Mixed search

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Abstract

In search theory, an important distinction can be drawn between models with directed search and with random search. In the present paper, we present a model where workers and firms meet through both directed and undirected search; we call this mixed search. We argue the model is qualitatively consistent with patterns of wages across job-to-job transitions. Finally, we present a simple calibrated example to illustrate the quantitative implications of the model.

Keywords: competitive search, random search, directed search, register data

JEL Classification Numbers: J30, J60

1 Introduction

Motivated by patterns of old and new wages across job-to-job transitions, we formulate a competitive search model with on the job search and idiosyncratic shocks to match productivity. A simple calibrated version of the model is used for simulations to illustrate the model's quantitative properties.

The key innovation of our model is the combination of random and competitive search in a single model of on-the-job search. In search theory, a distinction can be made between models of competitive or directed search on the one hand, and on the other hand models with random or undirected search. In this paper, we present a simple model of mixed search, where workers are able to direct their search some of the time, and at the same time meet employers through random matching.

The second feature of the model is stochastic match quality. The productivity of a filled job is a stochastic function of the firm's investment level, with realized match productivity revealed to both parties after the match is formed. In our model, with linear preferences and efficient contracts, some job-to-job transitions will be associated with wage cuts, as workers in unproductive matches affected by adverse productivity shocks move to jobs with higher expected productivity, which may pay a lower wage than their current job.

While some job-to-job transitions are characterized by large wage jumps most transitions are associated with small upwards adjustment in wages. Such small wage movements are consistent with the pure job ladder predicted by competitive search where workers current wage directly influences optimal search strategies, with low earning workers directing their search efforts to submarkets with a higher labor market tightness tightness and lower posted wages. At the same time, a substantial number of job to job transitions are accompanied by a decrease in wages. Using data on 11 countries, Jolivet et al. (2006) find between 18 and 38% of job-to-job transitions are associated with a wage cut, using administrative register data on workers and firms we find similar patterns. The model we present aims to account for these patterns. In addition, the model is consistent with a positive relationship between firm size and productivity, positive association between wage and job duration, negative duration dependence of hazard rates of jobs.

Related work includes the model in Garibaldi & Moen (2010) of competitive on-the-job search, though the present paper considers only the case where the productivity distribution is endogenous. There is a significant literature on the effects of job specific productivity shocks on job creation and destruction in random matching models (Mortensen & Pissarides 1994), while Guerrieri (2007, 2008) also feature models of competitive search with shocks to worker effort/training costs. The mixture of random and competitive search is in some ways a labor market analogy to search theroetic models of product markets with informed and uninformed consumers, e.g. Lester (2011). Finally, there is a large literature using microdata to estimate job search models (see, e.g. Postel-Vinay & Robin 2002, Cahuc et al. 2006); this paper follows Postel-Vinay & Turon (2010) in looking specifically at wage dynamics in an explicit search theoretic framework.

The paper is organized as follows: Section 2 presents the model and defines equilibrium, while the properties of the model are discussesd in section 3. section 4 presents some data on wage dynamics of job-to-job transitions; we argue the patterns are qualitatively consistent with the model's prediction. To assess the model's quantitative predictions, section 5 presents calibrated versions of the model with different degrees of random vs directed search. Section 6 presents some preliminary concluding remarks.

2 Model

Workers and firms face two matching technologies: directed search and random matching. Match quality is stochastic and revealed to both workers and firms immediately after a match is formed.

Firms pay investment costs K_i before they enter the market; we assume firms constantly post one vacancy. We denote by π_i the share of vacancies posted by type i firms. The total number of workers in the economy is normalized to 1: we denote by u_j^i the measure of workers employed in firms of type j with realized match productivity flow y_i, and let u_0^0 denote the unemployment rate.

The model is one of submarkets, where a market is composed of all firms paying a certain wage w and all workers who are searching in that market. The matching function in each submarket is

$$m = Au^{\beta}v^{1-\beta} \tag{1}$$

Workers and firms have two channels of matching. Workers search actively, choosing which submarket to direct their search to to maximize the value of search, matching according to the matching function in (1). At the same time, at an exogenous rate a, workers randomly meet firms, making a random draw from the distribution of vacancies, accepting all job offers with a contract greater than the current joint NPV of the job.

In the current model setup, the arrival rate of job offers through random matching, a, is simply an exogenous poisson rate, similar to the job offer arrival rate λ in Burdett & Mortensen (1998) when job offer arrival rates are identical for employed and unemployed workers. It would be straightforward to consider a an endogenous outcome of a separate matching technology, where a would depend on the aggregate market tightness $\Theta = \frac{v}{1-u_J}$ through a second matching function $a = m^R(\Theta)$, we keep a exogenous for the time being.

Match productivity is uncertain at the time of hiring. A firm investing $K = K_i$ at the time of entry generates a production flow $y = y_i$ in productive matches. With probability δ_{ij}, the hire is a mismatch, yielding a lower productivity flow $y_j < y_i$, for $j = 0, ..., i-1$. With probability $1 - \sum_j \delta_{ij}$ the match is productive, yielding the higher production flow y_i corresponding to the firm's investment level, for the duration of the match.

Wage contracts specify a wage w, which is independent of realized productivity, and on-the-job search possibilities, which are conditional on realized match productivity.

Once a match is formed (and the contract is signed), true match quality is revealed to both the worker and the firm. There is no recall employment, i.e. workers cannot return to their old employer, but have to keep searching while employed in the unproductive match with the new employer.

The realized joint value of a match to a worker and a firm will only depend on realized match productivity, however the realized division of surplus between the worker and the firm will depend on both type of firm/investment level as well as realized productivity, as the wage w[1] is determined before true productivity is revealed.

