Efficiency gains from narrowing banks: a search-theoretic approach

Fabien Tripier*

February 27, 2012

Abstract

In view of the recent proposals on banking reform in the wake of the recent global economic crisis, this paper identifies some efficiency gains associated with narrow banking using an approach based on search theory. It is herein shown that the optimal allocation of resources can be decentralised through competition between narrow banks (which take deposits from households) and finance houses (which make loans to entrepreneurs), whereas such a decentralisation is not feasible for commercial banks (which both take deposits and make loans). When a non-financial agent (such as a household) bargains with a commercial bank, it succeeds in appropriating a share of the value associated with the financial services provided to other non-financial agents (such as entrepreneurs) because commercial banks are affected by search frictions on both the loan and credit markets. This cross-market sharing prevents commercial banks from sharing the value of financial services with non-financial agents efficiently, and can be the origin of credit rationing and multiple equilibria. Because the use of narrow banking suppresses this cross-market sharing, it makes the competitive equilibrium efficient.

JEL Codes: C78; D83; G21

Keywords: Banking; Matching; Bargaining; Multiple Equilibria

*University of Nantes, LEMNA. Email: fabien.tripier@univ-nantes.fr.

1 Introduction

The proposals for narrow banking seek to protect the real economy from the inherent instability of the financial sector by narrowing the scope of the activities of deposit-funding banks.[1] According to these proposals, narrow banks should only collect deposits, and be prevented from investing in risky activities of business financing,[2] while the financing of non-financial agents should be undertaken by other institutions, known as finance houses. Because all the retail deposits collected by narrow banks would be secured on safe and highly liquid assets, a large part of the current elaborate supervisory and regulatory structure would be unnecessary. The main expected benefits of narrow banking are thus the reduced costs of financial regulation, together with a reduction in the moral hazard that is currently a problem in banking.[3] The narrow banking proposal has received some criticism, and some doubts have been raised about the feasibility of this reform and of its ability to deliver its objectives.[4] Among these criticisms, the efficiency gains afforded by the integration of financial services (such as the deposit-taking and lending services) within the same institution (precisely termed a *bank*) have been brought forward. Because of the synergies between deposit-taking and lending, narrow banking implies *efficiency losses* that require a trade-off to be made between stability and efficiency. I herein

[1] The origin of the narrow banking proposal is explained by Phillips (1995) and Bossone (2002). The narrow banking proposal has been proposed most notably by Litan (1987) in response of the savings and loan crisis of the 1980s, but the underlying idea was not new even then. A similar proposal was put forward during the great depression of 1930s, but abandoned in favour of the creation of deposit insurance.

[2] Chow and Surti (2011) compared narrow banking with two other proposals: the Volcker Rule and a Retail Pinge-fence. These three proposals redefine the scope of risky activities of regulated and deposit-funded banks, but differ on the degree of reduction in the scope of banking activities. The Volcker Rule and a Retail Pinge-fence are less strict since they allow deposit-funding banks to carry out certain type of risky investments for the business sector, in the form of lending.

[3] De Grauwe (2009), Kay (2009), Chamley and Kotlifoff (2009) and Phillips and Roselli (2009) have recently advocated the narrow banking proposal in order to reduce the risk of large taxpayer bailouts. They argue that the extremely high costs of bailouts of financial institutions after the 2007/08 crisis, together with the ineffectiveness of this policy in avoiding a severe economic contraction, call for a radical reform of the structure of the financial industry.

