free entry-condition), \( S(p_t|x_t) = J(p_t|x_t) + W(p_t|x_t) - U \).

Besides, as the wage is negotiated with a Nash bargaining, we have the following sharing rule:

\[
W(p_t|x_t) - U = \gamma S(p_t|x_t) \quad \text{and} \quad J(p_t|x_t) = (1-\gamma)S(p_t|x_t), \quad \text{with} \ \gamma \ \text{the bargaining power of the worker.}
\]

So from the equations of \( U, W, J \) and the sharing rule, we can find an expression for the present surplus as a function of the future expected surplus:

\[
S(p_t|x_t) = p_t x_t - z + \beta(1-s)[\lambda E_t \int_R^{x_h} S(p_{t+1}|y)dG(y) + (1-\lambda)S(p_{t+1}|x_t)] - \beta \theta t q(\theta t) \gamma S(p_t|x_h)
\]

This equation of the total surplus leads to the job destruction condition. Indeed, the reservation productivity corresponds to the micro productivity which cancels out the total surplus, so \( S(R) = 0 \). Hence the job destruction condition is written:

\[
p_t R = z + \beta A \theta^1 - r x - \beta(1-s)\lambda E_t \int_R^{x_h} S(p_{t+1}|y)dG(y)
\]

This condition takes into account the possibility of labor hoarding. Indeed, at equilibrium, the reservation productivity \( p_t R \) is equal to the opportunity cost of employment (the first two terms of the equation) minus the possibility of future profit (the last term), which represents the labor hoarding. So the labor hoarding decreases the reservation productivity at equilibrium.

### 2.2 The relationship between job finding and separation rates volatilities and the critical role of \( z \)

#### 2.2.1 The impact of the opportunity cost on the separation rate volatility

Hagedorn and Manovskii (2008) demonstrate the key qualitative role of \( z \) on the job-finding standard deviation. However, the worker’s opportunity cost of employment also affects the separation rate standard deviation through two opposite effects. Recall, from equation

\[ p_t R = \ldots \]

...
(2), that a match desappears when the joint surplus of this match falls to zero, i.e. when the present value of total productivity falls to the level of the unemployment value. In order to better understand the two effects, we rewrite (following Pissarides (2000)) the unemployment value in equilibrium as:

\[ U_t = \frac{1}{r} (z + \frac{\gamma}{1-\gamma} c\theta_t) \]

Equation (2) becomes:

\[ p_t R + \beta (1-s) \lambda E_t \int_R^{x_h} S(p_{t+1} y) dG(y) = \frac{1}{r} (z + \frac{\gamma}{1-\gamma} c\theta_t) \tag{3} \]

The first effect of \( z \) on the separation rate volatility is direct. From equation (3), before any shock, the higher \( z \), the higher \( R \) and then the higher the separation rate \( s + (1-s) \lambda G(R) \). Consider a negative aggregate productivity shock. For a given decrease in \( p \), the reservation productivity and the separation rate will increase. However, what matters for the volatility of this rate is not its jump in absolute but in percentage terms. When the separation rate, before the shock, is already high, its increase in proportion and thus its volatility will be low. Since the separation rate before the shock is an increasing function of \( z \), the direct effect results in a separation rate standard deviation that decreases in \( z \).

The second effect is indirect. Before some shock, from the job creation and separation conditions (equations (1) and (3)), the higher \( z \), the higher \( R \) and the lower the flow profit associated to a vacancy. Hence, the number of vacancies posted is low which implies a low equilibrium market tightness \( \theta \). From equation (3), the weak market tightness reduces in turn the level of \( R \) and thus the separation rate. Since the separation rate is low before the shock, a given decrease in \( p \) entails a strong jump in the separation rate in proportion. Hence, this “feedback effect” results in a separation rate standard deviation that is increasing in \( z \).

The total effect of \( z \) on the separation rate standard deviation is \textit{a priori} ambiguous. The simulations of the next section will show that the two effects cancel each other out for low values of \( z \) while for higher values of this parameter, the feedback effect dominates the direct effect, making the separation rate volatility an increasing function of \( z \).

### 2.2.2 The trade-off versus the amplification mechanism

In this sub-section, we will examine how the volatility of the separation rate affects the job finding standard deviation.