In the following, W_i^p denotes the expected NPV of contracts posted by type i firms, W_i^r refers to expected NPV of contracts for workers and firms who meet through random matching[2].

[1] which in this model is simply a transfer from the firm to the worker

[2] As a result of how we model wage setting in random matches, these will be equal in equilibrium though they are determined independently

2.1 Workers

We assume workers search efficiently on the job, that is, there are no internal coordination problems in the firm (Moen & Rosén 2004), meaning workers search on the job so as to maximize joint income. A further implication of this assumption is that the worker's reservation wage is given by the value of his current job. Unemployed workers receive (exogenous) unemployment benefits y_0.

A worker employed in a firm with flow value of production y_i, the worker searches for jobs so as to maximize gains from search:

$$rM_i = y_i + s(M_0 - M_i) + \max_k p(\theta_k)(W_k^p - M_i) + a \sum_{j>i} \pi_j (W_j^r - M_i) \tag{2}$$

Here, the first term y_i represents flow value of production on the job [3], the second term is expected loss from separation. The third term $p(\theta_k)(W_k - M_i)$ is the expected gain from directed job search; $p(\theta_k)$ is the arrival rate of job offers with posted contracts W_k^p to workers. The final term here represents the expected gain from job search through random matching. At rate a, a worker and employer meet at random, with the working accepting all wage offers which are more valuable than their current job.

From equation (2) it is clear that different combinations of job offer arrival rates and expected values of posted contract give the same value M_i. Rearranging, we get the indifference curve for each worker type - combinations of θ, W yielding the same expected gain from job search $p(\theta_k^c)(W - M_i)$

$$\begin{aligned} \theta_i &= g_i(W; M_i, M_0) \\ &= p^{-1}\left(\frac{rM_i - y_i - s(M_0 - M_i) - a\sum_{j>i}\pi_j(W_j^r - M_i)}{W_k^p - M_i}\right) \end{aligned} \tag{3}$$

The relationship between advertised wage and θ:

$$\theta(W) = \min_i (g_i(W; M_i, M_0)) \tag{4}$$

The joint value of a type J job is given by

$$\begin{aligned} rM_J &= y_J + s(M_0 - M_J) \\ M_J &= \frac{y_J + sM_0}{r+s} \end{aligned} \tag{5}$$

We assume workers employed in the most productive firms do not search at all, never ever, not even through random matching.

[3] unemployment benefits for $i = 0$

2.2 Firms

Like workers, firms have two ways of meeting workers, through directed search (wage posting) and through random matching. We assume firms continuously post a single vacancy (the "fishing line" assumption). Firms of each type (investment level) posts wage contracts to maximize the value of the vacancy taking into account that the expected arrival rate of workers to vacancies depends on posted wages. Firms are also randomly matched with workers at rate $\tilde{a}$; we assume efficient contracts so workers will accept the job if the value of the contract W_i^r is greater than the joint value of their current job.

The value of a vacancy can then be written

$$rV_i = q(\theta_i^c)(E[M|K = K_i] - W_i^p)] + \tilde{a}_i(E[M|K_i] - W_i^r) \tag{6}$$

where $\tilde{a}_i$ is the rate that type i firms hire workers through random matching:

$$\tilde{a}_i = \Theta^{-1} a h_i$$

where Θ is the ratio of vacancies to searching workers in the economy as a whole and h_i is the probability that a worker is hired once a match is made. Once a match happens, the probability that the worker is hired is equal to the probability that their current job has a joint value less than W_i^r. Wages in random matches are assumed to be set through bilateral bargaining, so workers will accept any job where there is a surplus to be shared, that is, where the joint value of their current job has a joint value less than $E[M|K_i]$, which will be the case as long as current match productivity is lower than y_i:

$$h_i = \frac{\sum_k \sum_{j<i} u_j^k}{1 - u_J}$$

At entry, firms choose investment level and corresponding flow value of production in productive matches $\{y, K\}$ to maximize net profits; we assume free entry in each market.

Inserting for the expected joint value of a job in a firm with investment level K_i

$$\begin{aligned} E[M|K = K_i] &= (1 - \sum_j \delta_{ij})M_i + \sum_j \delta_{ij} M_j \\ &= M_i + \sum_j \delta_{ij}(M_j - M_i) \end{aligned} \tag{7}$$

equation (6) can be written

$$rV_i = q(\theta_i)[M_i + \sum_j \delta_{ij}(M_j - M_i) - W_i^p] + a\Theta^{-1} \frac{\sum_{j=0}^{i-1} u_i}{u_J} [M_i + \sum_j \delta_{ij}(M_j - M_i) - W_i^r]$$

We argue any equilibrium where more than one investment level is chosen in equilibrium will be characterized by a job ladder where workers always wish to search for jobs in the submarket immediately above the one they currently work in. That is, only type $i-1$ workers direct their search toward type i firms. The argument is the same as in the working paper version of Garibaldi and Moen 2010 (with the extension of endogenous productivity distribution). To see why this should be the case, consider the case where there are two investment levels to choose from. Moreover, assume parameters are such that in the absence of on-the-job search, all firms would choose the lowest investment level[4], and suppose we were to open for on-the-job search. Then the "type 2" market has to open; if this were not the case, any firm entering the market as a type 2 firm would attract the search of all type 1 workers, achieving infinite profits, which clearly cannot be an equilibrium. Second, the market of unemployed workers and type 2 firms will be empty, as no type 2 firms would want to search for unemployed workers (by assumption).

