Skill Erosion during Unemployment as a source of Inefficiency

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Abstract

This paper analyzes whether the presence of skill erosion during unemployment calls for policy intervention. Comparing labor market outcomes in the decentralized and the constrained-efficient allocation reveals that skill erosion during unemployment is a source of inefficiency. The intuition behind this finding is the externality related to job creation: firms do not fully take into account how their hiring decisions affect the possibility that workers' skills deteriorate, and hence the composition of the labor force in terms of workers who have and who have not lost some of their skills. I show that in an environment characterized by random search, Nash bargaining, and aggregate uncertainty, constrained-efficient allocations cannot be achieved under skill erosion independently of workers' bargaining power. This is in contrast to the finding in a framework without skill loss, as shown by Hosios (1990). Consequently, in the presence of skill deterioration policy intervention is required to restore constrained-efficiency. The optimal labor market policy takes the form of an employment subsidy. This reflects job creation being too low in the decentralized allocation. Moreover, I find that the optimal employment subsidy is procyclical.

Keywords: unemployment, human capital depreciation, inefficiency, optimal labor market policy

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1 Introduction

The Great Recession has brought back the specter of long-term unemployment. For example, in the United States nearly half of the unemployed had been out of work for more than six months by the end of 2011. This increase in long-term unemployment has been of great concern to policy makers. One of the reasons is the wide held belief that being unemployed for a long period of time leads to the depreciation of a worker's human capital[2]. Ben Bernanke, chairman of the Federal Reserve, shares this view as becomes clear from his testimony before the U.S. Senate in March 2012:

> "*...One concern we do have, of course, is the fact that more than 40 percent of the unemployed have been unemployed for six months or more. Those folks are either leaving the labor force or having their skills eroded. Although we haven't seen much sign of it yet, if that situation persists for much longer then that will reduce the human capital that is part of our growth process going forward.*"[3]

But how should the presence of skill erosion affect the design of policy? This paper tries to provide a better understanding by looking at this issue through the lens of a search and matching model à la Diamond-Mortensen-Pissarides with aggregate uncertainty, risk-neutral agents and skill loss during unemployment.

I model skill erosion by assuming that workers face the risk of losing a fraction of their productivity when being out of work. So workers who have suffered from skill deterioration are less productive upon reemployment than workers who have not been affected by it. At the same time, workers can regain their initial skill level while being employed through learning-by-doing. Moreover, the risk of skill erosion is increasing in workers unemployment duration. Thus, the long-term unemployed are more likely to have lost some of their skills than the short-term unemployed.

Allowing for human capital depreciation during unemployment has several implications. First of all, workers are no longer homogeneous. The labor force now contains both workers who have and who have not lost some of their skills. Importantly, the composition of the labor force in terms of worker types is determined endogenously through its dependence on the firms' hiring decisions. The mechanism is the following. Firms' hiring decisions affect workers' job finding probability, and hence average unemployment duration. Given that the probability of skill loss is increasing in workers' unemployment duration, hiring affects the share of workers who have lost skills, and hence the composition of the labor force. Note that this implies that during periods when the average job finding probability is low, i.e. during recessions, the labor force's production potential decreases because the relative share of workers with eroded skills increases. Second, the composition of the unemployment pool affects job creation. Given that the labor market is characterized by random search, firms cannot choose which worker type they attract when opening a vacancy. Thus, the composition of the unemployment pool in terms of worker types affects the expected productivity of a newly hired worker, and hence vacancy posting.

[2] Empirical evidence for human capital depreciation is provided by e.g. Keane and Wolpin (1997)

[3] Question and answer session of the Senate Banking Committee hearing on the 1st of March 2012 with Federal Reserve chairman Ben Bernanke testifying on monetary policy and the U.S. economy: http://www.reuters.com/article/2012/03/01/usa-fed-bernanke-idUSL2E8E13KI20120301.

To detect if skill erosion during unemployment is a source of inefficiency, I analyze how labor market outcomes in the decentralized allocation differ from labor market outcomes in the constrained-efficient allocation. Comparison of hiring decisions in both allocations reveals that skill erosion is a source of inefficiency. The intuition behind this finding is the following. In addition to the congestion externality, a composition externality is present: firms do not fully take into account how their job creation affects the labor force's composition in terms of worker types. More precisely, firms only internalize their effect on workers' productivity through job creation when this worker stays at the firm. So firms do not taken into account that by employing a worker, this worker keeps her skills or regains her skills, and hence there is an additional high productive worker available when the match ends. At the same time, firms also ignore that through employment they prevent workers from being exposed to skill erosion, and hence being less productive upon reemployment. In other words, there are not fully internalized gains in terms of output from job creation related to workers' productivity being affected by their employment status.

Before turning to the policy implications, I analyze if constrained-efficiency can be obtained in the absence of policy intervention. As long as the wage setting mechanism is such that both the congestion externality and the externality related to skill loss can be internalized, there is not necessarily a role for policy. Given that I assume that wages are set every period through Nash bargaining, I examine, in the spirit of Hosios (1990), if there exists a parameter condition for the worker's bargaining power which restores efficiency. I find that such a condition only exists in the absence of aggregate uncertainty. When aggregate uncertainty is present and workers' bargaining power is constant across states, there is no longer a parameter restriction for workers' bargaining power which offsets both the congestion and the composition externality. Thus, in the presence of aggregate uncertainty, some kind of policy intervention is required to restore constrained-efficiency.

The optimal labor market policy which offsets the composition externality resulting from the presence of skill erosion during unemployment takes the form of an employment subsidy. On average, more jobs have to be created to attain efficient labor market outcomes than implied by the laissez-faire economy. Surprisingly, I show that the optimal subsidy is procyclical. In other words, in the presence of skill erosion, random search and period-by-period Nash bargaining, employment should be subsidized less during a recession. This finding reflects that the composition externality matters more during booms than during recessions. The intuition behind it is the following. The social cost of letting a worker lose skills is captured by the difference between the output produced by a worker with eroded skills and the expected output produced if another, random worker from the unemployment pool were to be hired. This expected output depends on the composition of the unemployment pool, and is decreasing in the relative share of unemployed workers with eroded skills. Therefore, when the fraction of unemployed workers with eroded skills increases, the difference between the output produced by a worker with eroded skills and the output produced by an average worker from the unemployment pool decreases. Thus, during recessions the social cost of letting a worker lose skills lowers. Through the same mechanism, it can be explained why the social cost of letting a worker lose skills increases during good times. In booms, the fraction of workers in the unemployment pool with eroded skills decreases. Therefore, the difference between the output produced by a worker with eroded skills and the expected output produced by an average worker from the unemployment pool increases. This implies that the social cost of letting a worker lose skills, measured by this expected output difference, increases. Since the private cost of letting workers lose skills is lower than the social cost, employment has to be subsidized. The procyclicality of this employment subsidy follows from the procyclicality of the social cost of letting a worker's skills erode.

The remainder of the paper is organized as follows. In section 2 I discuss the related literature. Section 3 outlines the model. The constrained-efficient allocation, and the analysis whether skill erosion during unemployment is a source of inefficiency, is presented in Section 4. Section 5 discusses how constrained-efficiency can be attained, and section 6 derives the optimal labor market policy. In section 7 simulation results are shown to get a better understanding of the labor market outcomes in the constrained-efficient versus the laissez-faire economy. Finally, section 8 concludes.

2 Related Literature

This paper is related to several strands in the literature. It relates to the broader literature analyzing the effect of skill loss on unemployment outcomes. Coles and Masters (2000) focus on the effect of skill erosion on steady state labor market outcomes. They find that multiple Pareto rankable steady state equilibria arise in a search model with loss of skills even in the presence of constant returns to matching. Moreover, they show that the Pareto optimal steady state equilibrium can be attained by implementing labor market policies which prevent long-term unemployment. Although different in its focus, the paper by Coles and Masters (2000) is related to this paper since they point to the externality which arises in the presence of skill erosion, i.e. the composition externality, as an explanation for the multiplicity of equilibria.

