

Equilibrium Intermediation and Resource Allocation With a Frictional Credit Market

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Abstract

We model an economy where financial intermediation is subject to search frictions. The economy may reduce the extent of these frictions by devoting human resources to intermediation.

More efficient credit markets (i.e., with less frictions) conduct to more efficient product markets via larger and more efficient firms. They are also conducive to a smaller size of the financial sector, as less resources need to be devoted to channeling funds between lenders and borrowers.

Moreover, we show that the amount of resources devoted to intermediation along the growth path is unaffected by the relative abundance of capital.

In contrast, improvements in the allocative efficiency in the product market produce a larger financial sector.

In a Solow growth version of the model, more efficient credit markets are associated with higher steady state incomes and capital, more demanding selection of firms and, thus, more efficient aggregate production and more homogeneous firms. Outside steady state, the rate of growth for any given capital level is larger the more efficient the financial sector.

In an AK version of the model, long run growth is decreasing in the amount of frictions in the credit market.

JEL Classification: G1, G24, E51, L11, L16

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1 Introduction

Motivation

Recent years have experienced a renewed and increasing interest in the implementation of search environments for the modeling of the non-Walrasian features of the credit and investment markets (Wasmer and Weil (2004), Silvera and Wright (2010), Wang, Besci, and Li (2005), den Haan, Ramey, and Watson (2003), Dell’Ariccia and Garibaldi (2005)).¹ Following Diamond (1990), this literature highlights, in an encompassing manner, the quantitative importance of information frictions, time usage, and a positive value of establishing creditor-borrower relationships. Implications of these models fit the data well (Dell’Ariccia and Garibaldi (2005), Petersen and Rajan (2002)). Differently from the existing literature on General Equilibrium with frictions in credit markets, in this article we allow for growth and impose aggregate resource constraints on **(1)** the amount of capital that can (and will) be used, and **(2)** the human resources that can be employed alternatively for financial intermediation or directly productive activities.

We believe that by doing so we improve on the traditional Mortensen and Pissarides (1994) framework that, when applied to financial intermediation issues, demands for an infinitely elastic supply of both firms and capital. In particular, in our context society endogenously determines the severeness of frictions. This is, by devoting more human resources into intermediation (sacrificing them from directly productive activities), finding finance becomes less of an obstacle for entrepreneurs, but doing so comes at a cost.

Furthermore, in our framework we are able to explore the bidirectional relationship between the finance and production sectors. On one hand, akin to earlier literature but modeling explicitly the intensive margin of production, we study how frictions and contractual inefficiencies in the credit market affect the minimum productivity requirement that projects need to possess in order to be financed. In this manner we connect within-industry mean and dispersion of firm’s size and productivity with the amount of frictions in the financial sector. On the other hand, having a notion of aggregate constraints allows

¹Not yet published papers include: Dell’Ariccia and Garibaldi (1998), Wang, Besci, and Li (2009), Silvera and Wright (2007), Jonathan and Koepl (2011).

us to study how inefficiencies in product markets affect the workings and relative size of the finance industry.

Finally, we show that the growth version of the model is compatible with qualitatively observed data: richer countries have more efficient financial institutions and more demanding firm selection, resulting in more homogeneous dispersion of firm productivities.

Implementation

For doing all this, we separate deposit from investment markets. Capital, being an homogeneous good, demands to have an homogeneous rental price. In our framework a Walrasian market for deposits serves this purpose.² Investments, on the other hand, are heterogeneous in their expected returns. The different rents of capital obtained by investing in heterogenous projects should be attributed not to capital ownership, but to the ability and effort devoted to finding and identifying quality investments.³ We deem as brokers the individuals who endeavor in such an activity. For the broker and the entrepreneur (the original owner of the investment opportunity) finding each other is a time-consuming activity, which we perceive natural to model using search theory.⁴

Results

Using our model we show: **(1)** that the share of GDP devoted to financial intermediation depends negatively on **(1.i)** the relative bargaining position of entrepreneurs (versus brokers), **(1.ii)** on how easy it is to find quality projects, and **(1.iii)** on the degree of inefficiency in the workings of the product market.

(2) Under reasonable conditions⁵, which we characterize, the relative abundance of

²The homogeneous nature of physical capital (machines) makes natural our assumption that all capital is used at all times.

³In certain occasions, the owners of capital may well coincide with the brokers, as is the case e.g. for some venture capitalists. This case is thoroughly studied by Silvera and Wright (2007).

⁴Blanchflower and Oswald (1998) report that raising funds is a principal obstacle to potential entrepreneurs. Further empirical support for this claim is provided by Evans and Jovanovic (1989), Evans and Leighton (1989), Holtz-Eakin, Joulfaian, and Rosen (1994), Gentry and Hubbard (2000), and Guiso, Sapienza, and Zingales (2004).

⁵These include (but are not restricted to) Cobb Douglas production functions and Dixit-Stiglitz preferences.

capital is irrelevant (even in the short run) for all allocative purposes. We show that **(2.i)** the productivity threshold for active firms, and **(2.i)** the distribution of agents between productive and financial activities are determined independently of the interest rate, and thus of capital.

This result is not only robust but (in our opinion) both surprising and useful. It is surprising because, at least in a context with fixed exogenous capital levels, one could have expected that very abundant capital would yield less stringent demands on product quality.⁶

Next, we show that **(3)** there are two ways in which financial markets may be inefficient, and how this affects product market outcomes. First, **(3.i)** less frictions in the investment market (i.e., being closer to a Walrasian environment) induces larger, more productive, and more homogeneous firms. Additionally, **(3.ii)** from an efficiency point of view the bargaining power of brokers can be either too large or too small. If it is too large, the share of GDP into intermediation is excessive, as is the speed at which entrepreneurs find finance: too many brokers provide too much liquidity, and there is an inefficiently small number of agents involved in directly productive activities.

(4) We look at growth both in a Neoclassical Solow-style framework and in an AK model. In a Solow growth version of the model, **(4.i)** more efficient financial markets are associated with higher steady state incomes and capital, more demanding selection of firms and, thus, more efficient aggregate production and more homogeneous firms. **(4.ii)** Outside steady state, the rate of growth for any given capital level, is larger the more efficient the financial sector is. Finally, **(4.iii)** in an AK version of the model less frictional credit markets generate higher growth rates.

⁶Most, if not all, of the literature on credit market search has focused on risk neutral agents and allowed capital to be determined in equilibrium once it is imposed that the interest rate equals the discount rate. As detailed below, albeit we assume risk neutrality for simplicity, we do not allow capital to jump or otherwise adjust instantaneously due to demand. In steady state with Ramsey-type consumers, both things are similar (albeit not identical), but given the capital adjustment restrictions, along the transition path they are not.

Literature

Our analysis relates to two several strands in the literature. First, a small number of papers studies the endogenous share of resources devoted to financial intermediation. In contrast to our model, however, these models do not consider time as a factor of production for new entrepreneurs, which we believe to be empirically relevant. Moreover, none of these papers studies the bidirectional link between efficiency in product and credit markets.

Second, we link credit market frictions to recent empirical findings on greater within-industry productivity dispersion and lower aggregate TFP in poorer countries (Hsieh and Klenow (2009)).⁷ These distortions matter: Hsieh and Klenow (2009) conclude that manufacturing TFP would increase by 30-50% in China and 40-60% in India if these gaps were reduced to the observed levels in the United States.⁸

While misallocation distortions may stem from a wide array of distorted prices faced by individual producers (as in Restuccia and Rogerson (2008)), we highlight credit frictions for prospective entrepreneurs as a specific source of allocative inefficiency. Recognizing credit frictions as a barrier to entry may potentially be useful in guiding empirical work, which depends on inferring misallocation distortions from existing firms (by measuring the residuals in first-order conditions, see Chari, Kehoe, and McGratten (2007)). Notably, the credit search approach may further be useful in this context to interpret heterogeneity in financing costs because search and bargaining implies that individual companies may face different financing costs; but that these differences need not be thought of as being due to non-economic factors. The approach may also be useful in theoretical applications where fixed costs play an important allocative role (e.g. Melitz (2003)-type models). Here, these costs are explicitly costs for obtaining credit; however, they are endogenous and respond to changes in the environment. In this manner, the fixed-cost nature of establishing a creditor-borrower relationship is how reducing credit frictions may be conducive towards more efficient product market outcomes. As such, our view is consistent with the broader

⁷Hsieh and Klenow (2009) measure the degree of misallocation by the size of gaps in marginal products of labor and capital across plants within narrowly defined industries.