Type i firms posting wage contracts maximize the value of vacancy given by (6) subject to the indifference curve (3) of type $i-1$ workers

The first order condition is:

$$\theta_i'(W^p) = \frac{q(\theta_i)}{q'(\theta_i)(E[M|K=K_i] - W^p)}$$

2.3 Solving for equilibrium

2.3.1 Equilibrium posted wages

Firms of type i set wage W_i^p to maximize the value of a vacancy given by (6) taking into account that type $i-1$ workers search on-the-job so as to maximize rM_{i-1}^c. To solve for W_i^*, use the first order condition for maximization of rM_{i-1}^c wrt W (from (2)):

$$\theta^{c\prime}(W) = -\frac{p(\theta^c)}{p'(\theta^c)(W - M_{i-1})}$$

and the first order condition for maximization of rV_i wrt W (from (6)):

$$\theta^{c\prime}(W) = \frac{q(\theta^c)}{q'(\theta^c)(E[M|K=K_i] - W)}$$

Solving for optimal posted contract of a type i firm, W_i^{p*}:

$$\begin{aligned} W_i^{p*} &= \beta E[M|K=K_i] + (1-\beta)M_{i-1} \\ &= \beta(M_i + \sum_j \delta_{ij}(M_j - M_i)) + (1-\beta)M_{i-1} \end{aligned} \tag{8}$$

[4]If this were not the case, allowing workers to search on the job would leave equilibrium unchanged, with only one investment level. However, as we aim to show that the job ladder property holds whenever there is more than one investment level, this case is not relevant.

2.3.2 Wage setting in random matching

Until now, we have made the implicit assumption that optimal posted wage contracts can be solved for without considering random matching, as the processes are separate. This simplifies the analysis considerably, however it is worth noting that this need not be the case. If the two processes are linked, e.g. if firms have to offer the posted wage to random matches, and random search is sufficiently important, i.e. when a is large, some higher type firms may find it optimal to post a contract worth less than W_i^{p*}, and attract only random matches. The argument is essentially the same as in Lester (2011). The existence of such "rip-off" firms would complicate solving for equilibrium, however the qualitative conclusions of our model would not be affected, as it would always be optimal for some firms to post wages W_i^{p*} as outlined above as long as $a < 1$.

For now, we bypass these consideration entirely, and impose the following wage setting in random matches. In random matches, wages are set through Nash bargaining under Hosios, with the worker's outside option is given by the joint value of the highest type job of which a worker would want to join the poaching firm. That is, unemployed workers meeting a type 2 firm will bargain over the distribution of M_2 using M_1 as their outside option. To rationalize this, one story can be that while a worker can observe type of the firm before bargaining takes place, the firm cannot observe the type of the worker (i.e. his current employer). This assumption simplifies the model as it ensure wages in jobs resulting from random matches pay the same wage as wages in jobs where the worker and firm meet through competitive search, that is, we will have a one-to-one relationship between firm type and wage.

$$W_i^{r*} = \beta(M_i + \sum_j \delta_{ij}(M_j - M_i)) + (1-\beta)M_{i-1} = W_i^{p*} \tag{9}$$

2.3.3 The free entry condition

In equilibrium, (2) must hold for all i. Inserting for $W_i^r, W_i^c, i = 1, ..., J$ gives

$$\begin{aligned} rM_i =& y_i + s(M_0 - M_i) + p(\theta_{i+1})\beta(M_{i+1} + \sum_{k=0}^{i} \delta_{(i+1)k}(M_k - M_{i+1}) - M_i) \\ &+ a\sum_{j>i} \pi_j(\beta(M_j + \sum_{k=0}^{j-1} \delta_{jk}(M_k - M_j)) + (1-\beta)M_{j-1} - M_i) \end{aligned}$$

these make up a system of equations implicitly defining job offer arrival rates as functions of the vector M of job asset values, the distribution of vacancies π and the parameters of the model.

$$p_i = p_i(M, \pi)$$

which also gives market tightness and arrival rates of workers to firms: $\theta_i = \theta_i(M, \pi)$, $q_i = q(\theta_i(M, \pi)) = q_i(M, \pi)$

Also, inserting for W_i^p, W_i^r, the equilibrium expected value of a type i vacancy

$$rV_i^* = \left[q(\theta_i) + a\Theta^{-1} \frac{\sum_{j=0}^{i-1} u_i}{1-u_J} \right] \times (1-\beta)(M_i + \sum_j \delta_{ij}(M_j - M_i) - M_{i-1})$$

is a function of M and the distribution of workers across jobs (u_js).

Free entry implies the zero profit condition must hold in each submarket, so in equilibrium, the value of a vacancy must equal investments:

$$V_i = K_i \tag{10}$$

2.3.4 Aggregate consistency

Equilibrium conditions: in steady state, the stocks of workers in each i, j-state have to be constant:

$$\dot{u}_i^j = 0$$

In addition, aggregate consistency requires the following definition of market tightness in the competitive market of type i firms to hold:

$$\theta_i = \frac{v_i}{\sum_j^J u_{i-1}^j}$$

3 Wages and mobility

The model presented above have empirically relevant implications for the distribution of durations of unemployment spells and job spells, and for transition rates between different kinds of jobs.

First, for any J different productivity levels, and any a, the model predicts that unemployment durations are exponentially distributed with parameter $\lambda_u = p(\theta_1) + a$. Moreover, job durations in each (i,j)-state are exponentially distributed with parameter λ_i^j:

$$\lambda_i^j = \lambda_0^2 = p(\theta_{i+1}) + a \sum_{k>i} \pi_k + s \equiv \lambda_i$$
$$\lambda_J = s$$

meaning job durations will be distributed as a discrete mixture of J exponentially distributed variables: with J productivity levels, job durations t are distributed according to the probability density function $\phi_J(t)$:

$$\phi_J(t) = \sum f_i \lambda_i \exp(-\lambda t)$$

where the f_i are employment shares

$$f_i = \frac{\sum_j u_i^j}{1-u^0}$$

summing up to one.