Skill erosion during unemployment has also been proposed as a mechanism which can help explaining the observed unemployment dynamics. Pissarides (1992) argues in the context of a stylized overlapping generations model that the presence of skill loss generates unemployment persistence. The intuition behind his finding is the quality of the unemployment pool affecting firms hiring decisions. The larger the share of unemployed workers who have lost some of their skills, the lower the expected productivity of a newly hired worker, and hence the lower firms' willingness to create jobs. During a recession, the quality of the unemployment pool deteriorates because average unemployment duration increases. Thus, firms' vacancy posting decreases more compared to an environment without skill loss. In my paper, I point to the same mechanism to explain why unemployment is more persistent in the presence than in the absence of skill erosion. More work on the effect of skill erosion on the persistence of unemployment is done by Esteban-Pretel (2005) and Esteban-Pretel and Faraglia (2010). Both papers quantify the extent to which skill loss can account for the persistence of unemployment fluctuations. Esteban-Pretel (2005) performs his analysis in a dynamic general equilibrium framework with large households, a labor market characterized by random search, endogenous separation, and skill loss. Esteban-Pretel and Faraglia (2010) introduce the same type of labor market frictions in a New Keynesian framework to analyze the response of unemployment to monetary shocks. The main difference between my paper and those papers is its focus. While Esteban-Pretel (2005) and Esteban-Pretel and Faraglia (2010) analyze whether skill loss can help explaining unemployment dynamics, I focus on whether labor market outcomes are constrained-efficient in the presence of skill loss. Moreover, those papers model skill erosion as firms having to pay a fixed training cost when hiring workers with eroded skills, whereas I follow Pissarides (1992) by assuming that workers with eroded skills are less productive upon reemployment.

Also in the turbulence literature skill loss during unemployment is present. The papers by Ljungqvist and Sargent (2004), den Haan et al. (2005), and Ljungqvist and Sargent (2007) differ in two main aspects. First of all, its focus is different. The turbulence literature tries to explain unemployment ouctomes in Europe versus the United States by looking at the interaction between the welfare system and skill loss upon unemployment. Second, the concept of skill loss in my paper differs from the concept used in the turbulence literature. In the latter, workers face the risk of losing skills at the moment of job loss because of a turbulent economic environment. Once workers are unemployed, their skills remain unchanged. On the contrary, in the environment described in my paper, workers' skills erode because of having spent long periods of time out of work. So workers' skills change while being unemployed.

This paper is also related to the literature analyzing optimal labor market policy in the presence of skill erosion. Shimer and Werning (2006), and Pavoni (2009) look at the implications for optimal unemployment insurance. The contribution of my paper relative to this strand of the literature lies in its focus on the optimal labor market policy necessary to offset the inefficiencies arising due to labor market frictions in an environment with risk-neutral workers and aggregate uncertainty.

3 Model

The model is a discrete time, dynamic stochastic general equilibrium model with two types of agents: workers and firms. Workers are assumed to be infinitely-lived and risk-neutral. They consume and participate in the labor market. They can be either employed or unemployed and searching for a job. Firms produce a homogeneous consumption good in a perfectly competitive market and use labor as input. They recruit workers by posting vacancies at a cost $\kappa > 0$ in a labor market characterized by random search à la Diamond-Mortensen-Pissarides. In this otherwise standard framework, I introduce skill erosion during unemployment and learning-by-doing while being employed. This implies that workers can lose a fraction of their skills when being out of work but they can regain their initial skill level when reemployed.

3.1 The labor market

Under the assumption of skill erosion during unemployment workers are heterogeneous in their skill level. This is because workers' skill level is affected by their unemployment duration. I assume that the longer the unemployment spell, the more likely it is that skills deteriorate. To keep the analysis simple, there are only 2 skill types: high (H) and low (L). A worker's skill type determines her productivity: high-skilled workers have high productivity, whereas low-skilled workers have low productivity. This implies that people who have suffered from skill loss are less productive upon reemployment than those who have not suffered from it. However, the low-skilled can regain their productivity through learning-by-doing while being employed.

The unemployment pool at the beginning of the period U contains both high and low-skilled job-seekers. The latter is defined as the sum of the high-skilled U^H and low-skilled unemployed U^L at the beginning of the period

$$U = U^H + U^L \tag{1}$$

I assume that the labor market is characterized by random search à la Diamond-Mortensen-Pissarides. So firms cannot direct their search to a particular worker type. This implies that when firms open vacancies at cost $\kappa > 0$, workers randomly drawn from the unemployment pool will show up for the job interviews. Since firms meet at most one worker at each round of job interviews, all interviews lead to successful hiring conditional on the match surplus being non-negative. The total number of interviews in every period is determined by the matching function. This function is assumed to be strictly increasing and concave in both arguments and to display constant returns to scale. It is given by

$$m(v, U) = B\ V^{1-\xi}\ U^{\xi}$$

where $1-\xi$ is the elasticity of vacancies, B represents the efficiency of the matching process, V is the total number of vacancies posted by firms, and U is the total number of job-seekers weighted by their search intensity. Because I assume that unemployment duration does not affect search intensity, the relevant measure of unemployment in the matching function is given by the total number of unemployed workers at the beginning of the period U (see equation 1).

The unemployment pool's heterogeneity enlarges the economy's state. In the standard search model without skill loss the state of the economy consists only of aggregate productivity A. However, in the presence of skill erosion the state also contains the number of high and low-skilled unemployed. This follows from vacancy creation being affected by the composition of the unemployment pool in terms of high and low-skilled workers. So the state s is given by $s = \left\{A, U^H, U^L\right\}$.

Defining labor market tightness as $\theta(s) \equiv \frac{V(s)}{U(s)}$, the probability for a firm posting a vacancy to meet a job-seeker can be defined as

$$q(s) \equiv \frac{m\left(V(s), U(s)\right)}{V(s)} = B\theta(s)^{-\xi} \tag{2}$$

where $q(s)$ is decreasing in labor market tightness. The job-finding rate or the probability that an unemployed worker gets a job interview is defined as

$$p(s) \equiv \frac{m\left(V(s), U(s)\right)}{U(s)} = B\theta(s)^{1-\xi} \tag{3}$$

where $p(s)$ is increasing in labor market tightness. Both worker types have the same job finding probability because unemployment duration has no effect on search intensity. When the match surplus is non-negative for both skill types, workers also have the same hiring probability. This follows from the assumption that each firm meets at most one worker at each round of interviews.

Defining the labor market flows requires an assumption about the timing of events. I assume the following. At the beginning of the period, workers' skill type changes: high-skilled workers who were unemployed in the previous period become low-skilled, and low-skilled workers who were employed in the previous period become high-skilled. Next exogenous separation takes place: a fraction γ of the matches breaks up. The *iid* shock ε to aggregate productivity is realized, where aggregate productivity follows an AR(1) process: $A = \rho A_{-1} + \varepsilon$. After observing the state of the economy, firms post vacancies and hire workers. Finally, production takes place using both the existing and newly hired workers.

Next, labor market flows are defined. High-skilled, low-skilled and total unemployment before hiring takes place evolve as follows. All low-skilled workers who were employed in the previous period have regained their productivity by the beginning of this period because of learning-by-doing. Thus, all last period's employed are high-skilled at the beginning of this period. Additionally, all the workers who were not employed last period have suffered from skill erosion and are all low-skilled. As a result, the high-skilled unemployed are only those people who were working last period and just lost their job. The low-skilled unemployed are all people who were not working last period. Total unemployment U in turn consists out of all workers besides the ones who were employed in the previous period and survived exogenous separation this period. Normalizing the total size of the labor force to 1, high-skilled, low-skilled and total unemployment are given by

$$U^H (N_{-1}) = \gamma N_{-1} \tag{4}$$

$$U^L (N_{-1}) = 1 - N_{-1} \tag{5}$$

$$U (N_{-1}) = 1 - (1 - \gamma) N_{-1} \tag{6}$$

As can bee seen from equations 4 and 5, both high-and low-skilled unemployment can be written as a function of previous period's employment. This follows from the assumption that workers' productivity deteriorates with probability 1 after having been unemployed for one period, and is restored after having worked for one period. Therefore, the economy's state is given by

$$s = \{A, N_{-1}\}$$

The state s only consist of two variables which simplifies the analysis. The setup can be extended to a framework where in each period the high-skilled unemployed become low-skilled with some probability $0 < l < 1$, and the low-skilled employed regain their skills with some probability $0 < g < 1$. This generalization of the model will not alter the main findings.

High-skilled employment consists of all workers who survived separation and the newly hired high-skilled workers. Since all individuals who were employed last period have regained their productivity, low-skilled employment is only made up of newly hired low-skilled workers. So employment of each type is given by

$$N^H(s) = (1-\gamma)\ N_{-1} + p(s)\, U^H(N_{-1})$$

$$N^L(s) = p(s)\, U^L(N_{-1})$$

3.2 Firms

Each firm consists of a single-worker production unit. Its output depends on the type of worker employed. Given that low-skilled workers are less productive than high-skilled workers, output produced in a match filled with the previous is lower than in a match filled with the latter. The production function of each worker type is given by equation 7 and 8 respectively

$$y^H(A) = A \tag{7}$$

$$y^L(A) = (1-\delta)\, A \tag{8}$$

Note that the productivity level of a high-skilled worker is normalized to one and that the productivity level of a low-skilled worker is given by $1-\delta$. The parameter δ can be interpreted as the rate of human capital depreciation.