When endogenous separations are integrated in the canonical MP class of models, one could intuitively expect that the added volatility of the separation rate would come at the expense
of the vacancies and job-finding volatilities, since the firms have now an additional margin on which they could adjust. Following a positive aggregate shock, the firms could either reduce the number of destructions or increase the number of vacancy creations, whereas in the constant destructions model, they are only able to adjust the number of creations.

More precisely, three effects, going in opposite directions, impact the job creation condition when endogenous separations are introduced. Recall that this condition is given by equation (1):

$$\beta(1-\gamma) \left( \frac{p_t(x_h - R)}{1 - \beta(1-s)(1-\lambda)} \right) = \frac{cA}{\bar{\theta}_t}$$

The first two effects are related to the discounted profit of a vacancy creation (the left-hand side of this condition) while the third effect impacts the discounted cost of such a creation (the right-hand side).

Suppose this time there is a positive aggregate shock (a rise in $p$). The first consequence is a direct increase in the flow profit associated to a vacancy and then in the discounted profit. This effect, which obviously raises the number of vacancy creations and the job-finding rate, is present and quantitatively identical in both versions. It is all the more powerful as the value for $z$ is high. Remember that the value of $R$ is given by the job destruction condition:

$$p_t R = z + \beta \bar{\theta}_t^{1-\alpha} \gamma S(p_t x_h) - \beta(1-s) \lambda E_t \int_{R}^{x_h} S(p_{t+1} y) dG(y)$$

$R$ is increasing in $z$. As emphasized by Hagedorn and Manovskii (2008), what matters for the job finding volatility is the profit’s response in percentage terms, which requires that the profit be low in equilibrium. Hence, with a high value for $z$, $R$ is equally high, the equilibrium profit is low and even a small increase in $p$ implies a strong increase in the profit in proportion and then a sharp job-finding volatility.

The second effect is a reduction in the reservation productivity $R$, which increases the flow and discounted profit linked to a new job. The strength of this effect, only standing in the endogenous separations model, is also increasing in $z$. The mechanism is the same as for the first effect: with a high value for $z$, $R$ is high, the equilibrium profit is low and even a small decrease in $R$ entails a harsh rise in the discounted profit in proportion.

At this stage, the job-finding volatility is increasing in $z$ for both versions of the model but higher in the endogenous separations framework because of the second effect. Consequently, adding some cyclical behavior in the separation rate amplifies the job finding volatility. This amplification is furthermore rising in $z$ since the second effect depends positively on this parameter.

The third effect is related to the discounted cost of vacancy posting. In each version, unemployment falls following a rise in $p$, which entails an increase in the tightness, a decrease
in the probability for the firm to fill a vacancy and then an increase in the expected duration of vacancy posting. The resulting rise in the discounted cost of vacancy posting reduces the incentive to create jobs. This effect thus decreases the job finding volatility in both frameworks. Nevertheless, it is stronger in the endogenous separations model, since the fall in unemployment following the rise in \( p \) is higher than in the constant destructions model. This third effect then implies that introducing cyclical separations comes at the expense of the job finding volatility and illustrates the intuitive trade-off between the job finding and separation rates volatilities.

Whether endogenous separations amplify or reduce the job finding volatility depends on the relative strength of the second and third effects. The value of \( z \) is prominent in this comparison since the quantitative impact of the second effect crucially rests on this parameter while the third effect is independent from it. For low values for \( z \), the second effect is weak and should be dominated by the third effect. In this case, the job finding volatility is lower in the endogenous separations model than in the constant separations model, implying the intuitive trade-off between job finding and separation rates volatilities. Alternatively, with higher values for \( z \), the second effect is strong and should more than offset the third effect. The job-finding volatility is now amplified by endogenous separations.

3 Baseline calibration and sensitivity to the opportunity cost value

We first present and discuss the calibration that allows our framework to replicate the unemployment standard deviation with the right contributions for the job finding and separation rates. We next illustrate the critical role of the worker’s opportunity cost of employment in the separation rate volatility and in the amplification mechanism.

3.1 Baseline calibration

3.1.1 Calibration strategy

We follow the strategy initiated by Hagedorn and Manovskii (2008) to calibrate the value of the worker’s opportunity cost of employment: we determine this parameter endogenously to replicate the standard deviation of the job finding rate (0.0687). We obtain a value of 0.8685 for \( z \).

The remaining parameters are determined in a way which is common to other papers with cyclical separations (see for example Pissarides (2007) and Fujita and Ramey (2011)).
The aggregate productivity is simulated with an AR(1) process $p_t = \mu p_{t-1} + \sigma_p$, with $\mu_p = 0.9895$ and $\sigma_p = 0.0034$. We follow here the process chosen by Hagedorn and Manovskii (2008) which is close to other specifications that exist in the literature.