The model predicts a negative relationship between the job hazard rate and the wage. For the reasons common in models with random search (higher reservation wage, lower acceptance rates) as well as for the reasons in competitive search (workers in high value jobs being in a sense more patient, direct their search to markets with higher queue lengths and wages).

The model predicts negative duration dependence of the job hazard rate, as higher type workers are overrepresented at higher durations. There are basically two forces driving this negative duration dependence. First, comparing productive matches in high and low type firms, high type workers will direct their search to submarkets with higher queue lengths, i.e. lower θ, due to competitive search. Second, workers who have been employed for longer have, on average, higher reservation wages, so they will accept a lower fraction of job offers achieved through random matches.

The model also predicts negative duration dependence conditional on wages, driven by unobserved heterogeneity in match productivity. At a given wage, workers with low realized value of productivity will search in submarkets with higher tightness and accept more jobs when matched with firms through random search compared to workers in more productive matches.

Next, we let Λ_{kl} denote the transition rate from jobs paying wage w_k to jobs paying wage w_l.

$$\Lambda_{kl} = \begin{cases} \delta_{kl-1} p(\theta_l) + a\pi_l \left(\sum_{j=0}^{l-1} \delta_{kj} \right) & \text{if } l \leq k \\ \left(1 - \sum_j \delta_{kj} \right) p(\theta_l) + a\pi_l & \text{if } l = k+1 \\ a\pi_l & \text{if } l > k+1 \end{cases}$$

To see how this compares to related models, consider first the special case with $\delta_{ij} = 0, \forall i, j$. When $a \rightarrow 0$, the probability of transitions more than one step up the wage ladder approaches zero. This corresponds to Garibaldi and Moen (2010) with endogenous productivity distribution, with a pure job ladder where workers always move up the job ladder one step at a time. When $A \rightarrow 0$, the model gives similar predictions to random search with wage posting, with $\Lambda_{jl} = \Lambda_{kl}$ for any two $j, k < l$. That is, with no directed search, the transistion probabilities to higher paying jobs depend only on the distribution of vacancies π.

Finally, consider the more general case when $a, A > 0$, that is, when both random and competitive search are important. Then $\Lambda_{kl} > \Lambda_{jl}$ when $k = l-1, j < l-1$. Put differently, directed search means that the relative transition probabilities to the submarket one step up the job ladder is higher than what would be predicted by the distribution of vacancies/jobs across submarkets.

4 Data

This paper uses administrative register data on workers and firms in an explicit search theoretic model. In this section, the sampling procedure linking labor market histories to wage data is described in some detail. Then, we consider some cross sectional properties of the data. Finally, we exploit the panel dimension of the data to look at individual wage dynamics. Data points towards a job ladder, with most job-to-job transitions leading to small upwards adjustment in wages. On the other hand, wage cuts are also relatively common.

Our principal source of data is Norwegian administrative registers on workers and firms. We use data from 2003 to 2007; while longer time series is available, we have chosen to limit the sampling period, as we will use moments of these data to calibrate a steady-state model and the steady-state assumption is unlikely to hold for longer time series. Unique person and firm identifiers allow use to construct two samples: one sample of individual labor market histories, recorded continuously, and one panel of individual wages and average firm productivity, recorded annually.

Job spells are constructed using data from the employment register. A job is defined as a unique person-firm pair, using recorded start and stop dates to determine duration of the job spell. Unemployment spells come from data on registered unemployment. In the Norwegian unemployment insurance system, registered unemployment can be full time or part time, permanent or temporary. To be counted as unemployed in our data, a person has to be registered as full time permanently unemployed for at least part of the unemployment spell.

The data source for hourly wages is Statistics Norway's "Wage Statistics". For private sector workers, the data source is questionnaires on individual workers filled out by all firms included in the annual sampling. Large firms are oversampled in the survey; in total, hourly wage data is available for 50-65% of workers in each industry each year. The dataset contains monthly earnings and contracted hours for each worker; full time equivalent monthly wages are computed using a standard working hours corresponding to 162.5 hours per month. All nominal figures, i.e. wages and accounting data, are deflated using the CPI.

As we are interested in how wages and productivity evolve as workers move out of unemployment and between jobs, it is important that we correctly identify job-to-job, job-to-unemployment and unemployment-to-job transitions. In particular, we have to be careful not to incorrectly classify an involuntary separation as a job to job transition. Two criteria are used to determine type of transition. First, we require that the recorded gap between stop of last spell and start of the next spell be less than 15 days. Second, we restrict the degree of overlap between spells, that the "new spell" should be ongoing for some time after the "old spell" has ended. Moreover, we exclude firms with observed mass layoffs, defined as a decrease in employment of 10% or greater, from the sample. The reason for doing this is that we believe that many of the observed job to job transitions from these firms may be involuntary, i.e. workers who have been fired and found new employment so fast, during the time of notice, that they are never recorded as unemployed. In order to avoid problems with changing firm identifiers leading to continuing jobs incorrectly being classified as job-to-job transitions, we require firms to appear in the employment register the calendar year before. Finally, we notice that the firm size distribution in the data is extremely skewed, with a few very large firms. To avoid these firms

Table 1: Descriptives, persons in sample

	Mean	Std. dev
Number of spells	1.862	1.291
Age (2003)	40.82	9.426
High school education	74.7%	
Higher education	25.3%	
Any unemployment	13.6%	
Any job to job transition	29.1%	
Any job to unemployment transition	4.37%	
Observations	317,715	

having an excess influence on population, the ten largest firms each year are excluded from the sample.