The firm's value function of having a worker of type $i = \{H, L\}$ employed is given by equation 9

$$J^i(s) \quad = \quad y^i(A) - w^i(s) + (1-\gamma)\, \beta E_s \left\{ J^H(s') \right\} \tag{9}$$

So a firm's value of having a worker of type i employed depends on the generated output, the wage cost $w^i(s)$ where wages are set through Nash bargaining as discussed in section 3.4, and the continuation value of the match. With probability $1-\gamma$ the match survives separation and the work-firm pair continues producing. Recall that all low-skilled workers regain their productivity after having been employed for one period. As a result, the continuation value of the match is the value of having a high-skilled worker employed.

The firm's value from posting a vacancy is given by

$$V^V(s) \quad = \quad -\kappa + q(s) \left[u^H(N_{-1})\, J^H(s) + u^L(N_{-1})\, J^L(s) \right] + (1-q(s))\, \beta E_s \left\{ V^v(s') \right\} \tag{10}$$

where $u^i(N_{-1}) \equiv \frac{U^i(N_{-1})}{U(N_{-1})}$ is the fraction of type i unemployed workers in the unemployment pool, with $\{i = H, L\}$.

Imposing the free entry condition $V^V(s) = 0$ implies that equation 10 becomes

$$\kappa \quad = q(s)\left[u^H(N_{-1})\, J^H(s) + u^L(N_{-1})\, J^L(s)\right] \tag{11}$$

Thus, firms create jobs such that the cost of posting a vacancy κ equals the expected gain of hiring a new worker. The latter is affected by the unemployment's pool composition because the composition determines the chance of a particular worker type showing up for the job interview. As a result, job creation is affected by the relative share of high and low-skilled workers searching for jobs.

When imposing free-entry, the firm's value of having a worker of type $i = \{H, L\}$ employed is given by

$$J^i(s) \quad = \quad \frac{\kappa}{q(s)} + G^i(s) \tag{12}$$

where

- $G^i(s) \equiv (y^i(s) - w^i(s)) - (\bar{y}(s) - \bar{w}(s))$ is the gain from employing a particular worker type compared to the gain of employing a worker randomly drawn from the unemployment pool.

- $\bar{y}(s) \equiv y^H(s)\, u^H(N_{-1}) + y^L(s)\, u^L(N_{-1})$ represents the expected output of a worker randomly drawn from the pool. Because workers are hired randomly, this also represents the expected output of a newly hired worker. It equals the weighted average of the output produced by each worker type. Note that because each worker type has the same hiring probability, each type's share in the unemployment pool is sufficient to determine this type's weight.

- $\bar{w}(s) \equiv w^H(s)\, u^H(N_{-1}) + w^L(s)\, u^L(N_{-1})$ is the expected wage cost for employing a worker randomly drawn from the pool.

In the standard model with homogeneous workers, the value of having a worker employed equals the expected hiring cost $\frac{\kappa}{q(s)}$. This is because a worker can be replaced by a newly hired worker at a cost $\frac{\kappa}{q(s)}$. However, in the presence of worker heterogeneity this is no longer the case as can be seen from equation 12. The reason is straightforward. A worker of a specific type can no longer be replaced by hiring a new worker because the newly hired is not necessarily of the same type. As a result, the value of employing a specific worker type contains an additional term represented by $G^i(s)$. This term captures the gain of having that specific worker type employed compared to having a random worker from the unemployment pool employed. Note that this gain depends on how likely it is that you can replace this worker by a worker of the same type. For example, a larger fraction of high-skilled workers in the unemployment pool lowers the gain of having a high-skilled worker employed because the probability that a newly hired worker would also be high-skilled is high.

The presence of skill erosion and learning-by-doing implies that job creation affects the productivity of the labor force. To provide more insight into how firms take the latter into account, I rewrite the vacancy creation equation as follows

$$\frac{\kappa}{q(s)} = \bar{y}(s) - \bar{w}(s) + (1-\gamma)\beta E_s\left\{\frac{\kappa}{q(s')} + G^H(s')\right\} \tag{13}$$

First of all, equation 13 shows that job creation is such that the cost of hiring (LHS) equals the expected output produced by the newly hired worker, taking into account the expected wage cost and the continuation value of the match (RHS). As mentioned previously, firms cannot directed their search to workers with a particular unemployment duration, and hence a particular productivity. Therefore, when firms post vacancies they take into account the probability that the match will start operating with a high or low-productive worker. Second, equation 13 points out that when firms create jobs they only take into account how their job creation affects workers' productivity when a worker continues being employed at the firm. Thus, firms take into account that if a match survives separation, a high-skilled worker will be producing. This might be because the worker was already high-skilled upon hiring or because by working the low-skilled worker regained her productivity. On the contrary, firms do not take into account that by employing a worker there will be another high-skilled worker searching for a job in case the match breaks up. Additionally, the firm does not take into account that job creation prevents workers from being unemployed, becoming less productive, and hence having the possibility to search for a job as a low-productive worker.

3.3 Workers

There is continuum of risk-neutral workers on the unit interval. Workers maximize their discounted expected utility which is given by

$$E_0\sum_{t=0}^{\infty}\beta^t\ \left(C_t + b\left(1-N_t\right)\right)$$

where C_t is consumption, β is the discount factor, and b is the value of home production that workers enjoy during their time spent in unemployment where $N_t \,\epsilon\, \{0,1\}$. Note that the value of home production is unaffected by the worker's unemployment duration. In other words, high and low-skilled workers enjoy the same value of home production.

The value function of a worker of type $i = \{H, L\}$ from being employed is given by

$$V^{E,i}(s) = w^i(s) + \beta E_s\left\{\left(1-\gamma+\gamma p(s')\right)V^{E,H}(s') + \gamma\left(1-p(s')\right)V^U(s')\right\} \tag{14}$$

For each worker type the value of being in a match depends on the wage and the continuation value of the match. The continuation value of the match is made up of three parts. With probability $1 - \gamma$ the match survives separation and the worker continues working at the same firm as a high-productive worker. With probability γ the match breaks up and the worker start searching for a new job. If she finds a new job, which happens with probability p, she will be employed as a high-skilled worker. If she does not find a job, she will remain unemployed.

The value function of high and low-skilled workers of being unemployed after hiring took place is given by

$$V^U(s) = b + \beta E_s \left\{ p(s') V^{E,L}(s') + (1 - p(s')) V^U(s') \right\} \tag{15}$$

The value of being unemployed after hiring took place is the same for both worker types. The reason is that once they can start searching again for jobs the high-skilled unemployed will already have lost a fraction δ of their skills, and hence will also be searching as low-productive workers.

3.4 Wages

3.4.1 Wage setting mechanism

Wages are negotiated in every period between worker and firm. Following the literature, I assume Nash bargaining. Consequently, total match surplus is split between worker and firm such that each of them gets a fraction of total match surplus depending on their bargaining power. Defining total match surplus as $M^i(s) \equiv V^{E,i}(s) - V^U(s) + J^i$ where $i = \{H, L\}$, the surplus for a worker and a firm of being in a match is given by

$$S^i(s) \equiv V^{E,i}(s) - V^U(s) = \eta M^i(s) \tag{16}$$

$$J^i(s) = (1 - \eta) M^i(s) \tag{17}$$

where η is the bargaining power of the households, $1 - \eta$ is the bargaining power of the firms, S^i is the surplus for a worker type i of being in a match, and J^i is the surplus for a firm of being matched with a worker of type i. Note that the surplus for a firm of being in a match equals the value of having a worker employed because free-entry drives the value of having a vacancy to zero.

Combining equation 14, 15 and 16, worker type i's surplus of being in match becomes

$$S^i(s) \quad = \quad w^i(s) - o(s) + (1 - \gamma) \beta E_s \left\{ S^H(s') \right\} \tag{18}$$

where $o(s)$ represents the outside option of the worker

$$o(s) \equiv b+\beta E_s\left\{p(s') S^L(s')-\gamma p(s') S^H(s')\right\} \tag{19}$$

The surplus of the worker of being in a match is a function of the wage, the worker's outside option and the continuation value of the match. The latter reflects that workers take into account that because of learning-by-doing they will be high-productive after having been in the match for one period. The outside option is defined by equation 19. It is the same for both worker types, and contains three parts. First of all, when having a job a worker cannot engage in home production b. Second, when being employed a worker cannot search for another job. However, if she had not been employed, her skills would have deteriorated, and hence she would have been searching for a job as a low-productive worker. The worker keeps this in mind. At the same time, the worker also takes into account that being in a job guarantees that she keeps her productivity or regains her productivity. Thus, she takes into account that when the match separates she will be able to search as a high-skilled worker. This reduces the outside option.