⁸Cf. Restuccia and Rogerson (2008), Alfaro, Charlton, and Kanczuk (2008), and Bartelsman, Haltiwanger, and Scarpetta (2008).

view that the credit sector, by mobilizing savings, allocating resources, and screening projects, plays a crucial role in shaping the development process of new products, firms, and sectors (Greenwood and Jovanovic (1990), Levine (1997), and Matsuyama (2007)).

We contribute to the analysis of aggregate consequences of financial frictions, surveyed by Matsuyama (2007). Li and Sarte (2003) provide evidence that changes in intermediation costs directly affect output. Cooley, Marimon, and Quadrini (2004) show that limited financial contract enforceability amplifies the impacts of technological innovations on aggregate output.⁹ More recently, Russ and Valderrama (2009) exploit the relative costs of bank and bond financing to explain how bank lending frictions may affect the firm size distribution through intra-industry reallocations. Here we instead exploit the value of creditor–borrower relationships.

The present paper is organized as follows. Section 2 describes the basic model employed in our analysis. Section 3 solves the model with an exogenous capital supply. Section 4 does comparative statics with respect to the efficiency of the financial sector (4.1), the rate at which firms are destroyed (4.2), the bargaining power of entrepreneurs (4.3) and the degree of efficiency of the product market (4.4). Sections 5.1 and 5.1 show examples of the basics of the models working in the contexts of (respectively) a Solow-Swan accumulation and AK growth model. Finally, section 8, concludes.

2 General Environment

The economy is populated by a mass one of agents who live in continuous time, and die at an exogenous rate δ . There is a flow of new arrivals that keep the population constant. Agents are all identical and can opt at any moment between two economic activities. They are either “entrepreneurs” or “brokers”.

All production takes place within firms that use only capital and entrepreneurial activity as inputs.¹⁰ Albeit all agents (and thus, all entrepreneurs) are identical, the projects differ in their productivities. Let a be an indicator of the productivity of projects, which is

⁹Cagetti and Nardi (2006) and Quadrini (2000) show that models with financial frictions and entrepreneurship may explain observed distributions of wealth.

¹⁰It would be trivial to extend the model to have labor as an additional input, without adding insights.

drawn from a distribution $G(a)$ (details below). A project with productivity a and using an amount of capital k generates a stream of income $F(k, a, Y)$, where $F_k(k; a, Y) > 0$, $F_a(k; a, Y) > 0$, $F_{ka}(k; a, Y) > 0$ and $F_{kk}(k; a, Y) < 0$, and Y measure of aggregate demand or market size. We may well have $F_Y(k; a, Y) = 0$, which would be the standard neoclassical assumption. We include Y for the sake of generality, and because it is what appears if agents have Dixit-Stiglitz preferences in a monopolistic competition environment, a framework that we will use later on.

Each unit of capital will obtain a flow rent r (to be determined in equilibrium) when used in firms. The flow of profits generated by a project is then $\pi(a, r, Y) = \max_k \{F(k, a, Y) - rk\}$, and the capital demanded by a project of productivity a if the return to capital is r is $k^d(a, r)$.

Furthermore, we will assume that $F(k, a, Y)$ is log linear in k , a , and Y . Thus, profits are a log linear function of r , a , and Y , and the ratio of capital income to profits is a constant:

$$\pi(a, r, Y) = (1 - e_k) e_k^{\frac{e_k}{1-e_k}} a^{\frac{e_a}{1-e_k}} r^{-\frac{e_k}{1-e_k}} Y^{\frac{e_y}{1-e_k}} \quad (1)$$

$$\frac{rk^d(a, r, Y)}{\pi(a, r, Y)} = \frac{e_k}{1 - e_k} \quad (2)$$

Where e_k , e_a and e_y are the (constant) elasticities of $F()$ with respect to k , a and aggregate demand respectively. $e_k \in [0, 1]$, $e_y \in [0, 1]$ and $e_a \in \mathfrak{R}^+$.

Examples of log-linear $F(k; a, Y)$ are: **(1)** a Dixit-Stiglitz economy, where $F(k; a, Y) = \varepsilon \left(\frac{a}{r}\right)^{\varepsilon-1} Y$ (with ε a measure of the elasticity of substitution between any pair of horizontally differentiated consumer varieties and $\varepsilon \equiv (\varepsilon-1)\varepsilon^{-1}\varepsilon^{-\varepsilon}$), or **(2)** a competitive economy with decreasing returns, where $F(k; a) = A(ak)^\alpha$ (where now α and A are measures of decreasing returns and aggregate productivity, respectively).

The project generates income while the entrepreneur is alive, and disappears when she dies. The capital does not disappear upon the death of the entrepreneur.

Let \bar{k} be the total amount of capital that is (inelastically) supplied. In terms of this section capital is owned by capital owners who are left unmodelled. In sections 5.1 and

5.1 we look at the dynamics of capital determination and allow capital to be owned by the agents, but we do not allow them to invest their own capital in their own project. We do this because we want to abstract from issues that arise from structure of the joint distribution of capital and talent in a context of financial market imperfections. It is not that we do not consider it interesting, very much the opposite, but we are sacrificing this for the sake of simplicity. We plan to study this issue in further work.

2.1 Deposit and Investment Markets

If capital markets were Walrasian, this would be a boring model. All firms would produce at the maximum possible productivity, all individuals would be “entrepreneurs”, and the interest rate would be determined to equalize aggregate capital demand to \bar{k} . We depart from this paradigm by introducing search frictions in the allocation of capital to projects. We do so by dividing the capital market in two separate markets, interconnected by brokerage services.

On one hand there is a Walrasian “deposit market” where capital offered by its owners in exchange for a market return r . There is also a “investment market” where entrepreneurs obtain the capital that they use in production. These markets can be thought as two different rooms, which differ along two dimensions.

(1) They have different actors, as entrepreneurs are barred from entering the “deposit” room, while capital owners are barred from the “investment” room. There is a third type of agents (“brokers”) that breach the gap between the markets. Being able of acting in both rooms, they demand capital in the deposit room, and they supply it in the investment room.

(2) The deposit market is frictionless, while the investment market is characterized by search frictions. Thus, in the “*deposit room*” all agents have instantaneous access to all other participants. This insures that capital (an homogeneous commodity) will be paid a rental price r per unit of time determined which is in equilibrium.

The “*investment room*” is considerably more interesting, as in there it takes time to find a partner. The larger the ratio of brokers to entrepreneurs searching for financing,

the faster that entrepreneurs gain access to capital, but for both sides (entrepreneurs and brokers) to meet each other takes (an endogenous amount of) time. The more resources that society devotes to intermediation (as brokers can not produce output), the faster that investment opportunities will find capital to fulfill them.

With this we try to capture the double nature of capital markets. On the one hand, financial capital is a rather homogeneous good from the point of view of its owners. Thus, the return of capital can not differ between them. But, on the other hand, the return of capital might be different in different investment possibilities, and the owners of these investment possibilities do not have immediate access to capital. All capital is used all the time, but not all investment opportunities are used at any given moment of time. Their owners invest time looking for capital, and a certain number of individuals invest their time in connecting them with the capital being left free by projects that died away.

It does not strike to us as outrageous to assume that “*the investment market*” is subject to frictions. This can be a metaphor of either the slow search process for partners who are able to attest the validity of the investment project, or the possibility of finding contractual solutions to informational asymmetries or moral hazard problems. As a matter of fact, entrepreneurs spend time looking for finance, and there exist a sector of the economy whose task is to intermediate between owners and users of capital.

The extra income that capital generates in a high productivity project (versus a lower productivity one) should not be attributed to a capital rent. It should be attributed to (1) the talent to generate the high productivity,(2) the talent to recognize that its productivity is high, and (3) the luck of being there and being able to access the capital that the entrepreneur needs, being the only one who is in such position. In our case brokers get a rent for intermediating and allowing capital to flow to its uses. They find themselves in a bilateral monopoly position vis a vis the entrepreneur in the match.

Thus, society determines the degree of frictions in the investment market by devoting more or less resources into intermediation: the larger the number of individuals who opt for financial intermediation, the smaller the number of entrepreneurs. Our paper is about the determination of how many resources are used in intermediation versus

directly in production. We emphasize the human resource constraint that society faces: more resources into intermediation imply less resources in production, but they facilitate production by bringing capital closer to its productive use.

2.2 Agents, Matching and Bargaining

The speed at which entrepreneurs meet brokers is determined by a matching function with constant returns to scale in the masses of searching entrepreneurs and brokers. The tightness of the capital market (θ) is defined as the ratio of the mass of entrepreneurs who are searching for finance to the mass of brokers. Notice that the numerator is smaller than the total mass of entrepreneurs, as some entrepreneurs already found finance and are producing. We denote by $p(\theta, \nu)$ to the function determining the rate at which entrepreneurs meet brokers, which is decreasing in θ . ν is a shift parameter denoting the general efficiency of the matching process; with $\frac{\partial p(\theta, \nu)}{\partial \nu} > 0$ and $\lim_{\nu \rightarrow \infty} p(\theta, \nu) = \infty$. A walrasian investment market would be characterized by $\nu \rightarrow \infty$. The rate at which brokers meet entrepreneurs is $\theta p(\theta, \nu)$, which grows with tightness.