We restrict the sample further by excluding all individuals who changed education level during the sample period. We include only persons who have at least completed high school. We require all individuals to have at least 5 years of continuous labor market experience[5] in the years before our sample 1998-2002. In order to avoid the impact of life cycle phenomena on wage dynamics, we exclude workers younger than 25 and older than 60. Observations with missing accounting or wage data are excluded from the sample, as are observations which could not be linked to spell data (e.g. in wage statistics, but not in demographics file).

In our empirical analysis, we implicitly assume that the observed wage correctly reflects the flow value of the job to the worker. This assumption is potentially problematic if workers receive some income from ownership of the firms, which is especially relevant for small firms and family firms. To deal with this, persons who at some point have registered income from self employment are excluded from the sample; and we require firms to have at least 20 full time equivalent workers in order to be included in the wage sample. For the same reason we exclude observations on wages for workers employed in the public sector (central or local government, public schools or health care); to avoid complications comparing wages in the public and private sector (compensating differentials, pension arrangements etc.)

The final sample contains data on 317,715 individuals. Table 1 contains some summary statistics on these persons. The average person is registered with 1.84 unique spells in the durations sample. Using the criteria outlined above, job-to-job transitions are substantially more common than job to unemployment-transitions: while 29.1% of persons are registered with one or more job-to-job transitions, only 4.51% of persons have one or more job-to-unemployment transition during the 5 years individuals are followed in our sample.

Considering first the cross sectional properties of the data, figure 1 shows density plots of individual wages for entrants from unemployment and for all

[5]defined as earning more the national insurance scheme basic amount, approximately equal to 9840 USD in 2003

Figure 1: Density plot, wages

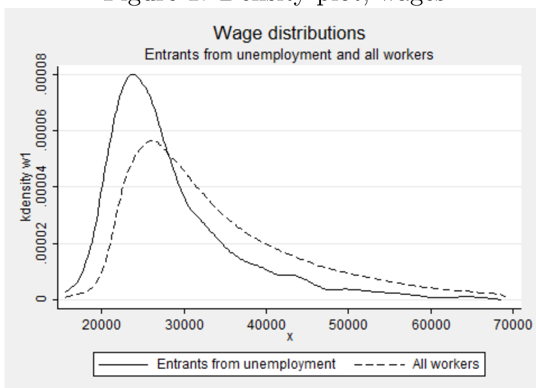


Table 2: Descriptives, by group

	Stayers		Movers, JTJ		Movers, UTJ	
	mean	sd	mean	sd	mean	sd
Monthly wage	34,505	10,328	35,681	11,039.9	27,906.3	10,948.5
$\Delta \ln(w)$	0.039	0.114	0.0389	0.182	-0.00596	0.176
Wage cut	24.6%		35.4%		49.1%	
Value added	114,793	84,535	108,755	77,225.7	85,993.2	90,512.2
$\Delta \ln(w)$	0.065	0.208	0.104	0.66	0.0143	0.755
Value added cut	24.6%		44.5%		50.4%	
Observations	528762		11068		554	

workers in the sample. The distribution of wages for entrants from unemployment lies to the left of the population distribution; it is also more highly skewed and has a lesser variance.

Table 2 summarizes the evolution of individual wages and firm productivity for three groups: these are all persons observed in "wage statistics" for two (or more) consecutive years, classified into three groups by whether they are stayers or movers, and if they are movers, whether the transition was job-to-job or with an intervening period of registered unemployment. The number of observations in the third column in particular is quite small: this is in part due to the requirement that the worker should be observed in the wage sample for two consecutive years and have an intervening unemployment spell ending with a transition to employment.

Compared to stayers, job-to-job movers have higher wage growth on average, however the variance of wage growth is also higher, so the share of transitions with a wage cut is higher, at 35.4%. Transitions with an intervening period of unemployment have, on average, a small negative wage growth. The last three rows of table 2 summarize levels and growth of a proxy for productivity, firm average value added per full time equivalent worker-month. The pattern here is fairly similar; entrants from unemployment work in firms with lower average productivity, and experience lower productivity growth compared to stayers and workers moving directly between jobs.

Looking closer at individual wage dynamics across time, models of on-the-job search give different predictions depending one whether one assumes random or competitive search. Competitive search equilibrium leans more towards a job ladder than random search models. With pure competitive search, the worker's decision of where to direct his search depends on his current wage. Thus, there will be a one-to-one relationship between wage in the old job and wage in the new job. Taking the model literally, there is zero probability of moving more than one step up the job ladder. Ranking vacancies from the highest to the lowest posted wage, the rank of his current wage uniquely determines the wage rank of the new job.

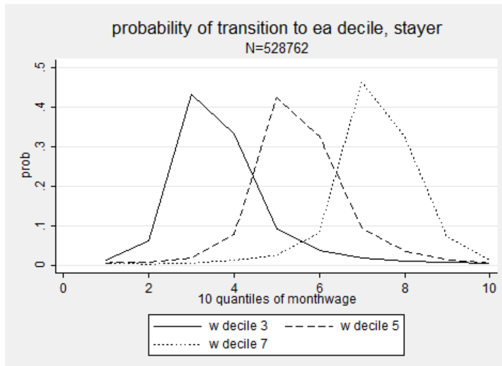
To see how this differs from the predictions of random search models, consider first models with posted wages, i.e.Burdett & Mortensen (1998). In this model, workers and firms match at random, with the worker accepting all wages above his reservation wage, given by his current wage. It follows that the distribution of wages in the new job conditional on a worker's wage in the old job is equal to the distribution of wages across vacancies truncated at his old wage. Ranking the vacancies from lowest wage to highest wage, the worker has an equal probability of moving to each job ranked higher than his current job.