The solution to the Nash bargaining problem leads to the following expression for the wage of a high and low-skilled respectively

$$w^H(s) = \eta A+(1-\eta) o(s) \tag{20}$$

$$w^L(s) = \eta(1-\delta) A+(1-\eta) o(s) \tag{21}$$

3.4.2 Implications

When both workers and firms have some bargaining power $\eta \epsilon (0,1)$, the following implications can be derived:

- *High-skilled workers are better off than low-skilled workers.* This can be shown as follows. From equation 18 it results that the difference in terms of surplus of being employed as a high or a low-productive worker only depends on the wage difference: $S^H(s)-S^L(s)=w^H(s)-w^L(s)$. The wage difference in turn can be derived from equation20 and21:$w^H(s)-w^L(s)=\eta\delta A>0$. Given that the wage of a high-skilled worker is strictly higher than that of a low-skilled worker when $\delta>0$, it follows that the surplus of being employed as a high-skilled worker is strictly higher than the surplus of being employed as a low-skilled worker.

$$S^H(s)-S^L(s)=\eta\delta A>0 \tag{22}$$

- *Firms are strictly better off employing a high-skilled worker than employing a low-skilled worker.* This can be shown as follows. The difference in the value for the firm of having one worker type employed compared to the other, can be derived from equation9. Combining this with the wage eqations 20 and21, gives

$$J^{H}(s) - J^{L}(s) = \left(y^{H}(A) - y^{L}(A)\right) - \left(w^{H}(s) - w^{L}(s)\right) = (1-\eta)\,\delta A > 0$$

- *All workers are willing to take a wage cut because the possibility of losing skills during unemployment worsens their outside option.* This can be shown as follows. As can be seen from equations 20 and 21, the wage depends on the outside option of the worker. Because of skill erosion, part of the outside option of the worker (see equation 19) is searching for a job as a low-skilled worker. In the absence of skill erosion, all workers would be of the high-productive type. As a result the outside option would be given by $o^{NS} = b + \beta E_s \left\{(1-\gamma)\,p(s')\,S^{H}(s')\right\}$. In the presence of skill erosion, the outside option, when taking into account the relation between the surplus of being high and low-skilled (see equation 22) is given by $o^{S} = b + \beta E_s \left\{p(s')\left[(1-\gamma)\,S^{H}(s') - \eta\delta A\right]\right\}$. So in the presence of skill erosion, and for $\eta > 0$, workers' outside option is strictly lower than in the absence of it: $o^{S} < o^{NS}$. As a result, when workers have bargaining power, wages of high-productive workers are strictly lower than what they would have been in a world without the possibility of skill deterioration during unemployment

$$w^{S} - w^{NS} = o^{S} - o^{NS} = -\eta\delta\beta E_s\{A\} < 0$$

By combining equations, wages can be written as

$$w^{i}(s) = \eta\left[y^{i}(s) + \beta E_s\left\{(1-\gamma)\,\theta(s')\,\kappa - \delta(1-\eta)\,p(s')\,A'\left(u^{H}(s') + \gamma u^{L}(s')\right)\right\}\right] + (1-\eta)\,b \qquad (23)$$

4 Skill erosion as a source of inefficiency

In order to detect whether skill erosion during unemployment is a source of inefficiency, I solve the social planner's problem and compare the social planner's job creation equation with the one of the decentralized economy. Note that given the assumption of risk-neutral agents, I abstract from any form of distributive inefficiency. I only focus on the inefficiencies arising due to labor market frictions.

4.1 Social planner's problem

The social planner's problem consists out of choosing the optimal amount of jobs to create such that the utility of the representative worker is maximized. The social planner is subject to the same technological constraints, the same pattern of losing and regaining skills, and the same labor market frictions as in the decentralized economy. However, the social planner takes into account how job creation affects both labor market tightness and the quality of the labor force. The planner's problem is defined as

$$V^P(s) = \max_v \left[C(s) + b(1 - N(s)) + \beta E_s \left\{V^P(s')\right\}\right]$$

subject to

$C(s) = AN^H(s) + (1-\delta)AN^L(s) - \kappa v(s)$	aggregate resource constraint
$N^H(s) = (1-\gamma)\ N_{-1} + p(s)\,U^H(N_{-1})$	high-skilled employment
$N^L(s) = p(s)\,U^L(N_{-1})$	low-skilled employment
$N = N^H(s) + N^L(s)$	total employment
$U^H(N_{-1}) = \gamma N_{-1}$	high-skilled unemployment
$U^L(N_{-1}) = 1 - N_{-1}$	low-skilled unemployment
$U = U^H(s) + U^L(s)$	total employment
$p(\theta(s)) = B\left(\frac{v(s)}{U(N_{-1})}\right)^{1-\xi}$	job finding probability
$A = \rho A_{-1} + \varepsilon$	aggregate technology

and the state of the economy is given by $s = \{A, N_{-1}\}$

4.2 Constrained-efficient versus decentralized allocation

In order to get a better understanding of how the presence of skill erosion during unemployment and learning-by-doing affects job creation in the constrained-efficient and the decentralized allocation, I focus on two different cases. First, I discuss the standard model with homogeneous workers. This model is obtained when the human capital depreciation rate is zero $\delta = 0$. Second, I discuss the case where skill erosion and learning-by-doing are present.

4.2.1 Case 1: no skill erosion ($\delta = 0$)

- Job creation in the constrained-efficient allocation

$$\frac{\kappa}{q(s)(1-\xi)} = y - b + \beta E_s \left\{ (1-\gamma) \frac{\kappa}{q(s')(1-\xi)} + (\gamma p(s') - p(s') + (1-\gamma)(1-\xi) p(s')) \frac{\kappa}{q(s')(1-\xi)} \right\} \tag{24}$$

Note : $-\frac{\partial p(\theta(s))}{\partial U(N_{-1})} = (1-\xi) \frac{p(\theta(s))}{U(N_{-1})}$

- Job creation in the decentralized allocation

$$\frac{\kappa}{q(s)(1-\eta)} = y - b + \beta E_s \left\{ (1-\gamma) \frac{\kappa}{q(s')(1-\eta)} + \eta (\gamma p(s') - p(s')) \frac{\kappa}{q(s')(1-\eta)} \right\} \tag{25}$$

Equation 24 and 25 represent job creation in the constrained-efficient and the decentralized allocation respectively. For both allocations job creation is such that the hiring cost (LHS) equals the gains from job creation (RHS). The latter consists of the output produced by the newly hired worker y, the loss in home production b, and the continuation value of the match.

The constrained-efficient and the decentralized allocation differ in two aspects. First, as is well-understood in the literature, the presence of search frictions gives rise to congestion effects. Job creation creates congestion for the other firms: posting a vacancy increases labor market tightness θ, and hence lowers the vacancy filling probability $q(\theta)$. So it becomes more diffult for other firms to find a worker. Because this congestion effect is internalized in the constrained-efficient allocation, the hiring cost, i.e. the cost of posting a vacancy κ taking into account the probability that this vacancy gets filled and the effect of vacancy posting on labor market tightness, is not necessarily the same in both allocations. Second, the structure of the continuation value of the match is different. In both allocations it is taken into account that with probability $1-\gamma$ the match survives separation and continues producing (first term in the continuation value). However, in the decentralized allocation it is not fully taken into account that with probability $\gamma p(\theta)$, the match separates and the worker finds a new job and continues producing (second term in the continuation value). It is also not fully taken into account that being employed prevents the worker from searching and finding another job (third term in the continuation value). In the constrained-efficient allocation an additional term shows up. This reflects another congestion effect. Employing a worker implies that if the match does not separate, there will be one unemployed worker less searching for a job. This increases labor market tightness, and makes it easier for the unemployed to find a job.

A well-known result in the literature, is that the wage setting mechanism is such that the congestion externality can be fully internalized. Constrained-efficiency is obtained when the bargaining power of the workers satisfies: $\eta = \xi$. This parameter condition is known as the Hosios (1990) condition.