2.2.1 Entrepreneurs

Entrepreneurs are in one of two states. They either search for a broker, or they produce in their firm. While they search for a broker they get no income. While they are producing they get income equal to their share (to be determined below) of $\pi(a, r)$

a is the productivity of their investment project. Which for simplicity we assume that they find instantaneously.¹¹ The distribution $G(a)$ from which a is drawn is known by all agents, but not the realization of a .

Entrepreneurs are unaware of the value of a before they meet the broker, which will inform them on its value. It is not important for us that they are *completely* unaware of the value of a , but given our assumption that they find a instantaneously, it simplifies matters substantially. We could as well allow them to have a signal on its quality. What we want is to capture that there are bad projects in the environment searching for credit, producing

¹¹ We could as well have assume the existence of a further state where entrepreneurs search for projects, but this would have given no further insight

congestion that affects negatively the good projects. To have completely uninformed entrepreneurs is the simplest way of modeling it.

Once an entrepreneur has met a broker, if they decide to go ahead with the project (more on it below), the entrepreneur will spend the rest of her life producing. She will get a constant flow of income until the moment of her death. At that time the capital will go back to its owners.

If the entrepreneur does not arrive to an agreement for production with the broker, both the project and the match are destroyed. This is, the entrepreneur can not look for another broker with the knowledge of the value of a (the project is destroyed). Also, the entrepreneur can not instantaneously draw another project and match with the broker she has just found (the match is destroyed). Project destruction simplifies bargaining considerably, while match destruction is consubstantial to having a proper search environment.

Not all values of a are going to be employed in production. If productivity is very low, the value of start searching for another project (in practice, for another broker) is larger than the value of producing. We will show that there exists a unique equilibrium threshold of productivities b such that the project is financed if and only if $a \geq b$.

2.2.2 Brokers

Brokers have the ability to access the deposit room, in order to provide funds for investment projects, but they can not create investment opportunities by themselves. They can always choose to become an entrepreneur, but in such a case they would need to find a broker. Even if it is costless to change occupations, agents can not be both entrepreneur and broker at the same time. That is what the notion of financial frictions is all about.

We assume that brokers can have relationships with multiple entrepreneurs without satiation. Their life is as follows: They search in the investment room for investment projects, which they find at a rate $\theta p(\theta, \nu)$. Once they find a partner with a productivity judged to be sufficiently large in order to go ahead with production, they bargain over the share of the lifetime output generated by the project that they get. We can think of

this as a once and for all payment. There is no further involvement of the broker in the business of *this* entrepreneur. After the transaction is over, the broker looks for another partner. For all practical purposes, being no different than it was before the meeting: there is no change of state for brokers. They *always* search. They do not care about how many transactions they have completed because they are risk neutral and they only look into the future. Their continuation value is constant.

2.2.3 Bargaining

Matched entrepreneurs and brokers are in a bilateral monopoly. We assume that they split the income from the project according to Nash bargaining. This implies that agents agree on choosing the efficient amount of capital (demanded by the entrepreneur and extracted from the deposit market by the broker). The entrepreneur's bargaining weight is $\beta \in (0, 1)$.

The outside options of broker and entrepreneur are very different. For the broker is zero, as she has no satiation in the number of entrepreneurs he can serve and her continuation value is constant.

The entrepreneur, on the other hand, commits her life into their project. If she starts looking for another project, no production could take place with this particular project. Thus, given assumed impossibility of using the process with another broker, what the entrepreneur gives up by accepting a proposal is the possibility of start searching for another project.

Essentially, thus, *ex ante*, the broker and the entrepreneur bargain over a productivity schedule that applies in the case of agreement, whereas the respective continuation values apply in the case of disagreement.

Notice that our assumption that the project has to be either used in the match or destroyed, is identical to assuming that they bargaining before knowing the productivity of the project on a an schedule of payments on which both sides commit.

3 Exogenous Capital Supply

In this section we solve the model assuming an exogenous and constant level of capital \bar{k} , taken as exogenous. In the next section we will look at the dynamic determination of capital.

3.1 Bellman Equations

Let's call V_0 and $V_1(a)$ to the values of an entrepreneur looking for capital, and one that is producing in a project with productivity a respectively.

Entrepreneurs looking for a broker have no flow of income, and do not know the value of their a . They meet brokers at a rate $p(\theta, \nu)$, and the minimum project productivity is b . Thus, their value function is just:

$$\delta V_0 = p(\theta, \nu) \int_b^\infty [V_1(a) - V_0] dG(a) \quad (3)$$

At the production stage they produce, sell, and pay annuities to the broker. Thus, their value is determined by:

$$\delta V_1(a, r, Y) = \pi(a, r, Y) - \rho(a, r, Y) \quad (4)$$

where $\rho(a, r)$ is the annuity of the payment made to the broker (to be determined in equilibrium).

Putting together equations (3) and (4):

$$\delta V_0 = \frac{p(\theta, \nu) [1 - G(b)]}{\delta + p(\theta, \nu) [1 - G(b)]} \int_b^\infty [\pi(a, r, Y) - \rho(a, r, Y)] \frac{dG(a)}{1 - G(b)} \quad (5)$$

The broker gets paid whenever she meets an entrepreneur with a productivity high enough, and we assume that the flow values of past engagements are perfectly ensured on a broker insurance market.

Thus the continuation value of being a broker (B) solves:

$$\delta B = \theta p(\theta, \nu) \int_b^\infty \Gamma(a) dG(a), \quad (6)$$

with $\Gamma(a) = \frac{\rho(a, r, Y)}{\delta}$.

3.2 Bargaining

Calling $S(a)$ to the total surplus generated by the match, the payments ρ supported by Nash bargaining are such that:

$$\begin{aligned} \beta S(a) &= V_1(a) - V_0 \\ (1 - \beta) S(a) &= \Gamma(a) \end{aligned}$$

This gives payment:

$$\rho(a, r, Y) = (1 - \beta) \{ \pi(a, r, Y) - \delta V_0 \} \quad (7)$$

with the value of an entrepreneur without a match being:

$$\delta V_0 = \frac{p(\theta, \nu) [1 - G(b)]}{\delta + p(\theta, \nu) [1 - G(b)]} \times \frac{\frac{\beta}{1 - \beta}}{\frac{\beta}{1 - \beta} + \frac{\delta}{\delta + p(\theta, \nu) [1 - G(b)]}} \times \int_b^\infty \pi(a, r, Y) \frac{dG(b)}{1 - G(b)} \quad (8)$$

It is worth looking at the components of equation 8: $\left(\int_b^\infty \pi(a, r, Y) \frac{dG(b)}{1 - G(b)} \right)$ is the expected flow of income *per period* generated by a project that is accepted. $\left(\frac{\frac{\beta}{1 - \beta}}{\frac{\beta}{1 - \beta} + \frac{\delta}{\delta + p(\theta, \nu) [1 - G(b)]}} \right)$ is the share of this income that the entrepreneur gets. This share increases with her bargaining power β and decreases with the percentage of time that she spend searching $\left(\frac{\delta}{\delta + p(\theta, \nu) [1 - G(b)]} \right)$. Thus, the product

$$\frac{\frac{\beta}{1 - \beta}}{\frac{\beta}{1 - \beta} + \frac{\delta}{\delta + p(\theta, \nu) [1 - G(b)]}} \times \int_b^\infty \pi(a, r, Y) \frac{dG(b)}{1 - G(b)}$$

is the expected income that an entrepreneur gets when their projects get accepted. Then,

given that $p(\theta, \nu)[1 - G(b)]$ is the rate at which entrepreneurs get projects that are accepted, it is easy to see that δV_0 is the present discounted value of the future income that they get.

The value of a broker can be rewritten (with equivalent interpretation) as:

$$\delta B = \frac{\theta p(\theta, \nu)[1 - G(b)]}{\delta + \theta p(\theta, \nu)[1 - G(b)]} \frac{\frac{1-\beta}{\beta}}{\frac{1-\beta}{\beta} + \frac{\delta}{\delta + \theta p(\theta, \nu)[1 - G(b)]}} \times \int_b^\infty \pi(a, r, Y) \frac{dG(b)}{1 - G(b)} \quad (9)$$

3.3 Equilibrium Conditions

Define m as the number of entrepreneurs in the economy. Then, the set of endogenous variables is $\{\theta, m, r, b, Y\}$. Their value has to be such that the following conditions hold

3.3.1 No Arbitrage between professions

Agents, being homogeneous, need to be indifferent between professions. Otherwise there would be possibility of arbitrage. Thus, in equilibrium $V_0 = B$.