More generally, while various models random search with wage setting through bargaining may yield a similar dependence of new wages on old wages, if we were to rank jobs by productivity instead of wage, the argument above extends to a wider class of models. In models where workers and firms meet randomly, workers will accept any job with a higher productivity than his current job. Again, ranking vacancies from least to most productive, workers should have an equal probability of moving to each job ranked higher than his current job.

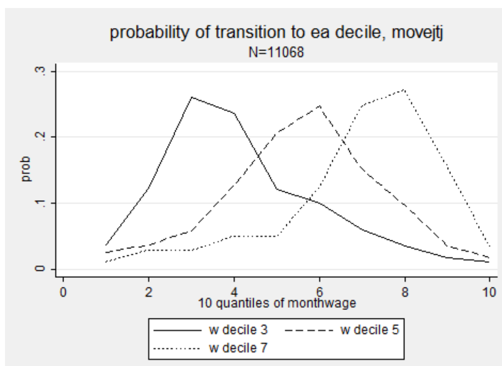
These predictions can be taken to the data using the linked samples of wages and durations described earlier. As the distribution of vacancies is unobservable, we use the distributions of wage and value added per worker in the population. In a stationary environment there is a unique correspondence between rank in the vacancy distribution and in the distribution across filled jobs, so the implications sketched out above should still hold.

Each job in the wage sample is classified into one of ten bins by decile of the distribution of monthly wages in the population. Then, we consider observations where we have wage data available for two consecutive years. For each decile of origin, $i = 1, ..., 10$, we caluculate the probability that a person with a wage at time t in a certain decile i of the population wage distribution is observed at time $t+1$ with a new wage in decile $j = 1, ..., 10$ of the same distribution by the ratio of transitions from i to j to the total number of transitions originating in decile i. Figure 2 plots results for deciles 3,5 and 7; i.e. for jobs in the lower, medium and higher ends of the wage distribution. Looking at the second panel in figure 2, most job to job transitions appear to be associated with small wage changes, with most jobs ending up in the next or the same decile of the wage

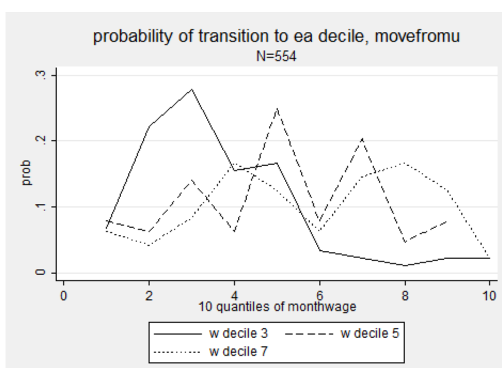
Figure 2: New wage rank, by rank of old job



(a) Stayers



(b) Movers jtj



(c) Movers utj

distribution. Compared to stayers, job to job transitions are more spread out.

Unobserved individual heterogeneity would cause the same pattern; but then we would expect this to be the case also for cases where the person has a registered unemployment spell between the two jobs. Comparing panels 2 and 3 of figure 2, we see that for transitions where there is an intervening period of unemployment between two jobs, there is less of a correspondence between the old wage and the new wages, and new jobs tend to be concentrated in the lower quantiles of the wage distribution.

Finally, models with state dependent bargaining power would predict similar patterns for individual wages. However, if we were to consider firm average wages instead of individual wages, these differences should average out. Figure 3 shows transition probabilities between deciles of the distribution of average firm wages. Looking at the second panel, workers whose old firm pays an average wage in decile j appear more likely to make a job-to-job transition to a firm in decile j or $j+1$ compared to higher deciles, which is difficult to explain using most models of random search with identical workers. To summarize, workers making job-to-job transition tend to move to jobs close to their current wage, consistent with the predictions of the model as presented in section 3

5 A calibrated example

From the discussion above, it is clear that the distribution of wages and durations is a nonlinear function of the model parameters, meaning it is difficult to express the associated likelihood function in closed form. Quantitative work on the model will likely require the use of simulation methods. In this section, we calibrate a simple version of the model with $J = 2$ productivity levels to illustrate the impact of random vs directed search on the data generating process.

To assess the quantitative implications of the model, we calibrate the model to match moments from microdata. With $J = 2$ productivity levels, the 13 parameters of the model are

$$\{y_0, y_1, y_2, K_1, K_2, r, s, \delta_{10}, \delta_{21}, \delta_{20}, a, A, \beta\}$$

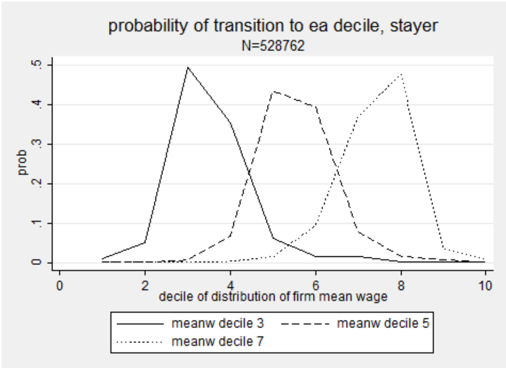
For empirical purposes, it may be necessary to place some additional structure on the δ_i^j. One possible restriction can be that the probability of a "good match" is the same in both types of firms:

$$\delta_{10} = \delta_{21} + \delta_{20}$$

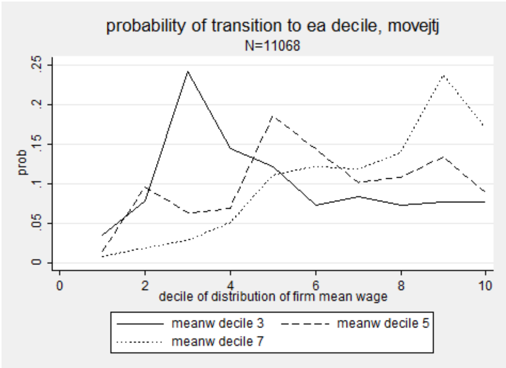
in this case the model has 12 structural parameters. if we moreover restrict match productivity should fall down the ladder one step at a time, i.e.