4.2.2 Case 2: skill erosion ($\delta > 0$)

- Job creation in the constrained-efficient allocation

$$\frac{\kappa}{q(s)(1-\xi)} = \bar{y}(s) - b$$

$$+\beta E_s\left\{(1-\gamma)\left(\frac{\kappa}{q(s')(1-\xi)}\right) + (\gamma p(s') - p(s') + (1-\gamma)(1-\xi)p(s'))\left(\frac{\kappa}{q(s')(1-\xi)}\right) + P(s')\right\} \quad (26)$$

where

$$E_s\{P(s')\} \equiv E_s\left\{(1-\gamma)\left(y^H(s') - \bar{y}(s')\right) + \gamma p(s')\left(y^H(s') - \bar{y}(s')\right) - p(s')\left(y^L(s') - \bar{y}(s')\right)\right\} \quad (27)$$

- Job creation in the decentralized allocation

$$\frac{\kappa}{q(s)(1-\eta)} = \bar{y}(s) - b$$

$$+\beta E_s\left\{(1-\gamma)\left(\frac{\kappa}{q(s')(1-\eta)}\right) + \eta(\gamma p(s') - p(s'))\left(\frac{\kappa}{q(s')(1-\eta)}\right) + D(s')\right\} \quad (28)$$

where

$$E_s\{D(s')\} \equiv E_s\left\{(1-\gamma)\left(y^H(s') - \bar{y}(s')\right) + \eta\gamma p(s')\left(y^H(s') - \bar{y}(s')\right) - \eta p(s')\left(y^L(s') - \bar{y}(s')\right)\right\} \quad (29)$$

Equation 26 and 28 represent job creation in the presence of skill erosion and learning-by-doing in the constrained-efficient and in the decentralized allocation respectively. Job creation is such that the expected hiring cost (LHS) equals the expected gains from it (RHS). The gains from job creation when $\delta > 0$ differ from the gains when $\delta = 0$. First, skill erosion leads to worker heterogeneity: there is uncertainty as to the type of worker that will be hired. This is reflected in both allocations by the term $\bar{y}$. Second, because of learning-by-doing and skill loss during unemployment, job creation affects the productivity of the labor force. This in turn affects the economy's production potential. In the constrained-efficient allocation the effect of job creation on workers' productivity, and hence on output, is captured by the term $E_s\left\{P\left(s'\right)\right\}$ (see equation 27); in the decentralized allocation these effects are represented by the term $E_s\left\{D\left(s'\right)\right\}$ (see equation 29). Both terms can be split up into three parts. The first and second part reflect job creation's expected output gain related to employment guaranteeing that workers are retrained or keep their productivity, whereas he third part reflects job creation's expected output gain related to employment preventing a worker from being unemployment, losing skills or remaining low skilled, and hence being able to search and start producing as a low-productive worker. Next, I will discuss each part in more detail:

- Part 1: with probability $1-\gamma$ the match survives separation. When the worker remains employed at the firm, the worker will be producing as a high-productive worker. This is because being employed guarantees that a worker keeps her productivity or regains her productivity when she was low-productive upon hiring. As a result, employing a high-productive worker generates a gain in terms of output: $y^H - \bar{y} > 0$. This gain is defined as the output produced by a high-productive worker y^H compared to the output that would have been produced if the firm had employed another worker $\bar{y}$. Given that firms hire workers in a random way, the output of a high-skilled worker is compared to the expected output produced by a newly hired worker $\bar{y}$. Note that if there are low-productive workers ($u^L > 0$), the output produced by a high-productive worker is strictly higher than that produced by a random worker $y^H > \bar{y}$ where $\bar{y} = y^H u^H + y^L u^L$. This effect of job creation on the economy's production is fully taken into account in the decentralized allocation as can be seen from equation 29.

- Part 2: with probability γ the match ends and with probability $p\left(\theta\right)$ the worker finds a new job in the same period as a high-skilled worker. So job creation makes sure that in case of separation, there will be another high-skilled worker searching for a job. This leads to an additional gain in terms of output: $y^H - \bar{y} > 0$.

- Part 3: having a worker employed, prevents a worker from being unemployed, losing skills or staying low-skilled, and hence being able to search as a low-productive worker. If this now low-skilled worker had found a job, which happens with probability $p\left(\theta\right)$, this worker would have produced y^L whereas if another random worker from the pool had been hired, the output would have been $\bar{y}$. For that reason, job creation leads to an additional gain in terms of output: $\bar{y} - y^L > 0$.

To detect if skill erosion is a source of inefficiency, I compare the constrained-efficient and the decentralized allocation (see equation 26 and 28 respectively). I find that job creation in the decentralized allocation is not necessarily the same as the job creation required to obtain constrained-efficiency. In the decentralized allocation, for $0 \leq \eta < 1$, the expected gains from job creation through the effect on workers' productivity are only fully taken into account when the worker stays employed at the same firm. The expected gains if a match separates or if a worker had not been hired (second and third term of equation 29 respectively) only partially show up in equilibrium. More precisely, the last two effects of job creation on output are only taken into account partially, depending on the worker's bargaining power η. Therefore, skill erosion during unemployment is a source of inefficiency. Note that in addition to the composition externality following from workers' productivity being affected by their employment status, congestion externalities are also present. The latter follows from the environment being characterized by random search.

Skill erosion during unemployment is a source of inefficiency because job creation involves a *composition externality*. Firms do not fully take into account how their job creation affects workers' productivity and hence the composition of the labor force in terms of high and low-productive workers. As discussed previously, firms' vacancy creation equation shows that firms only take into account how they affect workers' productivity when these workers stay in the firm. So a firm does not internalize its effect on a worker's skills if this worker leaves the firm (second term) or if this worker had never been hired by the firm in the first place (third term). The last two terms are still partially accounted for in equilibrium through the wages when workers have some bargaining power. This follows from workers taking into account that being in a job affects their skills and hence their outside option, as discussed in section 3.4.

4.3 Job creation and the unemployment pool's composition

Both in the constrained-efficient and the decentralized allocation (see equation 26 and 28 respectively), the composition of the unemployment pool affects job creation through two channels. First, the unemployment's pool composition at the moment of vacancy posting affects job creation because the expected output of a newly hired worker is a function of its composition. Second, the expected future composition of the unemployment pool also influences job creation. This results from the expected future composition affecting the expected future output gains following from workers' productivity being affected by their employment status (see equation 27 and 29 respectively). As mentioned previously, these gains from job creation are determined by comparing the expected output of a particular worker type $E_s\left\{y^i\left(s'\right)\right\}$ with the output that is expected to be produced if that worker were to be replaced by another worker $E_s\left\{\bar{y}\left(s'\right)\right\}$. The effects of both channels are as follows:

- *Job creation, and hence labor market tightness θ, are lower when the unemployment pool has a larger share of low-skilled workers.* This follows from the composition of the unemployment pool at the moment of vacancy posting affecting the expected output of a newly hired worker. The larger the share of low-productive workers, the lower the expected output of a newly hired worker: $\frac{\partial \bar{y}(s)}{\partial u^L(s)} = \frac{\partial\left[y^H(s)-u^L(s)\left(y^H(s)-y^L(s)\right)\right]}{\partial u^L(s)} = \frac{\partial\left(1-u^L(s)\delta\right)A}{\partial u^L(s)} < 0$. Note that a decrease in the gain of vacancy posting, lowers the RHS of equations26 and 28. So a drop in labor market tightness θ, and hence an increase in the vacancy filling probability $q\left(\theta\right)$, is required to restore equality.

- *Job creation's expected output gain related to employment guaranteeing that workers are retrained or keep* *their productivity, is increasing in the expected future share of low-skilled workers in the unemployment pool*:

$$\frac{\partial E_s\left\{y^H(s') - \bar{y}(s')\right\}}{\partial E_s\left\{u^L(s')\right\}} = \frac{\partial E_s\left\{u^L(s')\left(y^H(s') - y^L(s')\right)\right\}}{\partial E_s\left\{u^L(s')\right\}} = \frac{\partial u^L(N)\, E_s\left\{\delta A'\right\}}{\partial u^L(N)} > 0$$

The expected output of a newly hired worker is decreasing in the fraction of low-productive workers in the unemployment pool $\left(\frac{\partial E_s\{\bar{y}(s')\}}{\partial E_s\{u^L(s')\}} < 0\right)$. So the difference between the output generated by a high-skilled worker and the output generated if another, random worker from the unemployment pool were to be employed, increases. The intuition is as follows. The larger the expected share of low-skilled unemployment, the smaller the expected chance that a high-productive worker will start producing if a new worker were to be hired. Consequently, the expected gain of guaranteeing that workers are retrained or keep their productivity, and hence the gain of job creation, is higher when fewer high-productive workers are expected to be available.

- *Job creation's expected output gain related to employment preventing a worker from being unemployed, losing skills or remaining low skilled, and hence being able to search and start producing as a low-productive worker, is decreasing in the expected future share of low-skilled workers in the unemployment pool*:

$$\frac{\partial E_s\left\{\bar{y}(s') - y^L(s')\right\}}{\partial E_s\left\{u^L(s')\right\}} = \frac{\partial E_s\left\{\left(1 - u^L(s')\right)\left(y^H(s') - y^L(s')\right)\right\}}{\partial E_s\left\{u^L(s')\right\}} = \frac{\partial\left(1 - u^L(N)\right) E_s\left\{\delta A'\right\}}{\partial u^L(N)} < 0$$

The expected output of a newly hired worker is decreasing in the fraction of low-productive workers in the unemployment pool $\left(\frac{\partial E_s\{\bar{y}(s')\}}{\partial E_s\{u^L(s')\}} < 0\right)$. So the difference between the output generated if another, random worker from the unemployment pool were to be employed compared to employing this worker who would have lost some of her productivity, decreases. The intuition is as follows. The expected gain of preventing a worker from being unemployment, losing skills or remaining low skilled, and hence being able to search and start producing as a low-productive worker, is lower the more low-productive workers are expected to be available. This is because the larger the expected share of low-skilled unemployment, the larger the expected chance that a low-productive worker will start producing if a worker were to be hired. Thus, the gains of creating jobs today and hence preventing to have another low-productive worker searching for a job in the next period, decreases, because it is expected to be very likely anyways that a low-productive worker will get hired.