The discount factor is equal to $\beta = \frac{1}{1+r}$, with $r = 0.04/12$. This corresponds to an annual interest rate of 4%, which is consistent with the data.

The wage bargaining is assumed to be symmetric, an assumption often made in the literature. This implies a value for $\gamma$ of 0.5. The elasticity of the matching function $\alpha$ is set at 0.5, the mid-point of Petrongolo and Pissarides (2001).

The choice of the other parameters is determined by the empirical series of the unemployment, the job finding and the separation rates. The empirical series come from the CPS $^1$. We calculated the unemployment, the separation and the job finding rates for the period 1976 - 2010. These monthly data were seasonally adjusted and then HP filtered.

The efficiency parameter of the matching function $A$ is determined in order to replicate the empirical level of the job finding rate, that is 30.48%. The cost of posting a vacancy ensures that the job creation always holds at sample mean, for a value of the tightness equal to 0.72 at steady-state (see Pissarides (2007)).

When it comes to the separation process, we have three endogenous parameters: the probability of being hit by a shock at the micro-level $\lambda$, the exogenous part of the separation rate $s$ and the standard deviation $\sigma_x$ of the distribution function of the micro productivity. $G(x)$ is a log normal distribution function, of zero mean. The highest match-specific productivity $x_{h}$ is set to 1, following here a classical assumption of this literature (see Pissarides (2007) or Mortensen and Nagypál (2007)).

The exogenous separation rate $s$ is set in order to replicate the empirical value of the separation rate. $\lambda$ is chosen to reproduce the cross-correlation of the unemployment and the vacancies, that is to ensure that the Beveridge curve holds. The last parameter $\sigma_x$ is adjusted to replicate the empirical standard deviation of the separation rate (0.0525).

The following table summarizes all the parameters.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\mu_p$</td>
<td>0.9895</td>
</tr>
<tr>
<td>$\sigma_p$</td>
<td>0.0034</td>
</tr>
<tr>
<td>$\beta$</td>
<td>$\frac{1}{1+0.04/12}$</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>0.5</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>0.5</td>
</tr>
<tr>
<td>$A$</td>
<td></td>
</tr>
<tr>
<td>$s$</td>
<td></td>
</tr>
<tr>
<td>$\lambda$</td>
<td></td>
</tr>
<tr>
<td>$\sigma_x$</td>
<td></td>
</tr>
<tr>
<td>$x_{h}$</td>
<td>1</td>
</tr>
</tbody>
</table>

$^1$This data was constructed by Robert Shimer. For additional details, please see Shimer (2007) and his webpage http://sites.google.com/site/robertshimer/research/flows.
### Parameters

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Baseline calibration</th>
<th>Target</th>
</tr>
</thead>
<tbody>
<tr>
<td>$z$</td>
<td>0,8685</td>
<td>job finding standard deviation</td>
</tr>
<tr>
<td>$\mu_p$</td>
<td>0,9895</td>
<td>HM (2008)</td>
</tr>
<tr>
<td>$\sigma_p$</td>
<td>0,0034</td>
<td>HM (2008)</td>
</tr>
<tr>
<td>$\beta$</td>
<td>0,9967</td>
<td>annual interest rate of 4%</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>0,5</td>
<td>Petrongolo-Pissarides (2001)</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>0,5</td>
<td>symmetric bargaining</td>
</tr>
<tr>
<td>$\mu_x$</td>
<td>0</td>
<td>Ramey (2008)</td>
</tr>
<tr>
<td>$x_h$</td>
<td>1</td>
<td>Pissarides (2007)</td>
</tr>
<tr>
<td>$A$</td>
<td>0,3592</td>
<td>job finding rate</td>
</tr>
<tr>
<td>$c$</td>
<td>0,1363</td>
<td>job creation</td>
</tr>
<tr>
<td>$s$</td>
<td>0,0149</td>
<td>separation rate</td>
</tr>
<tr>
<td>$\lambda$</td>
<td>0,1</td>
<td>Beveridge curve</td>
</tr>
<tr>
<td>$\sigma_x$</td>
<td>0,0357</td>
<td>separation rate standard deviation</td>
</tr>
</tbody>
</table>

#### 3.1.2 Discussion of results

The next table give the standard deviations and the elasticity of each rate with respect to productivity for the baseline calibration:

<table>
<thead>
<tr>
<th></th>
<th>Unemployment</th>
<th>Job finding rate</th>
<th>Separation rate</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Standard Deviation</strong></td>
<td>Data</td>
<td>0,125</td>
<td>0,0687</td>
</tr>
<tr>
<td></td>
<td>Model</td>
<td>0,1174</td>
<td>0,0675</td>
</tr>
<tr>
<td><strong>Productivity</strong></td>
<td>Data</td>
<td>-5,914</td>
<td>3,786</td>
</tr>
<tr>
<td></td>
<td>Model</td>
<td>-6,142</td>
<td>4,8188</td>
</tr>
</tbody>
</table>

Two comments emerge from the previous table.

First, our baseline calibration does not imply any degree of real wage stickiness. The elasticity of the real wage with respect to productivity is equal to 0,96. Thus, this calibration makes the MP framework with cyclical separations able to solve the unemployment volatility puzzle without resting on a still debated real wage rigidity.

Secondly, the value of $z$ required to replicate all standard deviations and the elasticities is less extreme in the endogenous separations model than in the constant separations one. Recall that Hagedorn and Manovskii (2008) determined this value for the latter and found an opportunity cost at 0,955. At first sight, the lower value for the endogenous separations framework seems quite intuitive. When there are only constant separations, the standard
deviation of unemployment is entirely explained by the job finding rate and a high value of 
\( z \) is called for. Instead, for the model with cyclical separations, half of the unemployment 
volatility is explained by the separation rate and the target for the job finding standard 
deviation is lower, and so is the required value for \( z \). Nevertheless, remember that the 
value for \( z \) found by Hagedorn and Manovskii applies in the case of a strong real wage 
rigidity (\( \gamma = 0.05 \) in there calibration which implies a real wage-productivity elasticity of 
0.45) that increases the job finding response. On the contrary, there is no wage stickiness 
in our baseline calibration. Thus, even if the target for the job finding standard deviation 
is lower, the absence of real wage rigidity makes the response of the job finding sharply 
depending on \( z \). When there is no wage stickiness, the assertion that the required value for 
\( z \) is automatically lower in the endogenous framework is then not so obvious. We show in 
the next sub-section that the fact that \( z \) is indeed lower is the result of the amplification 
mechanism.

3.2 Sensitivity to the worker’s opportunity cost value

We now give some illustration of the crucial role of the worker’s opportunity cost of em-
ployment in both the separation rate standard deviation and the amplification mechanism.

3.2.1 The strong dependence of the separation rate volatility on \( z \)

We first determine the impact of the worker’s opportunity cost of employment on the 
separation rate volatility by maintaining \( \sigma_x \) at the level allowing to replicate the empirical 
separation rate standard deviation for the baseline \( z \) (i.e. \( \sigma_x = 0.0357 \) for \( z = 0.8685 \)).
We observe a high dependence of the separation rate volatility on the value of $z$. There is a striking similarity with the results obtained by Hagedorn and Manovskii (2008) for the job finding volatility: both the job finding and separation rates standard deviations are very sensitive to $z$ and strongly convex as $z$ approaches 1. The constant value for the separation rate standard deviation for $z$ below 0.6 means that the feedback and direct effects depicted in Section 2.2.1 cancel each other out. On the contrary, from $z = 0.6$ to $z$ near 1, the positive and convex relation traduces the domination of the feedback effect for this range of values.

Figure 2 now gives the relation when $\sigma_x$ is adjusted for each value of $z$ to get the separation rate standard deviation as close as possible to its empirical counterpart. Table 3 reports the corresponding values of $\sigma_x$. 

Figure 1 : Separation rate volatility - $\sigma_x$ constant.
Previous papers introducing cyclical separations in the MP framework commonly conveyed the idea that the empirical separation rate standard deviation is always attainable by simply adjusting $\sigma_x$. The main conclusion from Table 3 is that this idea is misleading. For values of $z$ below 0.7, we are not able to reproduce the right standard deviation, whatever the value of $\sigma_x$. It is interesting to note that the separation rate standard deviation for $z = 0.4$ is more than six times lower. A value for $z$ at 0.4 corresponds to an opportunity cost of employment for the worker that only includes unemployment benefits (see Shimer (2005)). This value for $z$ is often retained in the calibration of constant separation rates MP models.