$$\begin{aligned} \delta_{10} &= \delta \\ \delta_{21} &= \delta(1-\delta) \\ \delta_{20} &= \delta^2 \end{aligned} \tag{11}$$

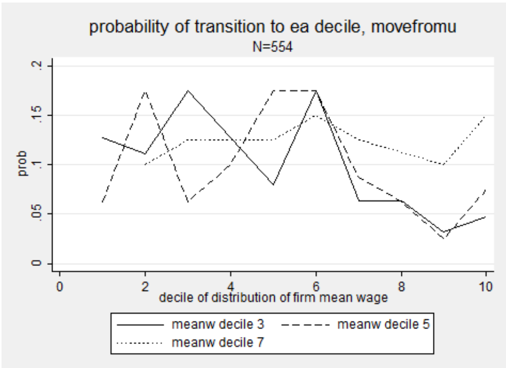
Figure 3: New firm average wage rank, by rank of old job



(a) Stayers



(b) Movers jtj



(c) Movers utj

the number of parameters is further reduced. This admittedly ad-hoc restriction should improve the prospect of identifying productivity shocks using the observed share of wage cuts in the data. Using (11), we are left with 11 structural parameters

$$\{y_0, y_1, y_2, K_1, K_2, r, s, \delta, a, A, \beta\}$$

The monthly discount rate r is set to 0.0043, corresponding to an annual discount rate of 5%, while the elasticity of the matching function β is set to 0.5 (Pissarides 2009, Petrongolo & Pissarides 2001). The utility of leisure, y_0 is fixed at 70% of y_1. Typical replacement ratio of unemployment benefits is around 0.6, placing a lower bound on y_0, while an upper bound of y_0 is given by y_1. The seven parameters $y_1, y_2, K_1, K_2, A, s, \delta$ are then set to jointly to match, conditional on a, the following moments from the data: mean wage of entrants form unemployment, average cross section wage in population, average duration of unemployment spells, average duration of job spells ending in a job-to-job transition, the average unemployment rate for the period (3.9%), and the share of jtj transitions with wage cuts.

A key parameter of interest in the model is a, representing the relative importance of random search. To illustrate how a changes the model, three versions of the model are calibrated, with low, medium and high values of a.

Table 3 summarizes the calibrated models. The first column contains the parameters of the version with a low value of a, with random search accounting for 10% of transitions from unemployment. The second column presents a model with 30% of unemployment to job transitions happening through random search, while in the third column a is set at 50% of the unemployment hazard rate[6].

Table 3: Calibration: parameters

	$a = 0.1\lambda_u$	$a = 0.3\lambda_u$	$a = 0.5\lambda_u$
a	0.0104	0.0311	0.0519
A	0.2575	0.2171	0.166
y_0	22.80	22.60	22.35
y_1	32.57	32.29	31.94
y_2	39.98	40.17	40.46
K_1	15,337	16,790	17,463
K_2	215,392	229,350	248,575
s	0.0042	0.0042	0.0042
δ	0.460	0.462	0.465
β	0.5	0.5	0.5

[6] In the calibration procedure, the job ladder assumption that that it should never be profitable for type 2 firms to search for unemployed workers is assumed to hold in searching for parameter values; this assumption has to be verified ex post. The assumption holds for $a = 0.1\lambda_u$ and $a = 0.3\lambda_u$, but fails to hold for $a = 0.5\lambda_u$. We nevertheless report the calibrations for $a = 0.5$ to give an impression of how equilibrium might look. For higher values of a, there were difficulties calibrating the model to the moments in table 4, and the chosen parameters were not consistent with the job ladder assumption.

Table 4: Calibration: targets

	$a = 0.1\lambda_u$	$a = 0.3\lambda_u$	$a = 0.5\lambda_u$	Target
Mean new wage "utj"	28.58	28.57	28.57	28.55
Mean wage pop cross section	33.78	33.76	33.75	33.77
Unemployment rate	0.039	0.039	0.039	0.039
Mean duration unemployment spells	9.63	9.63	9.63	9.64
Mean duration before jtj trans	55.9	55.9	55.9	55.9
Share of jtj transitions with wagecuts	0.35	0.35	0.35	0.35
Average firm size across filled jobs	607	607	607	607

Next, in order to illustrate how the three calibrations differ, the parameter values in table 3 are used to simulate a sample of $N = 100,000$ persons. Each simulated individual is assigned an initial state $\{i, j\}$ from the steady state distribution, and each employed worker is assigned to a firm. Next, durations are drawn from the appropriate exponential distribution. Unemployed workers can transition to type 1 or type 2 job, while employed workers can move to a new job or to unemployment.