From 8 and 9 it follows that:

Result 1 *No arbitrage between professions implies that credit market tightness equals the ratio of bargaining powers:*

$$\theta = \frac{\beta}{1 - \beta} \quad (10)$$

This expression for the liquidity in investment markets is akin to the one encountered by Wasmer and Weil (2004) in an environment with homogeneous projects. Notice that for bargaining purposes our firms *are also homogeneous*. This is, we assume that if there is no agreement it is as if they never met. There is no effect of the value of the current a in the outside possibility of any agent. Thus, it is just the expected future values that matter, and in the same manner for both agents. The shares that they get from the surplus need to adjust to make both professions equally attractive. As a consequence, θ does not depend on the degree of frictions in the credit market or of inefficiencies in the product market, which simplifies the analysis enormously.¹²

¹²This would not happen if there were no commitment at the bargaining stage. This is, if the entrepreneur could (out of equilibrium) refuse to go ahead with the match and search for another partner

The intuition of the result is straight forward. The more bargaining power entrepreneurs have (β), the more attractive entrepreneurial activities are. Longer search periods without production (less liquidity) are necessary to equalize value across activities. The decrease in liquidity comes across as a consequence of an increase in the ratio of searching entrepreneurs to brokers.

Notice that this increase may take place either by increasing the number of brokers (the amount of resources that society devotes to financial activities) or by increasing the threshold of productivity (which would result in more rejected projects and a longer expected search period for entrepreneurs).

3.3.2 Human resource constraint: Size of financial sector

All agents have to be employed. Either as brokers (in the denominator of θ), or as entrepreneurs without a match (in the numerator of θ), or as entrepreneurs running their business, and using their desired capital. It is straight forward to prove that in steady state the share of entrepreneurs who are *not* in a match (and are thus searching) is $\frac{\delta}{\delta + p(\theta, \nu)[1 - G(b)]}$

Thus, tightness in the investment market (the ratio of searching entrepreneurs to brokers) is

$$\theta = \frac{\delta}{\delta + p(\theta) [1 - G(b)]} \frac{m}{1 - m}$$

Given that from result 1 we know the value of θ , and the fact that all individuals are equal (the average income of entrepreneurs and brokers are their values, which are identical), it follows that the share of finance in GDP ($1 - m$) is:

$$\begin{aligned} 1 - m &= (1 - \beta) \frac{\delta}{\delta + \beta p(\theta, \nu)[1 - G(b)]} \\ &= (1 - \beta) \left[1 - \frac{p(\theta, \nu) [1 - G(b)]}{\delta + p(\theta, \nu) [1 - G(b)]} \times \frac{\frac{\beta}{1 - \beta}}{\frac{\beta}{1 - \beta} + \frac{\delta}{\delta + p(\theta, \nu)[1 - G(b)]}} \right] \end{aligned} \quad (11)$$

Notice that this implies that more human resources devoted to financial activities

but knowing now the value of a , their outside options would be different, and in general θ would depend on b and (perhaps) r . We believe that to assume away this possibility is worth the cost, as the model becomes so much more tractable.

imply (ceteris paribus) a larger threshold productivity b , and vice versa. The reason is as follows. The waiting time for entrepreneurs is determined by arbitrage (result 1), if b increases, the number of rejections also increases, which means that the share of *searching* entrepreneurs also increases, which demands of an increase in the size of the financial sector. A larger financial sector allows society to be more selective in the quality of the projects. While the resources devoted to finance produce do not produce output directly, they allow to improve the productivity of firms by reducing the opportunity cost of searching for a better project.

3.3.3 Individually optimal search rule: productivity threshold.

Optimal Search. The productivity threshold b is such that it is not rational *for the individuals in the match* to break the match and look for better projects if $a \geq b$, and it is rational to do so otherwise: $S(b) = 0$.

Agents in a match go ahead with a project if and only if the match has positive value. For the marginal match, the value is zero, i.e. $S(b) = 0$, which given that the continuation value of the broker is independent of events in this match, becomes: $\delta V_1(b) = \delta V_0$.

We can re-write the value of an entrepreneur within a match as:

$$\delta V_1(a) = \delta V_0 + \beta [\pi(a, r, Y) - \delta V_0] \quad (12)$$

which is quite intuitive. The larger the bargaining power of brokers and the larger the income generated by this specific project, the more than they get with respect to what they would get if they decide to start all over and go back into search. Thus, it is clear that projects will be accepted if and only if the stream of profits that they generate are larger than the value of going back into search. From where it follows that b is such that $\pi(b, r, Y) = \delta V_0$.

Using 8 we can then rewrite this condition as:

$$\frac{\pi(b, r, Y)}{\int_b^\infty \pi(a, r, Y) \frac{dG(b)}{1-G(b)}} = \frac{p(\theta, \nu) [1 - G(b)]}{\delta + p(\theta, \nu) [1 - G(b)]} \times \frac{\frac{\beta}{1-\beta}}{\frac{\beta}{1-\beta} + \frac{\delta}{\delta + p(\theta, \nu) [1 - G(b)]}} \quad (13)$$

Notice that the RHS of 13 lies in the interval $[0, 1]$ and equals the present discounted value of the share of the income that goes to the entrepreneurs. It grows with the rate at which they get finance for suitable projects because (1) this brings forwards the rewards and (2) increases their share of the income generated by improving their outside option. Thus, it is decreasing in the value of the threshold b , and equals zero as it approaches its upper limit.

The LHS of 13 is the ratio of marginal to average profits. Using 1 this is a function of b which is independent of both r and Y . We will deem such a function as $H(a, \epsilon)$:

$$H(b, \epsilon) \equiv \frac{\pi(b, r, Y)}{\int_b^\infty \pi(a, r, Y) \frac{dG(a)}{1-G(b)}} = \frac{(b)^\epsilon}{\int_b^\infty (a)^\epsilon \frac{dG(a)}{1-G(b)}} \in (0, 1) \quad (14)$$

where ϵ is the elasticity of profits to a : $\epsilon = \frac{e_a}{1-e_k}$

It is intuitive to expect $H(b, \epsilon)$ to be a non-decreasing function of b . If a were bounded by above at value \bar{a} , it is certain that the ratio would not be decreasing in a neighborhood of \bar{a} . Its maximum value is 1, and it is achieved as a approaches the limit. Thus, the following assumption:

Assumption 1 *The distribution of a is such that the ratio of marginal to average profit (H) is a non-decreasing function of b : $\frac{\partial H(b, \epsilon)}{\partial b} \geq 0$*

Notice that assumption 1 is a restriction on the distribution of productivities. It is by no means a stringent assumption. Not only because it is clearly intuitive, but also because it holds for many (if not all) of the commonly used distributions. For instance, if a is bounded and uniformly distributed, it is the case. More interestingly, if a is distributed with a Pareto, this takes the peculiar form of $H(b, \epsilon)$ being independent of b , which facilitates algebra enormously. From now on we will always assume that assumption 1 holds.

3.3.4 Capital Market Clearing. Determination of r

All capital needs to be used, as the deposit market is Walrasian. Additionally, all matches broker-entrepreneur share surplus according to Nash, which itself implies that capital is

chosen optimally within each match. Thus, total capital demand will be the summation of $k^d(r, a)$ over all active firms. Calling it K^d , all capital is used if and only if $K^d(r, b, m) = \bar{k}$.

All capital receives the same remuneration r , which is fixed by the Walrasian Auctioneer of the Deposit Room in order to equal capital supply \bar{k} with capital demand. Demand comes from all active firms. Each of them demands according to productivities $k^d(a, r)$. Thus:

$$\frac{p(\theta) [1 - G(b)]}{\delta + p(\theta) [1 - G(b)]} m \int_b^\infty k^d(a, r) \frac{dG(a)}{1 - G(b)} = \bar{k}$$

where the LHS is capital demand, the sum of all capital demanded by projects that are in production (m times the percentage of projects that find finance times their average capital demand). Multiplying both sides by r , and using 2, we can rewrite this as:

$$\frac{p(\theta) [1 - G(b)]}{\delta + p(\theta) [1 - G(b)]} m \int_b^\infty \frac{e_k}{1 - e_k} \pi(a, r, Y) \frac{dG(a)}{1 - G(b)} = r\bar{k} \quad (15)$$

The Walrasian nature of the deposit market insures that this condition will instantaneously hold. In steady state the value of capital demand is straight forward. It gets a bit more complicated when capital is growing, but we delay this discussion to section 5.1.