5 Attaining constrained-efficiency

In the absence of skill erosion during unemployment, congestion is the only source of inefficiency. Hosios (1990) has shown that the congestion externality is internalized when workers' bargaining power equals the elasiticity of unemployment in the matching function: $\eta = \xi$. This parameter condition restores constrained-efficiency both in the absence and in the presence of aggregate shocks. In the presence of skill erosion, the same parameter condition internalizes the congestion externality.

- *In the presence of skill erosion, the congestion externality is internalized for the same parameter condition for workers' bargaining power η as in the absence of skill erosion during unemployment: $\eta^* = \xi$.* This can be explained as follows. The congestion effect refers to job creation affecting labor market tightness θ, which affects the hiring probability, and hence the probability that output is generated. The same parameter condition internalizes the congestion externality because there is no interaction between the congestion effect and the unemployment pool's composition. More precisely, the unemployment's pool composition does not interact with the hiring probability, because a worker's hiring probability is independent of a worker's skill level. As result, when labor market tightness changes, the hiring probability for each worker type is affected in exactly the same way. Therefore, the effect of job creation on labor market tightness, and hence on the probability that a match starts producing, is independent of the composition of the unemployment pool.

Following Hosios (1990), I analyze if there exists a condition for the workers' bargaining power for which both the congestion and the composition externality are internalized.

- *In the absence of aggregate shocks, there exists a parameter condition for workers' bargaining power η such that constrained-efficiency is obtained.* This can be shown by solving for workers' bargaining power such that job creation in the decentralized allocation equals job creation in the constrained-efficient allocation.

- *In the presence of aggregate shocks, there exists no longer a parameter condition for workers' bargaining power η such that constrained-efficiency is obtained. Internalizing both the congestion and the composition externality requires workers' bargaining power to be state-dependent.* Comparing the constrained-efficient and the decentralized allocation (see equation 26 and 29 respectively), shows that for the parameter condition $\eta^* = \xi$, only the congestion externality is internalized. Internalizing the composition externality requires a value of $\eta^* = 1$. Because both conditions are mutually exclusive, there exists no longer a bargaining power η constant across states, for which the decentralized alloction replicates the constrained-efficient allocation. Obtaining constrained-efficiency without policy intervention is only possible when the worker's bargaining power η is allowed to be a function of the economy' s state s.

6 Optimal labor market policy

When workers' bargaining power η is state-independent and the economy is subject to aggregate shocks, the congestion and the composition externality cannot be internalized at the same time by imposing a parameter restriction on η. Therefore, I analyze the labor market policy which can restore constrained-efficiency in this environment. I assume that financing goes through some lump sum taxation. Thus, there is no distortion arising from the financing method and the policy maker can always replicate the constrained-efficient allocation. This allocation can be implemented by using a single instrument because there is only one decision to be made, namely job creation.

6.1 Optimal employment subsidy

When employment is subsidized/taxed $\Phi(s)$, firms' value of having a worker employed becomes

$$J^i(s) = y^i(A) - w^i(s) + \Phi(s) + (1-\gamma)\beta E_s\left\{J^H(s')\right\}$$

Job creation in the decentralized allocation is now given by

$$\frac{\kappa}{q(s)(1-\eta)}{=}\bar{y}(s){+}\Phi(s) - b$$

$$+\beta E_s\left\{(1-\gamma)\left(\frac{\kappa}{q(s')(1-\eta)}\right) + \eta\left(\gamma p(s') - p(s')\right)\left(\frac{\kappa}{q(s')(1-\eta)}\right) + D(s')\right\} \tag{30}$$

where $D(s)$ is defined by equation29. Note that because the employment subsidy/tax affects total match surplus, wages are also affected. The solution to the Nash bargaining problem is

$$w^i(s) = \eta\left(y^i(s) + \Phi(s)\right) + (1-\eta)\,o(s)$$

It can be seen from the above equation the in the presence of an employment subsidy $\Phi(s) > 0$, wages are higher than in the absence of it. The positive effect of employment subsidies on wages has also been emphasized by Mortensen and Pissarides (2001).

The employment subsidy/tax $\Phi(s)$ for which job creation in the decentralized allocation (equation 30) is the same as job creation in the constrained efficient allocation (equation 26) is defined as

$$\Phi(s) \equiv \frac{(\eta-\xi)}{(1-\xi)(1-\eta)\,q(\theta(s))}\kappa{+}\beta E_s\left\{\frac{(\eta-\xi)(1-\gamma)(\eta p(\theta(s'))-1)}{(1-\xi)(1-\eta)\,q(\theta(s'))}\kappa + \delta(1-\eta)\,p(\theta(s'))\,A'\left(1-(1-\gamma)\,u^L(N)\right)\right\}$$

Since the congestion externality is already well-understood in the literature, I focus on the case where the congestion externality is fully internalized and only the composition externality remains. As can be seen from the above equation, for $\eta = \xi$, the optimal employment subsidy $\Phi(s)$ becomes

$$\Phi(s) = \delta(1-\xi)\beta E_s\left\{p(\theta(s'))\,A'\left[1-(1-\gamma)\,u^L(s')\right]\right\} \tag{31}$$

6.2 Properties of the subsidy

When skills erode during unemployment ($\delta > 0$), it is optimal to subsidize employment: $\Phi(s) > 0$. Thus, in the decentralized allocation not enough jobs are created to obtain constrained-efficiency. This results from the nature of the inefficiency. Skill erosion and learning-by-doing are sources of inefficiency because two gains from job creation are not fully taken into account: job creation's expected output gain related to employment guaranteeing that workers are retrained or keep their productivity in the case a match separates, and job creation's expected output gain related to employment preventing a worker from being unemployed, losing skills or remaining low skilled, and hence being able to search and start producing as a low-productive worker. The optimal employment subsidy captures precisely the parts of these gains that are not taken into account in the decentralized allocation. This can be seen easily from equation 32 which is obtained from rearranging the expression for the subsidy given by equation 31.

$$\Phi(s) = (1-\xi)\beta E_s\left\{X(s')\right\} \tag{32}$$

where $X(s) \equiv p(s)\left[\gamma\left(y^H(s) - \bar{y}(s)\right) + \left(\bar{y}(s) - y^L(s)\right)\right]$

Moreover, the subsidy is increasing in job creation's expected output gains ignored in the decentralized allocation: $\frac{\partial \Phi(s)}{\partial E_s\{X(s')\}} > 0$. Consequently, the subsidy has the following properties:

- *The optimal employment subsidy is increasing in the human capital depreciation rate* δ:$\frac{\partial \Phi(s)}{\partial \delta} > 0$. This is because the expected output gains both from retraining and preventing a worker from losing skills are increasing in the fraction of skills that the worker regains and would have lost: $\frac{\partial E_s\left\{y^H(s') - \bar{y}(s')\right\}}{\partial \delta} = \frac{\partial \delta E_s\left\{A' u^L(s')\right\}}{\partial \delta} > 0$ and $\frac{\partial E_s\left\{\bar{y}(s') - y^L(s')\right\}}{\partial \delta} = \frac{\partial \delta E_s\left\{A'\left(1-u^L(s')\right)\right\}}{\partial \delta} > 0$

- *The optimal employment subsidy is decreasing in the expected share of low-productive workers in the unemployment pool*: $\frac{\partial \Phi(s)}{\partial E_s\{u^L(s')\}} < 0$. As discussed in section 4.3, the expected gains from retraining and prevention of skill erosion depend on the output expected to be generated if another, random worker from the unemployment pool were to be employed. The expected gains from retraining increase with the expected share of low-productive workers in the unemployment pool, whereas expected gains from prevention of skill deterioration decrease with the expected share of low-productive workers in the unemployment pool. Whether the optimal employment subsidy is increasing or decreasing in the expected share of low-productive workers, depends on the respective weights of each gain in the subsidy. Because the probability that a match separates is small $\gamma < 1$, the weight on the expected gains from prevention of skill erosion is larger than the weight on the expected gains from retraining, as can be seen from equation 32. Therefore, the employment subsidy is decreasing in the expected share of low-productive workers in the unemployment pool.

- *The optimal employment subsidy is increasing in the expected job finding or hiring probability*: $\frac{\partial \Phi(s)}{\partial E_s\{p(\theta(s'))\}} > 0$. This can be explained as follows. The employment subsidy captures the gains from job creation related to preventing workers from losing skills and retraining workers such that when a match separates a high-productive worker is looking for a job. However, these gains from job creation only matter when the worker would have been hired after losing skills and when the fired retrained-worker would have been hired again. This is because the gains from retraining and prevention of skill loss are only realized when the worker whose skills are affected, actually starts producing with this specific productivity level. The higher the expected job finding probability, the higher the chance that future output will end up being affected by the change in skill type. Therefore, both the gains from retraining workers and the gains from preventing workers from losing skills, are increasing in the expected job finding probability.