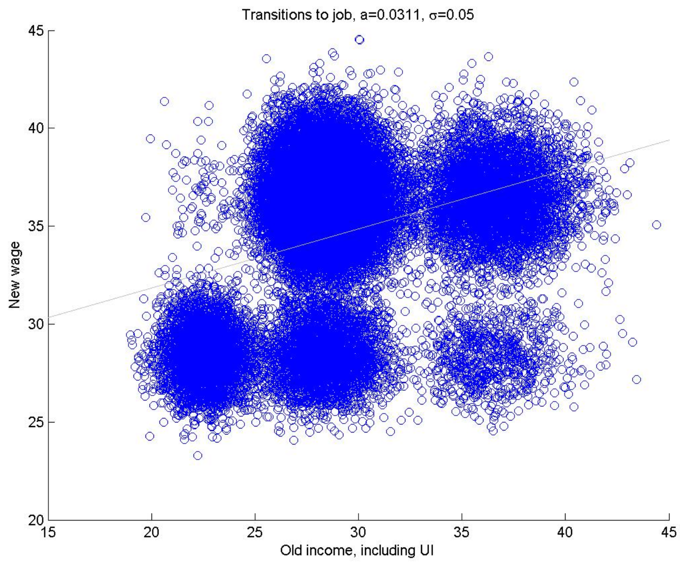
In the simulations, we assume log-additive classical measurement error in wages, with observed wages given by $w_i = w_j \exp(\varepsilon_i)$, with $\varepsilon_i \sim N(0, \sigma)$. Clearly, the magnitude of σ will be important for the results, however as long as we are primarily interested in how changes in a affect the simulated data, it should be sufficient to use the same σ for all simulations. For now, we set $\sigma = 0.05$.

Table 5: Simulated moments, $\sigma = 0.05$

	$a = 0.1\lambda_u$	$a = 0.3\lambda_u$	$a = 0.5\lambda_u$	Target
Variance wage, entrants from unemployment	2.3424	3.5371	4.668	69.043
Skewness wage, entrants from unemployment	0.8399	1.9281	2.1299	1.688
Variance wage, cross section	100.5233	101.4392	102.8714	108.986
Skewness wage, cross section	-1.9228	-1.891	-1.8482	1.089
Average job duration	161.2	162.5	163.9	74.9
$\rho(dur, w_0)$	0.335	0.3321	0.328	0.038
Average productivity y	36.4029	36.4489	36.5472	107.572
Variance productivity y	12.8018	14.0336	15.8098	6518.443
$\rho(n, y)$	0.9931	0.9942	0.9953	0.146
$\rho(y, w)$	0.908	0.9096	0.9115	0.229
$\rho(w_1, w_0)$, job-to-job movers	0.0024	0.0026	-0.0013	0.817

While the calibrated example is perhaps too simplistic to realistically capture wage dynamics, the simulations might nonetheless shed some light on the effects of a on the shape of wage dispersion and wage dynamics. Table 5 contains a summary of simulation results. The simulated distributions of wages of entrants from unemployment have approximately has a too low variance, however the

Figure 4: Transitions to jobs, simulated data



skewness is close to what is found in the data. Higher a increases the positive skew of the entrant wage distribution. Meanwhile, the distribution of wages in the cross section is more spread out, but the shape is wrong, with a strong negative skewness.

The essentially zero correlation between old and new wage is, together with the negative skew of the wage distribution across filled jobs, perhaps the most troubling feature of table 5. However, this is likely a direct consequence of the model having only two productivity levels. To illustrate this, figure 4 plot simulated transitions to job ($a = 0.3\lambda_u$), including transitions from unemployment, with the previous income defined as unemployment benefits y_0 observed with measurement error as for wages.When these are included in the sample, we notice a pattern more similar to the data discussed in section 4, with an apparent job ladder where the bulk of workers search for jobs one step above the ladder.

6 Conclusions

We have formulated a model of mixed search, where workers search on and off the job through both directed and undirected search. We argue the model is qualitatively consistent with patterns of wages of job-to-job transitions. A calibrated example is presented to illustrate the model.

References

Burdett, K. & Mortensen, D. T. (1998), 'Wage differentials, employer size and unemployment', *International Economic Review* **39**(2), 257–273.

Cahuc, P., Postel-Vinay, F. & Robin, J. (2006), 'Wage bargaining with on-the-job search: Theory and evidence', *Econometrica* **74**(2), 323–364.

Garibaldi, P. & Moen, E. R. (2010), 'Job to job movements in a simple search model', *American Economic Review* **100**(2), 343–374.

Guerrieri, V. (2007), 'Heterogeneity, job creation and unemployment volatility', *The Scandinavian Journal of Economics* **109**(4), 667–693.

Guerrieri, V. (2008), 'Inefficient unemployment dynamics under asymmetric information', *Journal of Political Economy* **116**(4), 667–708.

Jolivet, G., Postel-Vinay, F. & Robin, J. (2006), 'The empirical content of the job search model: Labor mobility and wage distributions in europe and the us', *European Economic Review* **50**(4), 877–907.

Lester, B. (2011), 'Information and prices with capacity constraints', *American Economic Review* **101**(4), 1591–1600.

Moen, E. R. & Rosén, A. (2004), 'Does poaching distort training?', *Review of Economic Studies* **71**(4), 1143–1162.

Mortensen, D. & Pissarides, C. (1994), 'Job creation and job destruction in the theory of unemployment', *Review of Economic Studies* **61**(3), 397.

Petrongolo, B. & Pissarides, C. (2001), 'Looking into the black box: A survey of the matching function', *Journal of Economic Literature* **39**(2), 390–431.

Pissarides, C. (2009), 'The unemployment volatility puzzle: Is wage stickiness the answer?', *Econometrica* **77**(5), 1339–1369.

Postel-Vinay, F. & Robin, J.-M. (2002), 'Equilibrium wage dispersion with worker and employer heterogeneity', *Econometrica* **70**(6), 2295–2350.

Postel-Vinay, F. & Turon, H. (2010), 'On-the-job search, productivity shocks, and the individual earnings process', *International Economic Review* **51**(3), 599–629.