[4] Bossone (2002) posed the question of the availability of sufficient amounts of high-quality and liquid assets in the economy that are eligible to be used as collateral for transaction deposits of narrow banking. Mishkin (1999) emphasised that if the narrow banking proposal is to be effective in improving the safety of the economy, it is not a *panacea* because it would not completely eliminate the need for financial regulation, nor would it fully prevent the need for financial bailouts. Wallace (1996) was more critical of the narrow banking proposal: "Let's eliminate the potential difficulties by eliminating illiquid banks. That may be a good idea, or it may be silly -as silly as a proposal to reduce automobile accidents by limiting automobile speeds to zero."

propose an alternative view by demonstrating that narrow banking brings *efficiency gains* in a search environment.[5] A key question is whether the specialisation of the banking sector improves or reduces the efficiency of the allocation of resources by banks. In the literature[6], the switch to narrow banking is generally associated with significant reductions in efficiency because of the substantial synergies that exist between the deposit-taking and lending activities associated with (*i*) the provision of liquidity and (*ii*) the production technology of banking services. In the case of (*i*), Diamond and Dybvig (1983) clearly demonstrated that banks are a good solution for creating liquidity by their transformation of illiquid assets into liquid demand deposits. In view of the fact that depositors can 'vote with their feet', Calomiris and Kahn (1991) identified an additional benefit of the integrated banking system, in that it creates a powerful form of incentive for bankers to be disciplined in their decisions related to risk. For Kashyap et al. (2002), the synergy proceeds from the fact that the services of deposit-taking and lending are very similar (both provide liquidity on demand in order to accommodate unpredictable needs), and use the same resources (overheads in the form of liquid-asset holdings). Provided that these two activities are not correlated, there is a synergy that makes commercial banks efficient because they only need a small amount of liquidity to act as a buffer for the two activities. In the case of (*ii*) Pulley and Humphrey (1993) studied the implications of narrow banking on the costs of financial services. The separation of insured deposits from most types of bank lending requires the non-joint production of financial services that are currently jointly provided. Narrow banking is costly when there are significant economies of scope between the production of deposit and loan services. Pulley and Humphrey (1993) demonstrated the significant benefits of sharing fixed costs in joint production.[7] They suggested that these costs could be offset by a reduction in the regulation costs associated with narrow banking. Other contributions, e.g.,

[5] With efficiency gains, there is no longer a trade-off between stability and efficiency. In order to focus on efficiency gains, I do not consider the issue of stability in this paper. In order to address efficiency and stability simultaneously, a more general search model of financial intermediation is required. The model developed herein should be seen to be a preliminary step in this direction.

[6] See Berger et al. (1999) for an overview of the literature on efficiency in banking, Strahan (2008) for a review of deposit-lending synergies, and Bossone (2002) for a discussion of the narrow banking perspective.

[7] They used data from large U.S. banks over the period 1978-1990 to measure the economies of scope.

Vander Vennet (2002), have concluded that the opposite is true, in that economies of scope legitimated the de-specialisation of financial institutions that took place in the United States during the 1990s.

In a search environment, I show that the switch to narrow banking is associated with efficiency gains, because commercial banks fail to share efficiently the value created by financial services with non-financial agents. A switch to narrow banking could deliver efficiency gains because it reduces the credit rationing of entrepreneurs and eliminates the possibility of multiple equilibria.[8] I use the DMP[9] matching model to analyse the deposit and loan markets in a search environment, in line with recent literature on search frictions in financial markets. Den Haan et al. (2003) and Wasmer and Weil (2004) developed theoretical search models of the credit market[10], while Dell'Ariccia and Garibaldi (2005) and Craig and Haubrich (2006) constructed rich data sets of credit flows, which can be interpreted in terms of search frictions. The model developed herein develops the ideas put forward by these authors on the modelling of the credit market[11] and extends the scope of this literature by considering search frictions between banks and households in deposit markets. Search frictions in the deposit market account for the financial costs borne by households, which were previously measured by Vissing-Jørgensen (2002), and the costs of switching financial intermediary, which are shown to be significant in the banking industry by Kiser (2002), Shy (2007), and Degryse and Ongena (2008).[12] With

[8]In a different setup, Shy and Stenbacka (2007) also showed some of the efficiency gains of narrowing banks. They introduced the use of the fraction of perfectly liquid accounts as a new regulatory instrument for improving the efficiency of the financial markets for depositors, and supported the creation of narrow banks that specialise in basic banking services for depositors. Laeven and Levine (2005) provided empirical evidence in favour of the efficiency gains of specialisation, which were interpreted by the authors to be the consequences of agency problems in financial conglomerates that outbalance the economies of scope associated with the diversification of banking activites.