3.3.5 Output determination

In a neoclassical set-up (π being independent of Y) this is just a residual. Otherwise it feed backs into the other endogenous variables. The determination of output (which, because \bar{k} is constant equals aggregate consumption expenditures) is as follows. First, output equals the summation of capital income ($r\bar{k}$) and the income generated by the agents. Second, agents discount the future at the same rate that they die and are replaced, and that all of them are equal at birth. Thus, it follows that the average income across all agent needs to equal expected lifetime present discount income of any of them.

It is easy to see (after some manipulation) that the value at birth for any agent is:

$$\delta V_0 = \frac{\beta p(\theta, \nu) [1 - G(b)]}{\delta + \beta p(\theta, \nu) [1 - G(b)]} \int_b^\infty \pi(a, r) \frac{dG(a)}{1 - G(b)} \quad (16)$$

Now, using equation (18), it is straightforward to see that the average lifetime income

equals the annuity of the profit of the marginal firm: $\delta V_0 = \pi(b, r, Y)$

Sometimes an individual gets no income, and sometimes she gets a positive income (her share of π). The average equals the marginal profit. Given that there is a mass one of agents, it follows that GDP equals:

$$Y = r\bar{k} + \frac{1}{\delta}\pi(b, r, Y) \quad (17)$$

3.4 Equilibrium Characterization and Solution

An equilibrium is then a vector $\{\theta, m, r, b, Y\}$ such that equations 10, 11, 13, 15 and 17 hold.

The solution algorithm is extremely simple. As we have seen, arbitrage between professions (result 1) pins down θ . Once this is determined, the b is determined via equation 13. Using their values, $1 - m$ is obtained from the human resource constraint 11. Finally, r and Y are obtained from 15 and 17. The equilibrium is characterized in the following results.

Result 2 *The threshold of productivity b solves:*

$$\frac{(b)^\epsilon}{\int_b^\infty (a)^\epsilon \frac{dG(a)}{1-G(b)}} = \frac{p(\theta, \nu) [1 - G(b)]}{\delta + p(\theta, \nu) [1 - G(b)]} \times \frac{\frac{\beta}{1-\beta}}{\frac{\beta}{1-\beta} + \frac{\delta}{\delta + p(\theta, \nu) [1 - G(b)]}} \quad (18)$$

There always exists at least one solution of 18, and if $H(b, \epsilon)$ is monotonously not decreasing, the solution is unique.

Proof: *As a function of b , the RHS is monotonously decreasing, valued in $(0, 1)$ and it approaches 0 as b approaches its upper limit (or infinity). $H(b, \epsilon)$ approaches 1 as b approaches its upper limit (or infinity).¹³ So, there must exist at least one solution. If $H(b, \epsilon)$ is monotonously non decreasing, the solution is obviously unique. ■*

¹³In order to insure that the two lines cross we need to assume that when a is valued at its lowest possible value, the LHS is lower than the RHS: $\frac{a^\epsilon}{\int_a^\infty a^\epsilon dG(a)} < \frac{\beta p(\theta, \nu)}{\delta + \beta p(\theta, \nu)}$. Which is obviously the case if, for instance, $a = 0$

Result 3 *Given the value of b determined in result 2. The number of brokers in the economy (and the share of finance in GDP) is:*

$$1 - m = (1 - \beta)(1 - H(b, \epsilon)) \quad (19)$$

By simply manipulating the previous equations, the return to capital r and total output are determined by:

Result 4 *Given b from result 2*

$$\begin{aligned} r\bar{k} &= \frac{e_k}{1 - e_k} \pi(b, r, Y) \\ Y &= \left[\frac{e_k}{1 - e_k} + \frac{1}{\delta} \right] \pi(b, r, Y) \end{aligned} \quad (20)$$

Furthermore, both r and Y are maximized whenever b is maximum:

$$\begin{aligned} \ln Y &= \frac{1}{1 - e_Y} \left\{ \ln \left[\frac{e_k}{1 - e_k} + \frac{1}{\delta} \right] - e_k \left[\ln \frac{e_k}{1 - e_k} - \ln \bar{k} \right] + e_a \ln b \right\} \\ \ln r &= \left[1 - \frac{e_k}{1 - e_y} \right] \left[\ln \frac{e_k}{1 - e_k} - \ln \bar{k} \right] + \frac{e_Y}{1 - e_Y} \ln \left[\frac{e_k}{1 - e_k} + \frac{1}{\delta} \right] + \frac{e_a}{1 - e_Y} \ln b \end{aligned} \quad (21)$$

Thus it follows that:

Result 5 *The allocative decisions of the economy θ , m and b are independent of \bar{k} .*

r is determined as a residual by equation (15), and has no effect whatsoever in the determination of usage of human resources. The size of the financial sector is fixed by arbitrage and the individually optimal determination of b .

This is because to have more or less capital (and thus the interest rate) does not affect the marginal to average profit ratio ($H(b, \epsilon)$), which makes the allocative decisions independent of r , and consequently of capital abundance. Any change in r affects in the same manner the average and the marginal entrepreneur, and thus it has no incidence in the productivity that equals them, b .

We find this a surprising and useful result, because of two reasons. First, because in the measure that is true that an increase in the interest rate affects proportionately in the same manner to average and marginal firms, the positive correlation across countries of income and financial sector size can not be simply by relative capital abundance, which would otherwise be a likely candidate. Other explanations would be needed. More on it later.

Second, because it is by no means absurd to expect that an increase of r does not affects disproportionately more the marginal than the average firm: both will suffer an increase of r proportionately to their capital usage, which itself is determined by their productivities. In most common models both (marginal and average firms) will be affected in the same manner.

In dynamic models with risk neutral agents, if capital is endogenously determined the interest rate needs to equal the discount rate to make the Euler equation hold. Capital would adjust instantaneously by either not eating at all, or by moving it to zero. Production would be fixed at a level where the marginal productivity of capital equals the discount rate. In such a case, the interest rate is fixed, and equation 18 does not depend on the predetermined level of capital (as it depends on it only through r), even if the profit function were not log-linear. The results of such an exercise, are then perfectly akin with the ones we obtain in this section. We believe that in order to study growth and long run outcomes, it is important to restrict the process of capital accumulation. In section 5.1 we do so by putting restrictions on the utility function.

4 Comparative Statics

In this section we look at the steady state effects of alterations in the fundamental parameters on the equilibrium resource allocation.

4.1 Effects of the frictions in the investment sector

A more efficient investment sector is one where with the same human resource allocation matches are done faster. This is captured by ν .

Result 6 *b and output are both increasing in the efficiency of the search process in the investment sector (ν). Furthermore, as ν approaches infinity the limit of b is its maximum possible value (or infinity if it is unbounded).*

The number of brokers, $(1 - m)$ is decreasing with ν .

Proof: *The RHS of 18 increases with p , which depends only on parameters (β and the general efficiency of the matching process, characterized by the function $p()$), so we can characterize it as a function of the rate at which entrepreneurs find brokers, p . Clearly, it approaches 1 as p approaches infinity. ■*

The more efficient the matching process is, the pickier the economy is in choosing which projects get financed. The reason is, obviously, that the opportunity cost of going back to search is smaller, the more efficient the matching is. If the market is walrasian (ν approaching infinity), only the best conceivable projects get financed.

Likewise, and perhaps more surprisingly, larger ν produce a smaller share of GDP devoted to finance. The reason is nevertheless straight forward: there are few brokers because they are not needed. Few of them are able to produce fast matches between entrepreneurs and capital, so it is more efficient to place them into productive activities. It does not mean that the total GDP devoted to finance is smaller, as this increases with ν . Only, that less resources are devoted to finance, and its *share* of GDP decreases.

4.2 Effects of the destruction rate

A faster death rate δ shortens the horizon of agents, and makes them less picky:

Result 7 *b is not increasing in δ , and strictly decreasing if $H(b, \epsilon)$ is strictly increasing in b .*

The number of entrepreneurs does no decrease with δ , and strictly increase if $H(b, \epsilon)$ is strictly increasing in b .

Proof: The RHS of 18 decreases with δ , while the LHS is independent of it and increasing if $H(b, \epsilon)$ is increasing in b . The movement of $1 - m$ is implied by 3 ■

The shorter the time that the entrepreneur has left before dying, the less important than quality is, and the more important that starting working soon becomes. Consequently, if agents have short lives they are less picky and finance projects of less quality: they should not spend them looking for projects.

Notice from result 3 that the number of brokers would *increase*, in spite of the fall in the productivity threshold. The reason is that albeit it is easier to get your project financed (thus needing less brokers per entrepreneur), you are also replaced faster by somebody who is unmatched. Thus, a larger destruction rate demands of a larger financial sector, because by definition many entrepreneurs (the new borns) are in need finance. As $\delta \rightarrow \infty$, the number of brokers approaches $(1 - \beta)$ in order to keep the tightness of the market constant.