The papers that introduce a separation margin usually calibrate $z$ around 0.7 that also includes the value of leisure forgone. Figure 2 illustrates that the values just above 0.7 are the first for which the adjustment of $\sigma_x$ allows to get the right separation rate volatility. This explains why the separation rate standard deviation seemed quite easy to reach in these papers.
3.2.2 The amplification in practice

A good way of illustrating how the amplification mechanism works and depends on $z$ is to compare the job finding standard deviations for, on the one hand, the MP framework with endogenous separations and on the other the framework with an exogenous and constant separation rate. Table 4 sums up the standard deviations of the job finding rate for each value of $z$.

In order to get the standard deviations of the job finding rate for the model with a constant separation rate, we set $s$ at 1.7% and $\lambda$ at 0. In this case, the match specific productivity is drawn at the first period and then does not change, because the probability to be hit by a match-specific productivity shock is now equal to zero. It means that the reservation productivity stays the same every period and that the separations are now completely exogenous.

Table 4: Job finding volatility.

<table>
<thead>
<tr>
<th>$z$</th>
<th>0.2</th>
<th>0.3</th>
<th>0.4</th>
<th>0.5</th>
<th>0.6</th>
<th>0.7</th>
<th>0.8</th>
<th>0.8685</th>
<th>0.9</th>
<th>0.955</th>
</tr>
</thead>
<tbody>
<tr>
<td>(1)</td>
<td>0.0094</td>
<td>0.0107</td>
<td>0.0131</td>
<td>0.0148</td>
<td>0.0191</td>
<td>0.0191</td>
<td>0.0241</td>
<td>0.0564</td>
<td>0.0785</td>
<td>0.2117</td>
</tr>
<tr>
<td>(2)</td>
<td>0.0095</td>
<td>0.0109</td>
<td>0.0136</td>
<td>0.0163</td>
<td>0.0222</td>
<td>0.0216</td>
<td>0.0279</td>
<td>0.0675</td>
<td>0.0930</td>
<td>0.255</td>
</tr>
<tr>
<td>(1) - (2)</td>
<td>0.0001</td>
<td>0.0002</td>
<td>0.0005</td>
<td>0.0015</td>
<td>0.0025</td>
<td>0.0038</td>
<td>0.007</td>
<td>0.0111</td>
<td>0.0145</td>
<td>0.0433</td>
</tr>
</tbody>
</table>

(1) : job finding volatility in the endogenous separation framework  
(2) : job finding volatility in the exogenous separation framework

For a worker’s opportunity cost below 0.5, the job finding standard deviations for the two frameworks are quite similar, meaning that the second and third effects depicted in Section 2.2.2 cancel each other out. For this range of values for $z$, introducing a cyclical behavior for the separations neither amplifies nor reduces the job finding volatility.

From $z = 0.5$, the job finding standard deviation is higher for the endogenous separations model than for the exogenous one. The second effect dominates the third effect. Introducing cyclical separations now amplifies the job finding response to aggregate productivity shocks. As it is shown in the table 4, this amplification is increasing and convex in the value of $z$. For $z = 0.5$, the standard deviations difference is only equal to 0.0015 while for $z = 0.8685$, this difference amounts to 0.0111, representing 20% volatility in more. As $z$ approaches 1, the amplification becomes considerable: at $z = 0.955$, the job finding standard deviation difference amounts to 0.0433, which is two thirds the empirical job finding standard deviation (0.0687).
4 Conclusion

In this paper, we have followed a calibration strategy close to Hagedorn and Manovskii (2008) by determining a value for the worker’s opportunity cost of employment that makes the MP framework with cyclical separations replicate the unemployment standard deviation and the right contributions of the job finding and separation rates. We find that this value for the opportunity cost is quite lower than the value required for the constant separation rate model. Moreover, the MP framework with cyclical separations solves the unemployment volatility puzzle with the symmetric Nash bargaining and no other mechanism added. It notably generates a sufficient amount of variability without resting on a real wage rigidity.

We also stress two interesting results. First, the separation rate standard deviation is an increasing and convex function of the opportunity cost. The separation rate volatility is then not driven only by the standard deviation of the idiosyncratic shock and not so obvious to reproduce. Secondly, for relatively high values of the opportunity cost, an amplification mechanism dominates the intuitive trade-off, making the job-finding volatility in the endogenous separations framework higher than in the constant separations model. The amplification is also an increasing and convex function of the opportunity cost and is at the source of the lower required value for the opportunity cost.
References