[9]For Diamond (1982), Mortensen and Pissarides (1994), and Pissarides (2000).

[10]A second set of contributions have considered search frictions on the over-the-counter markets, such as the interbank market. Rocheteau and Weill (2011) surveyed these contributions and provided a general setup for analysing financial markets with search frictions, as originally proposed by Duffie et al. (2005).

[11]The basic structure of the credit matching model is an aggregate matching function and a Nash bargaining process on interest rate. This structure was used by Wasmer and Weil (2004) in association with search frictions on the labour market, by Besci et al. (2005) with heterogenous borrowers, by Petrosky-Nadeau and Wasmer (2011) in a model of search frictions on three markets (labour, credit, and goods), and by Chamley and Rochon (2011) with loan rollover. Note that Den Haan et al. (2003) did not consider the Nash solution for the interest rate, but an agency contract with moral hazard, and introduced liquidity shocks for lenders.

[12]See Sharp (1997) and Zephirin (1994) for models of switching costs in the deposit markets.

the DMP model, these financial costs are made endogenous because they are determined by the market tightness, which is defined to be the ratio of the search effort made by banks to the masses of non-financial agents who are seeking banks. When banks devote more resources to their search activities, the financial costs of the search for non-financial agents is reduced.

Banks transform the deposits of households into loans for entrepreneurs. An initial matching process takes place between households and banks in the deposit market, and a second matching process then occurs between entrepreneurs and banks on the loan market. If banks are taken to be commercial banks, they may be assumed to search simultaneously on the two frictional markets. In contrast, if banks are more specialised, narrow banks search in the deposit market, while finance houses search in the loan market. In the latter case, finance houses buy liquid assets supplied by narrow banks in a perfectly competitive interbank market. There is no public sector in this economy, and these assets are therefore defined to be sufficiently liquid and safe to be possessed by narrow banks. Using this simplification, I omit part of the debate on the organisation of the banking industry under the narrow banking proposal[13] in order to focus on the specific impact of search frictions on bank competition through the use of interest rates. If interest rates are exogenous, the organisation of the banking industry is not relevant, and the two possible models result in the same allocation of resources. I then define the optimal allocation of resources in this search environment and discuss the ability of competitive banks to replicate this allocation when interest rates are both endogenous and result from the outcomes of a Nash bargaining process.

In the case of specialised banks, competition is efficient if the bargaining power of agents is equal to their contributions to the matching process. This condition, also known in the literature on matching as the Hosios (1990) condition, efficiently internalises the trading externalities

[13] The narrow banking proposals differ significantly where it is necessary to be explicit on the types of assets that narrow banks should be allowed to hold (public vs. private) and on the organisation of banks (could the narrow bank and the finance house be different parts of the same entity?) See Phillips (1995), Bossone (2002), and Chow and Surti (2011) for a comparison of the alternative proposals.

associated with the matching function in the bargaining process.[14] In commercial banks, this condition is no longer sufficient, and competition is inefficient. When a commercial bank bargains on the interest rate in one of the two markets, its contribution to the Nash criteria include the marginal value of the asset on the two markets of *deposit and loan*. In contrast, the contribution to the Nash criteria of a specialised bank is restricted to the marginal value of the asset in only one of the two markets: either *deposit or loan*. This fact distorts the bargaining process with commercial banks because a non-financial agent (for example, the household) may receive a share of the bank's value of the financial relationship together with the other non-financial agent (for example, the entrepreneur). This mechanism, which I refer to as cross-market sharing, is specific to commercial banks. With specialised banks, a non-financial agent receives a share of the bank value from its own financial relationship but not from the financial relationship with the other non-financial agent. The bargaining process between the specialised bank and the non-financial agent does not take place across the markets because it is restricted to the value provided to the non-financial agent by the financial services concerned. Therefore, the bargaining processes on interest rates can efficiently decentralise the optimal allocation only for the case of specialised banks.