4.3 Effects of the bargaining power of entrepreneurs

Result 8 *There exists a value of β called $\hat{\beta} : 1 - \hat{\beta} = -\frac{\theta}{p(\theta, \nu)} \frac{\partial p(\theta, \nu)}{\partial \theta}$ such that $\hat{\beta}$ maximizes b (and thus, Y). If $\beta < \hat{\beta} \rightarrow \frac{db}{d\beta} > 0$, and if $\beta > \hat{\beta} \rightarrow \frac{db}{d\beta} < 0$.*

An increase of β decreases $1 - m$ if $\beta < \hat{\beta}$. If the value of β is much larger than $\hat{\beta}$, it is possible than an increase of β might increase $1 - m$

Proof: *It is straight forward once we realize that the derivative of the RHS of 18 with respect to θ (which is just a monotonous transformation of β) is*

$$\frac{\delta[1 - G(b)]}{(\delta + \beta p(\theta, \nu)[1 - G(b)])^2} \frac{p(\theta, \nu)}{1 - \beta} \left[1 - \beta + \frac{\theta \frac{\partial p(\theta, \nu)}{\partial \theta}}{p(\theta, \nu)} \right]$$

The relationship of $1 - m$ with β is more complicated. From 19 it follows that

$$\frac{d(1 - m)}{d\beta} = -[1 - H(b, \epsilon)] - (1 - \beta) \frac{\partial H(b, \epsilon)}{\partial b} \frac{db}{d\beta}$$

Thus, from result 8 it follows that if $\beta < \hat{\beta} \rightarrow \frac{d(1-m)}{d\beta} < 0$. It can only be increasing if $\frac{db}{d\beta}$ is a very large negative number, which is conceivable only if $\beta \gg \hat{\beta}$. ■

Increasing β has two effects on the PDV of the value of the entrepreneur. On one hand, it increases the share of the profits that she appropriates, increasing the value of $\frac{\frac{\beta}{1-\beta}}{\frac{\beta}{1-\beta} + \frac{\delta}{\delta + p[1-G(b)]}}$ and the value of being an entrepreneur. On the other hand, it increases the value of θ , and thus reduces the speed at which entrepreneurs find brokers, which reduces when do they expect to start receiving income $\left(\frac{p(\theta, \nu)[1-G(b)]}{\delta + p(\theta, \nu)[1-G(b)]} \right)$ and how much do they receive as it decreases their outside option and $\frac{\frac{\beta}{1-\beta}}{\frac{\beta}{1-\beta} + \frac{\delta}{\delta + p[1-G(b)]}}$. Given that the LHS is an increasing function independent of β , it is clear that b is maximized when the value of being an entrepreneur V_0 is maximized for any given b . This is just a reflexion of the Hosios condition. More power to the entrepreneurs produces a search externality, as there will be more of them, which will decrease their speed of finding finance. Their value (and the marginal productivity) is maximized when the elasticity of the matching function equals their bargaining share.

The effect of β on the number of brokers has two components. On one hand a larger β increases the value of being an entrepreneur, and to compensate it tightness needs to increase (equation 10) and a smaller percentage of human resources are devoted to brokerage. On the other hand b changes. If it increases, this by itself increases the demand of brokerage services (as more agents get rejected), while if it decreases this also decreases $1 - m$.

If we think of β as a reflection of the contractual and institutional arrangements favouring one or the other side of the market, it is clear that it can be either excessive or too small.

4.4 Comparative statics of the degree of product market efficiency

Result 9 *The minimum productivity threshold b (and consequently Y) are increasing in the elasticity of profits to talent (ϵ), irrespectively of the shape of $H(b, \epsilon)$.*

The number of brokers increases with ϵ .

Proof: *The LHS of 18 decreases with ϵ , while the RHS is independent of it and is decreasing in b . Notice that this is so even if it were the case that $H(b, \epsilon)$ were decreasing in b for some range. ■*

The more important talent is, the more that the society is going to be selective in the projects it chooses to finance; and for doing so it uses more resources into financial intermediation. The intuition for this result is as follows. Imagine that you have the marginal project. If ϵ were larger what you would get if you decide to go ahead (which is proportional to the marginal profit) will decrease *relative* to what you would get if you keep searching for a new project (which is proportional to average profit). Thus, if talent is more important, you would choose to go back into search. The more elastic than profits are to talent, the more that matters how different projects are, and you become more picky. If talent is very important, by no means you will want to go ahead with a mediocre project.

With a higher threshold of productivity more projects are rejected. Implying that there is a larger share of entrepreneurs with no finance. In order to keep θ constant it is necessary to have a larger number of brokers.

Clearly, this means that it is important the interpretation that we can give to ϵ . A larger value of ϵ is akin to have a more heterogeneous distribution of projects. As such it is quite intuitive that the more heterogeneous projects are, the more that we become picky in their adoption, as we aspire at more. Nevertheless, ϵ allows for interpretations that go beyond the distribution of talent. In particular we will argue that it represents the degree of efficiency in the product market.

We capture inefficiencies in product markets in a broad sense via redistribution of a firm's income that is implied by the inefficiencies. This is, we focus on the types of inefficiencies that will "transfer income" from high productivity firms to low productivity firms. Such an implication may result from political interconnectedness in face of a government budget constraint, or random taxation (as has been done in the literature on inefficiencies).

Thus, we consider a friction that does not affect capital demand of individual firm (in terms of the random tax, this corresponds to a random payment/transfer of money). This inefficiency will, however, affect b .

Consider the following tax and transfer scheme. The net profits of a firm are:

$$\hat{\pi}(a, r) = \pi(a, r)^{1-\tau} \tilde{\pi}^\tau \quad (22)$$

Where τ measures the degree of progressive redistribution between efficient and non-efficient firms, and $\tilde{\pi}$ is perceived by the agents as lump-sum. $\tau = 0$ means no progressive redistribution, and $\tau < 1$.

Clearly, balanced budget requires:

$$\int_b^\infty \pi(a, r) \frac{dG(a)}{1-G(b)} = \int_b^\infty \hat{\pi}(a, r) \frac{dG(a)}{1-G(b)}$$

This follows Benabou (2002), albeit in a very different context. In our environment τ is a measure of the allocative inefficiencies in the economy. Higher τ transfers profitability from efficient to inefficient firms, and thus usage of resources. Thus, it reflects a larger degree of inefficiency in the *product market*.

Notice that in our context τ simply decreases the elasticity of profits to productivity:

$$\begin{aligned} H(b, \epsilon, \tau) &= \frac{\hat{\pi}(b, r)}{\int_b^\infty \hat{\pi}(a, r) \frac{dG(a)}{1-G(b)}} \\ &= \frac{\pi(b, r)^{1-\tau}}{\int_b^\infty \pi(a, r)^{1-\tau} \frac{dG(a)}{1-G(b)}} \\ &= \frac{(b)^{\epsilon(1-\tau)}}{\int_b^\infty a^{\epsilon(1-\tau)} \frac{dG(a)}{1-G(b)}} \end{aligned} \quad (23)$$

Thus it follows that

Result 10 *A decrease of the allocative inefficiencies of the product sector (decrease of τ) produces larger steady state values of b and Y and a decrease of m*

In other words, a more efficient product market discriminates better between bad and good firms. There exists a private interest in investing more in differentiating the treatment of good and bad firms, and the economy does this by generating a larger financial sector: more brokers allow many rejections to take place without decreasing the speed at which entrepreneurs meet brokers.

Notice that the causality is the opposite than the one that usually it is assumed. It is not that a larger financial sector generates an efficient product sector, but the reverse. The large financial sector is large precisely because the productive sector is efficient. If it were inefficient, it would not be very important where capital is placed, as inefficient firms do quite well. There would be neither the need nor the demand to devote resources to place capital in a better firm. Human resources would be devoted to production, not to move capital towards more efficient firms.

It is precisely the fact that the product sector is discriminating and competitive (efficient), what induces a large financial sector, because it is more important to use capital well. It generates the need and the demand of devoting human resources towards determining the use of capital: a larger financial sector.

5 Example

In this section we use our model to introduce capital market frictions into a growth context, where capital supply is a state variable. Given that capital supply is irrelevant for allocative decisions (result 5), it is almost straight forward to do so.

We first solve explicitly for a model with the following functional assumptions:

- (1) that the production function of firms is Cobb-Douglas: $F(a, K, Y) = 2\sqrt{aK}$
- (2) that τ characterizes a re-distributive scheme as in section 4.4, and
- (3) that a follows a Pareto with minimum value \underline{a} and parameter γ

It would be easy to solve implicitly for a less restrictive set of assumptions, but it would not yield further insights, and would complicate matters unnecessarily.