- *The employment subsidy is increasing in expected aggregate technology*: $\frac{\partial\Phi(s)}{\partial E_s\{A'\}} > 0$. This results from the output generated by each worker type is a function of aggregate technology. Therefore, the expected output gains both from retraining and preventing a worker from losing skills are also a function of aggregate output: $\frac{\partial E_s\{y^H(s')-\bar{y}(s')\}}{\partial E_s\{A'\}} = \frac{\partial \delta E_s\{A' u^L(N)\}}{\partial E_s\{A'\}} > 0$ and $\frac{\partial E_s\{\bar{y}(s')-y^L(s')\}}{\partial E_s\{A'\}} = \frac{\partial \delta E_s\{A'(1-u^L(N))\}}{\partial E_s\{A'\}} > 0$

- *The optimal employment subsidy is procyclical.* This follows from the previous three properties of the subsidy.

7 Labor market outcomes

In this section, I analyze how differences between job creation in the decentralized and the constrained-efficient allocation translate into differences between labor market outcomes such as unemployment. This is done for the outcomes both in the absence and in the presence of aggregate shocks. Moreover, it is shown how labor market outcomes in the decentralized allocation differ from outcomes in the constrained-efficient allocation for different degrees of skill loss, when skill erosion is the only source of inefficiency.

7.1 Parameterization

I adopt the following parameterization strategy. I parameterize the model for the baseline case where $\delta = 0$ and $\eta = \xi$. When $\delta = 0$, there is no skill loss during unemployment and the only source of inefficiency is congestion. So for the parameter condition $\eta = \xi$, labor market outcomes in the decentralized and the constrained-efficient allocation are the same. To analyze how labor market outcomes in the decentralized allocation differ fromout-comes in the constrained-efficient allocation for different rates of human capital depreciation δ when skill erosion is the only source of inefficiency, I keep all the parameter values of the baseline case constant and alter only the value of δ.

I set the length of a period to one quarter given that it is assumed that workers become productive in the same period. The latter is an assumption which has become standard in the recent business cycle literature. Because the baseline case ($\delta = 0$) corresponds to the standard search model, I can use parameter values commonly used in the literature. I parameterize for the US economy and set the discount factor β to 0.99; the elasticity of vacancies in the matching function $1 - \xi$ is set to 0.5 and the bargaining power of the workers equals 0.5 in order to offset the congestion externality; the separation rate γ is 0.1; home production is set to $b = 0.5036$ which implies a replacement ratio of 0.54. Finally I set steady state employment and the steady state vacancy filling probability to 0.95 and 0.7 respectively. After having pinned down the values for these varaibles, I obtain a value for the vacancy posting cost κ and the matching efficiency B from the model's equilibrium conditions: $\kappa = 0.4334$ and $B = 0.6773$.

In what follows, I keep the values of all parameters, besides skill loss δ, fixed at their value obtained from the baseline parameterization. These values are summarized in the table below. This allows me to analyze both how labor market outcomes are affected by skill erosion, and how these outcomes change for different degrees of human capital depreciation when the rest of the economic environment is kept constant.

Parameter	Parameter	Value
Discount factor	β	0.99
Elasticity of vacancies in matching function	$1 - \xi$	0.5
Bargaining power workers	η	0.5
Separation rate	γ	0.1
Home production	b	0.5036
Vacancy posting cost	κ	0.4334
Matching efficiency	B	0.6773

7.2 Labor market outcomes in the steady state

Figure 1 shows the steady state labor market outcomes in the decentralized (inefficient) versus the constrained-efficient allocation for different degrees of skill loss. Steady state unemployment after hiring took place is depicted in the upper left corner. First of all, it can be seen that in the constrained-efficient allocation unemployment is *decreasing* in the human capital depreciation rate δ. The intuition behind this finding is that unemployment in the presence of skill erosion is undesirable because workers who suffer from skill erosion during unemployment produce less output upon reemployment than workers who have not suffered from skill erosion. The more skills erode, the more costly unemployment becomes in terms of output loss. Second, for all values of δ, unemployment in the constrained-efficient allocation is *lower* than in the decentralized allocation. Recall that the skill erosion during unemployment is the only source of inefficiency in the decentralized allocation. As discussed in section 4, in the decentralized allocation not all the gains from job creation are taken into account. Consequently, unemployment is higher than in the constrained-efficient allocation. However, also in the inefficient allocation, unemployment is decreasing in the human capital depreciation rate δ. The latter can be explained by workers being willing to accept a lower wage because their outside option worsens when they face the risk of losing skills during unemployment. This mechanism has been discussed in section 3.4.2, and is shown in figure 2.

Figure 1 also shows that in both allocations the share of low-productive workers in the unemployment pool $\left(u^L\right)$ is decreasing in the human capital depreciation rate δ. This in turn results in a larger share of employed workers being high-skilled $\left(n^H = N^H/N\right)$. Finally, the bottom right corner of figure 1 shows that the job

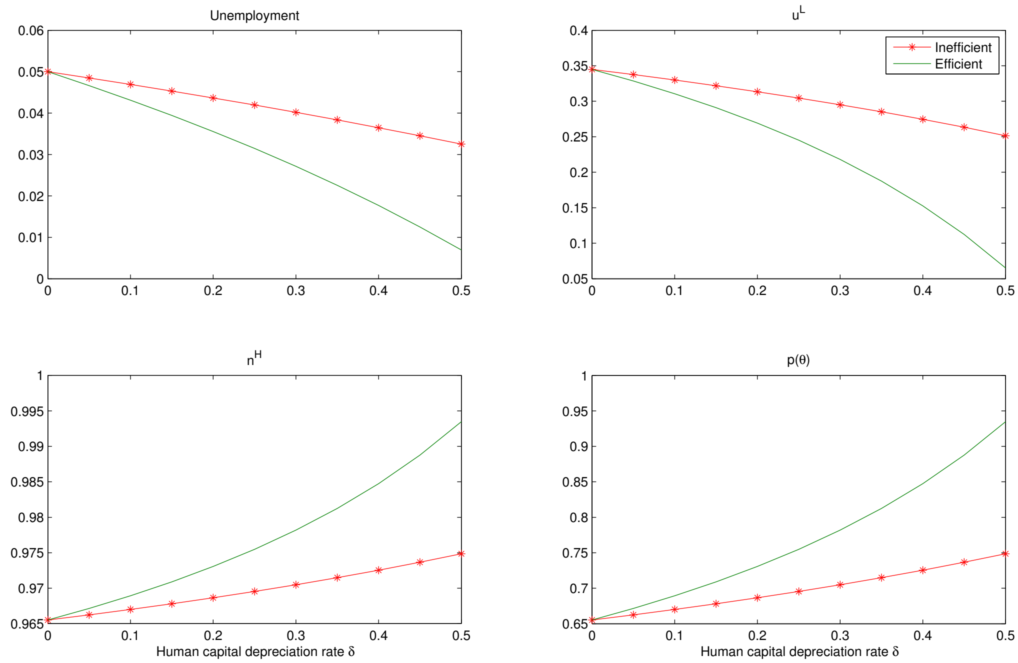

Figure 1: Labor market outcomes in the decentralized (inefficient) versus the constrained-efficient allocation for different degrees of skill loss

finding probability $p(\theta)$ is increasing in the human capital depreciation rate. This pattern supports both the lower unemployment rate and the lower share of low-productive workers in the unemployment pool.

As discussed in section 6, the composition externality can be offset by implementing the appropriate employment subsidy. Figure 3 shows that this subsidy is increasing in skill loss.

7.3 Labor market outcomes in the presence of aggregate shocks

This section analyzes the economy's response to a 1% decrease in aggregate technology relative to its steady state level, where aggregate technology follows and AR(1) process and $\rho_a = 0.95$. Unemployment outcomes are represented as the deviation from its steady state value expressed in percentage points. All the other variables are represented as relative deviations from steady state expressed in percent. Recall that different values of the human capital depreciation rate δ represent different steady states.