The following configurations are herein characterised in analytical terms for equilibrium with commercial banks. The loan market tightness and the size of the production sector are insufficient when: (*i*) the loan interest rate is bargained under the Hosios (1990) condition and the deposit interest rate is set to its optimal value; and (*ii*) the deposit interest rate is bargained under the Hosios (1990) condition and the loan interest rate is exogenous. A third case of interest is the occurrence of multiple equilibria when (*iii*) the deposit interest rate is bargained, but not under the Hosios (1990) condition, and the loan interest rate is exogenous. I use a set of numerical simulations to illustrate these configurations.

The remainder of the paper is structured as follows. In Section 2, I provide an overview

[14] Moen (1997) showed how it can emerge endogenously in matching models.

of the model. The search behaviours are defined in Section 3, and the normative properties of the equilibrium are presented in Section 4. In Section 5, I provide some conclusions.

2 Overview of the model

In the present study, time is considered in discrete steps and the model is deterministic. The utility and production technologies are linear and the discount rate for the present is $0 < \beta < 1$.

The economy is populated by $\overline{n}_t^d > 0$ households. Each household holds one asset that delivers one unit of money at each period. Households can use this asset either outside the banking industry or within in. Because of the interest payable, the marginal utility of the asset ρ_t^d, is higher if the household deals with a bank than if it does not ρ_t^c; with $\rho_t^d > \rho_t^c > 0$. When a household participates in the banking industry, it deposits its asset in a bank. Not all households participate, because participation is not costless. It takes time and resources to obtain information on the banking industry (on banks and on products) and on household characteristics, and consequently to make a good match between a bank and a household. When a household is matched with a bank, the financial relationship is durable because it would be costly for the agents to separate and to seek another partner on the market. Banks decide on their per-period search effort $v_t^d > 0$ (for the marginal cost $c^d > 0$), and the mass of households searching for banks is denoted $u_t^d > 0$ (they pay the per-period search cost $1/c^h > 0$). The number of new financial relationships between households and banks is determined by an aggregate matching function $m^d \left(v_t^d, u_t^d\right)$ that has constant returns to scale and is increasing in both arguments. The deposit market tightness is defined by the ratio $\alpha^d = v_t^d / u_t^d$. Any increase in the deposit market tightness raises the delay for banks in collecting deposits from households. With an exogenous probability of $0 < \delta^d < 1$, the financial relationships are terminated.

The mass of households matched with a financial intermediary constitutes the saving in the economy. On the other side of the economy, there is a population of $\overline{n}_t^\ell > 0$ entrepreneurs who have no funds and seek loans to start production. Similarly to the case for households, it takes time and resources for each party to obtain information on the other and then to make a good match between a bank and an entrepreneur. When an entrepreneur is matched with a bank, the financial relationship is durable for the same reason as for the relationships between households and banks. Banks decide on the per-period search effort $v_t^\ell > 0$ (for the marginal cost $c^\ell > 0$) and all unmatched entrepreneurs $u_t^\ell > 0$ search for a bank because the search is assumed to be costless for entrepreneurs. The mass n_t^ℓ (that satisfies $n_t^\ell = \overline{n}_t^\ell - u_t^\ell > 0$) of financed entrepreneurs produces $z_t > 0$ units of final good and pays $\rho_t^\ell > 0$ as loan interests. The production of unmatched entrepreneurs is zero. The number of new financial relationships between entrepreneurs and banks is determined by an aggregate matching function $m^\ell \left(v_t^\ell, u_t^\ell\right)$, which has constant returns to scale and is increasing in both arguments. The credit market tightness is defined by the ratio $\alpha^\ell = v_t^\ell / u_t^\ell$. By increasing the credit market tightness, the delay in finding funds for entrepreneurs is decreased. With an exogenous probability of $0 < \delta^\ell < 1$, the financial relationships are terminated.