It is very easy gross profit function, capital demand, net profit and $\tilde{\pi}$ are respectively:

$$\begin{aligned}
\pi(a, r) &= \frac{a}{r} \\
k^d(a, r) &= \frac{a}{r^2} \\
\hat{\pi}(a, r) &= \left(\frac{a}{r}\right)^{1-\tau} \tilde{\pi}^\tau \\
\tilde{\pi} &= \left(\frac{\gamma-(1-\tau)}{\gamma-1}\right)^{\frac{1}{\tau}} b
\end{aligned} \tag{24}$$

Given that b is bounded by below by \underline{a} , it is rather straight forward to check that results 2 and 3 imply the following result.¹⁴

Result 11 *There exists a level of taxes $\tilde{\tau} = \frac{1-(\gamma-1)\frac{1}{\beta}\frac{\delta}{p(\frac{\beta}{1-\beta}, \nu)}}{1+\frac{1}{\beta}\frac{\delta}{p(\frac{\beta}{1-\beta}, \nu)}} \in (0, 1)$ such that*

$$\begin{aligned}
1 - G(b) &= \begin{cases} 1 + \frac{1}{\beta} \frac{\delta}{p(\frac{\beta}{1-\beta}, \nu)} \frac{\tau+\gamma-1}{1-\tau} & \text{if } \tau \leq \tilde{\tau} \\ 1 & \text{if } \tilde{\tau} \leq \tau \end{cases} \\
b &= \begin{cases} \underline{a} \left[\beta \frac{p(\frac{\beta}{1-\beta}, \nu)}{\delta} \frac{1-\tau}{\tau+\gamma-1} \right]^{\frac{1}{\gamma}} & \text{if } \tau \leq \tilde{\tau} \\ \underline{a} & \text{if } \tilde{\tau} \leq \tau \end{cases} \\
1 - m &= \begin{cases} (1 - \beta) \frac{1-\tau}{\gamma} & \text{if } \tau \leq \tilde{\tau} \\ (1 - \beta) \frac{1}{1+\beta \frac{p(\frac{\beta}{1-\beta}, \nu)}{\delta}} & \text{if } \tilde{\tau} \leq \tau \end{cases}
\end{aligned} \tag{25}$$

The threshold of quality is increasing in ν and decreasing in τ , but it can not be smaller than \underline{a} . Thus, there is a degree of inefficiency of the product sector beyond which all projects are accepted.

The Pareto distribution insures that the marginal to average profit ratio is a constant¹⁵ that does not depend on b ($H(b, 1 - \tau) = \frac{\gamma-(1-\tau)}{\gamma}$). Consequently, it fixes the share of the financial sector (equation 19), which decreases with the degree of inefficiency. When all projects are financed ($\tau \geq \tilde{\tau}$), the degree of efficiency in capital markets (ν) *decreases*

¹⁴This is under the assumption that $(\gamma - 1)\frac{1}{\beta}\frac{\delta}{p(\frac{\beta}{1-\beta}, \nu)} \leq 1$. Otherwise the results are obvious.

¹⁵A way of understanding 13 is by noticing that with a Pareto distribution γ is a monotonously increasing function of the ratio of mean to standard deviation of a , as $\gamma = 1 + \sqrt{1 + \frac{\mu^2}{\sigma^2}}$. The smaller is the variance with respect to the mean, the more complex is the task of being above the threshold, as the more skewed is the distribution. Thus, if we understand complexity in this way (as the ratio of variance to mean of a), increasing it would demand to put more human resources into brokerage. The more difficult the problem, the more resources into solving it.

the size of the financial sector because of the reasons that we met in section 4.1: the more efficient capital markets are, less brokers are needed.

We can then go on to define total factor productivity as:

$$A = \begin{cases} b \left(1 + \frac{\tau}{\gamma-1}\right) = \underline{a} \left[\beta^{\frac{p(\frac{\beta}{1-\beta}, \nu)}{\delta}} \frac{1-\tau}{\tau+\gamma-1} \right]^{\frac{1}{\gamma}} \left(1 + \frac{\tau}{\gamma-1}\right) & \text{if } \tau \leq \tilde{\tau} \\ \underline{a} \frac{\gamma}{\gamma-1} \frac{\beta^{\frac{p(\frac{\beta}{1-\beta}, \nu)}{\delta}}}{1+\beta \frac{p(\frac{\beta}{1-\beta}, \nu)}{\delta}} & \text{if } \tilde{\tau} \leq \tau \end{cases} \quad (26)$$

Using result 4, given any level of capital \bar{k} , we get:

$$r = \frac{\sqrt{A}}{\sqrt{\bar{k}}} \quad (27)$$

$$Y = 2\sqrt{A}\sqrt{\bar{k}} \quad (28)$$

It is illustrative of some of our main points to make the comparative static exercises of section 4 in this example. Noticing how each exogenous variable affects total factor productivity (and thus on interest rates and output).

A decrease in the amount of frictions in the capital markets (increase of ν) increases TFP, but via two different mechanisms. It generates more efficient firms (via raising b), but also makes them smaller, thus increasing the productivity of capital. In the specific case of the Pareto distribution while we are in an interior solution ($\tau < \tilde{\tau}$) only the first of these mechanisms is active (because m is fixed). Nevertheless, when $\tilde{\tau} < \tau$ it is possible to notice the second mechanism: the size of the brokerage sector decreases, which results in more firms being created, and thus smaller size and larger productivity. There is no increase in the quality, as even projects less efficient than \underline{a} would be financed if they were going to exist, but there is a decrease in average size, and this increases productivity. In the general case both effects would be present, an increase in the quality of firms, *and* an increase in the size of human resources devoted to productive activities, which results in smaller (more productive) firms.

A decrease of the efficiency of the final goods market (higher τ) has two effects that go in opposite directions. On one hand it decreases the quality of firms (lower b), thus decreasing TFP. On the other hand there are more human resources devoted to productive

activities (m increases). Thus, firms are smaller and each of them uses less capital, increasing TFP (this is the effect of $\frac{\tau}{\gamma-1}$). In our example it is obvious that the first effect dominates¹⁶, but we know from result 10 that it does so for any other distribution of qualities or profit function. The interesting bit is to notice the two forces at work: **(1)** A more efficient final goods market generates the demand of brokerage services that allows to generate an improvement in the allocation of talent. **(2)** But this gain is partially offset because by putting less human resources into productive activities the number of firms is necessarily smaller and albeit they are more efficient they are also larger and each of them with lower marginal productivity of capital.

The cross country evidence shows that income and size of the financial sector are positively correlated. It has been suggested that this correlation is, at least partly, a consequence of differences in the efficiency of capital markets across countries. In this view countries with more efficient capital markets would allocate resources better, thus being richer. They would also use more resources into finance, as there would be more incentives to do so. This view is, at the light of our model, deeply misled.

The reason is that if what makes a country rich were that it has a more efficient capital market, this would also imply that it would need to place less resources into finance. Thus, its financial sector would be smaller, not larger. The direction of causality that our model suggest goes almost in the opposite direction. A more efficient final goods sector is what generates at the same time a larger financial sector and a more efficient economy. In an efficient final goods market, there are strong incentives to use capital in the right firms, so the economy *invests* heavily in moving the resources fast towards them. This investment is done in the form of a larger financial sector. Thus, capital is used more efficiently because it is used by better (albeit larger) firms.

According to our model, if the fundamental differences between countries were in the degree of efficiency of the financial sector (ν), we would observe the opposite (a negative correlation). To find a positive correlation between income and $1 - m$ using the model it would be necessary that the fundamental exogenous differences between countries are

$$^{16} \frac{\partial A}{\partial \tau} = -\underline{a} \left[\beta^{\frac{p(\frac{\beta}{1-\beta}, \nu)}{\delta}} \right]^{\frac{1}{\gamma}} \frac{1}{\gamma-1} \left(1 - \frac{1-\tau}{\gamma} \right)^{-\frac{1}{\gamma}} \left(\frac{1-\tau}{\gamma} \right)^{\frac{1}{\gamma}} \frac{\tau}{1-\tau} < 0$$

either in the degree of efficiency of the Good Market sector, or in the bargaining power of managers (β) provided that this one is inefficiently high ($\beta > \hat{\beta}$)

5.1 Growth

The irrelevance of capital abundance for the decision taking process makes very easy to solve a dynamic version of the model.