Figure 4 represent the response of unemployment and employment both for the inefficient decentralized and the constrained-efficient allocation. Both in the decentralized and in the constrained-efficient allocation the percentage point increase in unemployment is larger, the higher the skill loss. Note that the percentage point increase in the constrained-efficient allocation is slightly larger than in the decentralized allocation. This can be explained by the gains from job creation, related to retraining a worker and preventing a worker from losing skills, being only partially taken into account in the decentralized allocation. As discussed in section 4.3, the

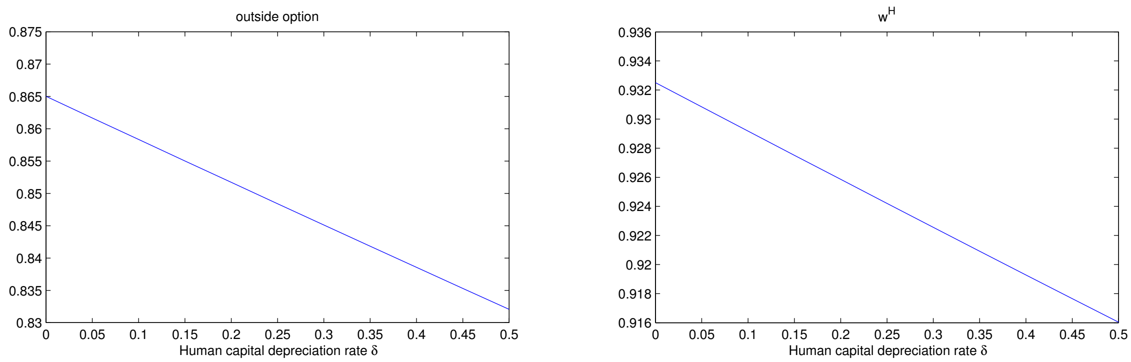

Figure 2: Wage and outside option of the high-productive workers

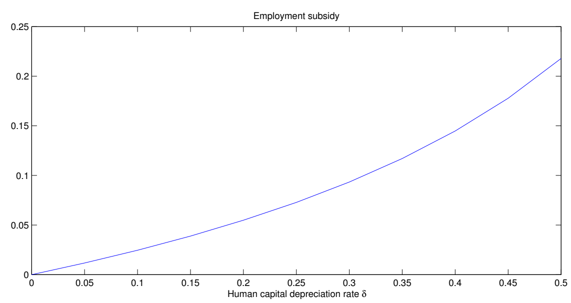

Figure 3: The optimal employment subsidy for different degrees of skill loss

gains from job creation related to preventing a worker from losing skills, decrease during a recession. Given that this is fully taken into account in the constrained-efficient allocation, job creation decreases more, and the percentage point increase in unemployment is larger. However, this does not imply that overall unemployment is higher in the constrained-efficient allocation, since steady state unemployment is lower in the constrained than in the inefficient allocation. Moreover, in both allocations unemployment is more persistent for higher values of δ. The latter can be explained by low-productive workers being less attractive the less productive they are. As a result, when the fraction of low-skilled workers in the unemployment pool increases during a recession, the incentive to post vacancies for firms decreases more for higher values of the human capital depreciation rate.

Figure 5 represents both the response of the optimal employment subsidy and its components. As discussed in section 6, the optimal employment subsidy is procyclical. When the shock hits the economy, the employment subsidy decreases relative to its steady state value, where this drop is larger for higher values of δ. In line with the properties of the employment subsidy discussed in section 6.2, the decrease in the employment subsidy is driven by the decrease in the job finding probability, and the decrease in the output gains from preventing a worker from being unemployed, losing skills, and hence being able to search as a low-productive worker $\left(\bar{y} - y^L\right)$, offsetting the increase in the output gains from retraining a worker $\left(y^H - \bar{y}\right)$. Note that despite the drop in aggregate productive, the output gains of retraining a worker becomes positive relative to its steady state value after less than 2 quarters. The reason is that a recession leads to a deterioration of the quality of the unemployment pool,

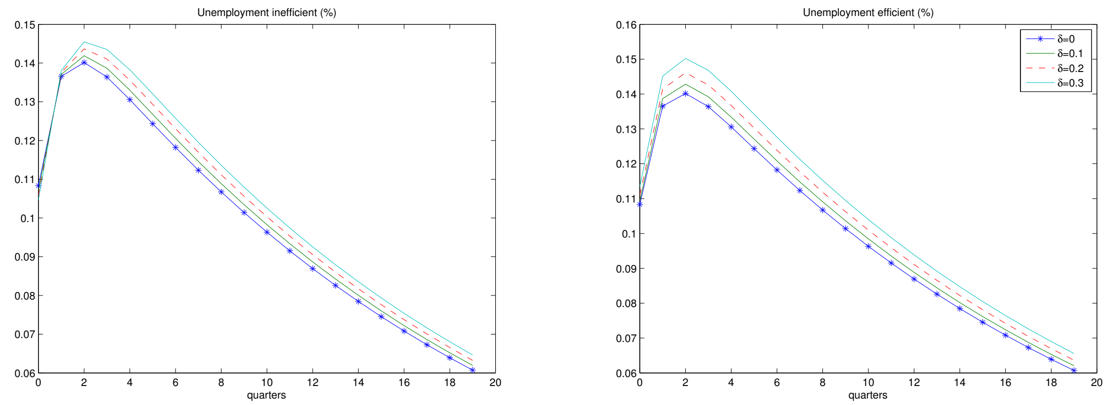

Figure 4: Unemployment outcomes in response to a negative persistent aggregate technology shock

and hence a decrease in $\bar{y}$ independent of the decrease in aggregate technology. Consequently, the difference in the output produced by a high-productive worker compared to another, random worker from the unemployment pool, increases.

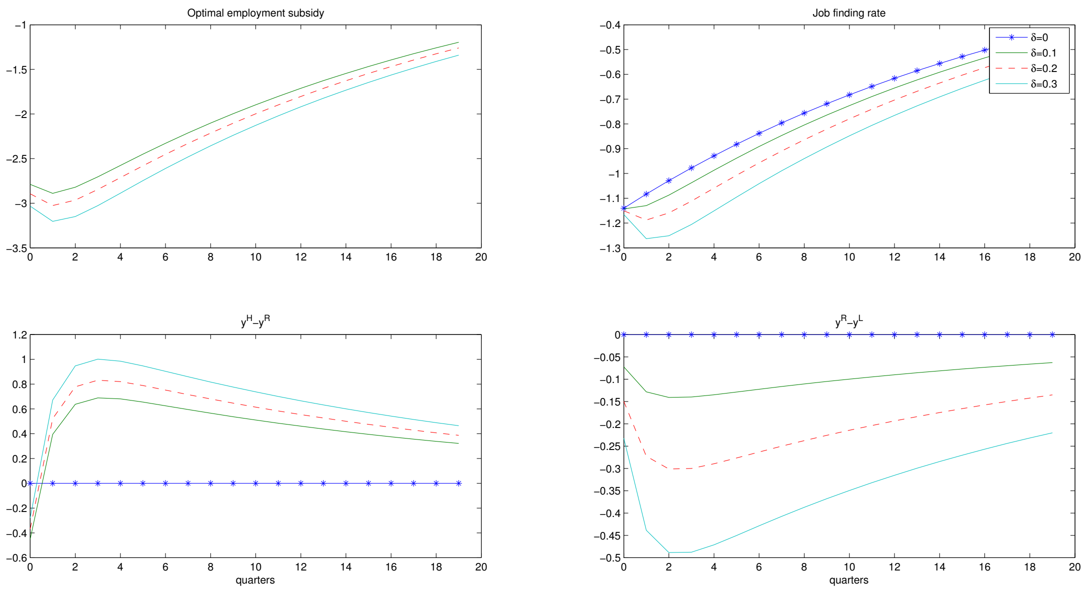

Figure 5: Optimal employment subsidy in response to a negative persistent aggregate technology shock

8 Conclusion

This paper shows that the presence of skill erosion during unemployment calls for policy intervention. The framework of analysis is a standard search and matching model a la Diamond-Mortensen-Pissarides with aggregate uncertainty, risk-neutral workers and skill loss during unemployment.

First of all, I show that skill erosion during unemployment is a source of inefficiency by comparing labor market outcomes in the constrained-efficient and the decentralized allocation. The intuition behind this finding is the following. In addition to the congestion externality following from random search, a composition externality is present: firms do not fully take into account how their job creation affects workers' unemployment duration, and hence the possibility that workers' skills deteriorate.

I find that when wages are set through period-by-period Nash bargaining, and aggregate uncertainty is present, constrained-efficiency can no longer be achieved by imposing a parameter restriction on workers' bargaining power when the latter is constant across states. This is in contrast to the finding in a framework where the unemployed are not exposed to skill loss, as shown by Hosios (1990). Consequently, in the presence of skill deterioration policy intervention is required to restore constrained-efficiency.

The optimal labor market policy which offsets the composition externality following from the presence of skill erosion during unemployment takes the form of an employment subsidy. This reflects job creation in the laissez-faire economy being too low to attain efficient labor market outcomes. Moreover, I show that the optimal employment subsidy is procyclical. This finding captures that the composition externality matters more in booms than during recessions. This in turn can be explained by the procyclicality of the social cost of letting a worker lose skills.

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