Interest rates are the outcome of individual Nash bargaining processes between banks and households for deposit interest rates, and between banks and entrepreneurs for loan interest rates. The bargaining powers are $0 < \eta < 1$ for non-financial agents and $(1 - \eta)$ for banks.

The banking industry transforms the deposits of households into loans for entrepreneurs. In order to focus on the effects of the different types of organisation of banks, I do not consider solvency regulations or reserve requirements on deposits, and assume that each unit of deposit gives one unit of loans. Two types of organisation of the banking industry are considered, namely integrated or specialised. These may be thought of in the following terms:

- In the integrated organization of the banking industry, commercial banks both take deposits from households and make loans to entrepreneurs.
- In the specialised organisation of the banking industry, narrow banks take deposits and supply funds through an an interbank market of short-term assets (at the unitary price

$\rho_t^b > 0$) to finance houses that then make loans to entrepreneurs.

These two types of organisation of competitive banks are compared with a third organisation, where allocations of resources are determined by a social planner.

3 Search and Markets Tightness

This section defines the search behaviors and markets tightness for exogenous interest rates.

3.1 The matching processes

The matching functions determine the flow of new relationships on the deposit and loan markets. The arguments of the matching functions are the search effort of banks, the mass of unmatched households that wish to participate in the deposit market, and the mass of unmatched entrepreneurs in the credit market. For market x, the matching function is

$$m^x\left(v_t^x, u_t^x\right) = \overline{m}^x \left(v_t^x\right)^{\varepsilon} \left(u_t^x\right)^{1-\varepsilon} \tag{1}$$

where $x = \{\ell, d\}$ denotes the loan market (ℓ) and the deposit market (d). The search effort of banks is v_t^x, the mass of unmatched non-financial agents is u_t^x, $\overline{m}^x > 0$ is a scale parameter specific to market x, and $0 < \varepsilon < 1$ is the elasticity of the matching function with respect to the search effort of banks, which is assumed to be the same on the two markets. The matching probabilities are $q\left(\alpha^x\right)$ for banks on the market x and $p\left(\alpha^x\right)$ for non-financial agents (households for $x = d$ or entrepreneurs for $x = \ell$). The matching probabilities are defined as follows

$$q\left(\alpha_t^x\right) = \frac{m^x\left(v_t^x, u_t^x\right)}{v_t^x} = \overline{m}^x \left(\frac{v_t^x}{u_t^x}\right)^{\varepsilon-1} = \overline{m}^x \left(\alpha_t^x\right)^{\varepsilon-1} = \frac{p\left(\alpha_t^x\right)}{\alpha_t^x} \tag{2}$$

and satisfy $q'\left(\alpha_t^x\right) < 0$ and $p'\left(\alpha_t^x\right) > 0$, where $\alpha_t^x = v_t^x / u_t^x$ measures the market tightness for $x = \{\ell, d\}$. The matching function induces trading externalities on financial markets.

3.2 Participation of households

To determine the equilibrium participation of households in the financial market, the value functions associated with the different states are defined. The value function of household outside the deposit market is denoted D_t^c and defined by

$$D_t^c = \rho_t^c + \beta D_{t+1}^c \tag{3}$$

If the household decides to participate to the banking industry, it is assigned the value function D_t^u, which is defined by

$$D_t^u = \rho_t^c - 1/c^h + p\left(\alpha_t^d\right)\beta D_{t+1}^m + \left(1 - p\left(\alpha_t^d\right)\right)\beta D_{t+1}^u \tag{4}$$