Consider first a toy dynamic version of the model. Assume that agents save (and invest) at an exogenous rate s , and that capital depreciates at a rate d (also exogenous). Assume further, that once a match has been made, firms can re-evaluate their capital needs as they see it fit given the prevailing interest rate. Calling k_t to capital at t :

$$\dot{k}_t = sY_t - dk_t \quad (29)$$

It follows that the steady state level of income is

$$Y^{SS} = \begin{cases} 2 \frac{s}{d} A & \text{if } \tau \leq \tilde{\tau} \\ 2 \frac{s}{d} a \frac{\gamma}{\gamma-1} \frac{\beta^{\frac{p(1-\beta, \nu)}{\delta}}}{1+\beta \frac{p(1-\beta, \nu)}{\delta}} & \text{if } \tilde{\tau} \leq \tau \end{cases} \quad (30)$$

While the equilibrium size of the financial sector would still be determined by 25. Thus, clearly if rich and poor countries were to differ by how efficient their financial sector is, there would be no correlation between financial sector size and income. If, on the other hand, they differ in the efficiency of the final goods sector, then the correlation would be positive.

AK endogenous growth model

It is almost straight forward to solve a model of endogenous growth with credit search fictions.

We maintain the assumptions of the Pareto distribution of a and the existence of an inefficiency in the goods market labelled by τ , but we change the utility and production functions.

We assume utility to be given by the standard Dixit Stiglitz aggregator. For simplicity the elasticity of demand is imposed to be $\frac{1}{2}$: $Y = \left(\int_0^N x_i^{\frac{1}{2}} di \right)^{1/\frac{1}{2}}$. Normalizing aggregate price to one, the implied demand for firm i is $x_i = Y p_i^{-2}$.

Finally, we assume that production is done with constant returns to scale and using only capital as input. A firm with a productivity quality a produces $x_i = 4ak_i$ units of output if it uses k_i units of capital.

It follows that profits and capital demand are the same that in the previous sections, and characterized in equation (24). Thus, the equilibrium productivity threshold and size of the financial sector are the same than in the previous model, and determined by (25). Furthermore, also as before, they are independent of capital supply.

Under the maintained assumption that capital accumulates according to (29), total factor productivity is still determined by (26), and interest rate and output are respectively.

$$r = A \tag{31}$$

$$Y = 2AK_t \tag{32}$$

and the growth rate of the economy is

$$\frac{\dot{k}}{k} = 2sA - 2d \tag{33}$$

In this context, more efficient financial and final markets bring more growth (via higher total factor productivity), but as before, the first one with a smaller financial sector while the second one with a larger one.

6 Extension: Competitive Search

We next check that our results do not depend on Nash Bargaining. Instead, the price of financial services is determined by competitive search.

The model is as in section 2 except that brokers post the price of capital that they

offered. They commit to that price and will provide services at that price independently of the productivity of the entrepreneur at whom they will service. In principle entrepreneurs could face a trade-off between a longer queue and paying a higher price. Nevertheless, in equilibrium only one queue is open, as all brokers are identical.

We call q the price of capital that entrepreneurs pay, and $q - r$ is the margin for the broker. q is the price that brokers post.

The value of an entrepreneur who gets capital at price q is

$$\delta V_E(a, q) = \pi(a, q) \quad (34)$$

An entrepreneur who does not have a match and *chooses* to search at a price q with a queue determined by θ would get a value $p(\theta) \int_b^\infty (V_E(a, q) - V_0) dG(a)$. Obviously she would choose the price and queue that maximize her value. Thus, if there were more than one queue where agents would look in equilibrium, in all of them there would need to be the same expected value:

$$\delta V_0 = p(\theta) \int_b^\infty (V_E(a, q) - V_0) dG(a) \quad (35)$$

Where the productivity threshold b in a queue with price q is such that:

$$V_E(b, q) = V_0 \quad (36)$$

Brokers choose the queue and price in order to maximize their income, but they know that they need to choose them such that (35) and (36). Otherwise, if q were higher or the queue length for entrepreneurs were shorter, there would be no entrepreneurs in the queue. Thus the problem of the brokers is to choose θ , q (and implicitly b) so that

$$\begin{aligned} \delta V_B &= \max_{\theta, q, b} \theta p(\theta, \nu) (q - r) \int_b^\infty k^d(a, q) dG(a) \\ s.t. & \quad \delta V_0 = p(\theta) \int_b^\infty (V_E(a, q) - V_0) dG(a) \\ & \quad V_E(b, q) = V_0 \end{aligned}$$

Given (1) and (2) we can rewrite it as:

$$\delta V_B = \max_{\theta, q, b} \theta p(\theta, \nu) \left(1 - \frac{r}{q}\right) \int_b \pi(a, q) dG(a) \quad (37)$$

$$s.t. \quad \delta V_0 = \frac{p(\theta, \nu)}{\delta} \int_b^\infty (\pi(a, q) - \delta V_0) dG(a) \quad (38)$$

$$\pi(b, q) = \delta V_0 \quad (39)$$

Brokers take V_0 as given, and choose the queue that maximize their own value, but subject to the restriction that it can not provide with less value to the entrepreneurs than V_0 , otherwise it would be empty.

7 Extension: Different property rights

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8 Discussion and Conclusions

We add to the existing literature of search frictions in financial markets the inclusion of two resource constraints that we consider important.

A capital resource constraint, as in principle the relative abundance of capital could be thought to affect the prominence of the frictions.

A human resource constraint, as human resources can be used either in directly productive tasks, or it can be devoted instead to alleviate the extend of the frictions and increasing the speed at which capital flows into investment projects. The trade off thus generated it is not obvious. In our model in equilibrium the speed at which projects find investment is fixed ($\theta = \frac{\beta}{1-\beta}$) at the unique level that equalizes the value of being an entrepreneur and a broker. Nevertheless there is a meaningful trade-off between the human resources devoted to intermediation and productivity. A larger share of brokers allows the economy to be pickier, the quality of the average project that gets financing gets better without affecting the speed at which entrepreneurs with good projects find financing.

Our first result is surprising: the capital resource constraints is irrelevant for the determination of the working of capital markets. This is, capital scarcity of course affects the price of capital, but it does not affect the speed at which capital is made available to investors. This is because when determining whether a project is worth financing, agents look ahead. They compare the return of investing in the project with what it would be obtain by starting from scratch looking for a new one. The price of capital affects proportionately the return of both projects (the current and the expected future one). Thus, it is irrelevant for the determination of which projects get financed.

The rest of our results relate to the proportion of human resources into finance and the average quality and heterogeneity of firms implied in the market. They allow us to interpret the workings of financial intermediation, and the process of financial development.

More efficient financial markets (in the sense of having less frictions), demand of less brokers. This might seem surprising at a first look, but it is quite sound when you think about it: if the market is efficient you need few brokers. Their role is to ease the frictions, and if they are absent, there is no room for them. In the limit, if finance were frictionless, there would be no brokers, only a Walrasian auctioneer. Nevertheless the economy is richer if there are less financial frictions, because the lower opportunity cost of throwing projects away implies that the quality demanded of projects is higher; and firms are more homogeneous as the tail of bad firms is cut shorter.

A larger destruction rate of firms has exactly the opposite effects. It decreases the marginal quality of firms (because it is not worth being picky) and it increases the size of the financial sector (as more firms are destroyed, and more new projects would need financing).

There is a Hosios condition determining the optimal bargaining share of entrepreneurs. If brokers have too much bargaining power any decrease of it would increase the average quality of firms and it would decrease the number of entrepreneurs (as it is less attractive to become one).

Finally, the more efficient the good market is, the more important relative differences

in quality of the firms are. This makes more important the allocation of capital into the “right” projects, which itself increases both the share of resources devoted to intermediation and quality of the marginal project financed.

There is well known evidence that financial and economic development go hand in hand. The share of GDP devoted to finance and GDP are highly correlated both across countries and across time. It is often argued that causality runs from the first to the second: more efficient financial markets foster larger share of GDP devoted to it and more efficient resource allocation. This line of reasoning would be wrong at the light of our model. A more efficient financial sector would indeed be associated with a richer economy with more homogeneous firms, but also with one with a *smaller* financial sector.

In order to correlate positively size of the financial sector with a richer economy, more efficient and more homogeneous firms our model points towards the opposite direction of causality. If the product sector is more efficient; if it is more important to be good, and is more discriminating against inefficient firms, then we would get both a large financial sector and large, efficient and homogeneous firms.

If we want our model to calibrate to the observed facts, in our model the reason why rich countries are rich is NOT because they have a more efficient financial sector. Our model points strongly in the opposite direction: rich countries are richer because they have a more efficient final goods sector, which generates the demand of a large financial sector. If we take as exogenous and fundamental the degree of efficiency of financial and good sectors, our model suggest than most of the variance across countries and time comes from the second, and not from the first.

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