The per-period utility flow ρ_t^c is reduced by the search cost $1/c^h$. To offset the search cost, the household has a probability $p\left(\alpha_t^d\right)$ of forming a match with a bank and of receiving the discounted value of the matched household βD_{t+1}^m. With a probability $1 - p\left(\alpha_t^d\right)$, the household fails to find a bank and is still searching for a bank at the next period, βD_{t+1}^u. The value function associated with the matched state for households D_t^m is defined by

$$D_t^m = \rho_t^d + \left(1 - \delta^d\right)\beta D_{t+1}^m + \delta^d \beta D_{t+1}^u \tag{5}$$

The household remains in the matched state with a probability of $\left(1 - \delta^d\right)$, in which case it is assigned the value βD_{t+1}^m, otherwise it switches to the unmatched state with probability δ^d, and is assigned the value βD_{t+1}^u.

At equilibrium, households are indifferent about whether they remain outside the banking industry or whether they engage with it by searching for a bank. This free entry condition implies that $D_t^u = D_t^c$, which is equivalent to

$$\begin{aligned} \frac{1}{c^h p\left(\alpha_t^d\right)} &= \beta\left(D_{t+1}^m - D_{t+1}^u\right) \\ &= \beta\left[\rho_{t+1}^d - \rho_{t+1}^c + \left(1-\delta^d\right)\frac{1}{c^h p\left(\alpha_{t+1}^d\right)}\right] \end{aligned} \tag{6}$$

for the value functions (3)-(4)-(5). The average cost of matching for households is equal to the per-period cost of search $(1/c^h)$ multiplied by the average delay of search $\left(1/p\left(\alpha_t^d\right)\right)$. At equilibrium, the matching cost i.e., the LHS of (6), must equal the discounted payoff of being matched i.e., the LHS of (6), which is defined by the discounted difference between the matched and unmatched value functions. This payoff is equal to the gap between the marginal utilities $\rho_{t+1}^d$and ρ_{t+1}^c, plus the value of the financial relationship $1/\left(c^h p\left(\alpha_{t+1}^d\right)\right)$, assuming it is not destroyed, with probability $\left(1-\delta^d\right)$.

3.3 Entrepreneurs

The value function of an unmatched entrepreneur is denoted L_t^u and defined by

$$L_t^u = p\left(\alpha_t^\ell\right)\beta L_{t+1}^m + \left(1 - p\left(\alpha_t^\ell\right)\right)\beta L_{t+1}^u \tag{7}$$

At time t, an entrepreneur may find a bank with probability $p\left(\alpha_t^\ell\right)$ and move to the matched state m in $t+1$ to be assigned the value βL_{t+1}^m. If not, it remains in this state with probability $1-p\left(\alpha_t^\ell\right)$, and is assigned βL_{t+1}^u. The value function of a matched entrepreneur is denoted L_t^m and is defined by

$$L_t^m = z_t - \rho_t^\ell + \left(1-\delta^\ell\right)\beta L_{t+1}^m + \delta^\ell \beta L_{t+1}^u \tag{8}$$

The entrepreneur receives a flow of revenues equal to $\left(z_t - \rho_t^\ell\right)$, the value of production less the interests. The entrepreneur remains in this state at time $t+1$ if the relationship survives with a probability of $\left(1-\delta^\ell\right)$. Otherwise, in the next period, the lending relationship is lost with a probability of δ^ℓ.

Let $u_t^\ell = \overline{n}^\ell - n_t^\ell$ be the mass of unmatched entrepreneurs, where $\overline{n}^\ell$ is the exogenous size of the entrepreneur population. The mass of financed entrepreneurs evolves as follows

$$n_{t+1}^\ell = \left(1 - \delta^\ell\right) n_t^\ell + p\left(\alpha_t^\ell\right)\left(\overline{n}^\ell - n_t^\ell\right) \tag{9}$$

and $n_t^\ell z_t$ is the total production of the entrepreneurs.

3.4 The Banking Sector

In this section, the programs of banks for the two types of organisation of the banking industry are solved, and the associated equilibria are defined.

3.4.1 The case of an integrated industry: commercial banks

The balance sheet of the representative commercial bank is: $n_t^\ell = n_t^d$, in which the bank lends all its deposits. The bank's profits are

$$\Pi_t = \rho_t^\ell n_t^\ell - \rho_t^d n_t^d - c\left(v_t^d\right) - c\left(v_t^d\right) \tag{10}$$

where ρ_t^ℓ is the loan interest rate, ρ_t^d the deposit interest rate, $c\left(v_t^x\right)$ the search costs[15] and v_t^x the search effort on the market $x = \{\ell, d\}$. The masses of deposits and loans are driven by the following laws of motion

$$n_{t+1}^x = \left(1 - \delta^x\right) n_t^x + q\left(\alpha_t^x\right) v_t^x, \text{ for } x = \{\ell, d\} \tag{11}$$

[15] Synergies between the deposit-taking and lending activities could be introduced into the search activities by considering a non-separable techonology of costs $c\left(v_t^\ell, v_t^d\right)$, with $\partial^2 c\left(v_t^\ell, v_t^d\right) / \left(\partial v_t^\ell \partial v_t^d\right) < 0$. I do not consider this case here because my focus is on the mechanisms associated with the rules for sharing the values of financial services.

To solve the maximisation program of the representative commercial bank, the value function is defined in a function of n_t^ℓ as the single state variable

$$\begin{aligned} B\left(n_t^\ell\right) &= \max_{n_{t+1}^\ell, v_t^d, v_t^\ell} \left\{\left(\rho_t^\ell - \rho_t^d\right) n_t^\ell - c\left(v_t^\ell, v_t^d\right) + \beta B\left(n_{t+1}^\ell\right)\right\} \\ &\quad -\lambda_t^\ell \left[n_{t+1}^\ell - \left(1-\delta^\ell\right) n_t^\ell - q\left(\alpha_t^\ell\right) v_t^\ell\right] \\ &\quad -\lambda_t^d \left[n_{t+1}^\ell - \left(1-\delta^d\right) n_t^\ell - q\left(\alpha_t^d\right) v_t^d\right] \end{aligned} \tag{12}$$

where λ_t^x are the two Lagrangian multipliers associated with the constraints on markets $x = \{\ell, d\}$ given by (11). The first order conditions with respect to the search efforts are

$$v_t^x : \frac{c_t^x}{q\left(\alpha_t^x\right)} = \lambda_t^x, \text{ for } x = \{\ell, d\} \tag{13}$$

with $c_t^x = \partial c\left(v_t^x\right)/\partial v_t^x$ being the marginal cost of the search effort for $x = \{\ell, d\}$. The marginal search cost c_t^x divided by the matching probability $q\left(\alpha_t^x\right)$ is the average matching cost for banks on market x. At the bank's optimum, this matching cost equals the marginal contribution of n^x to the bank's value, which is measured by λ_t^x for assets on markets $x = \{\ell, d\}$.

The first order condition with respect to the amount of loans at the next time step is

$$\begin{aligned} n_{t+1}^\ell : \lambda_t^\ell + \lambda_t^d &= \beta \frac{\partial B\left(n_{t+1}^\ell\right)}{\partial n_{t+1}^\ell} \\ &= \beta\left[\left(\rho_{t+1}^\ell - \rho_{t+1}^d\right) + \left(1-\delta^\ell\right)\lambda_{t+1}^\ell + \left(1-\delta^d\right)\lambda_{t+1}^d\right] \end{aligned} \tag{14}$$

The term on the LHS is the full marginal value of loans, i.e., the direct contribution of a unit of loan to the bank's value, which corresponds to λ_t^ℓ, plus its indirect contribution to the bank's value through the additional deposit that it requires, which corresponds to λ_t^d. The term on the RHS is the payoff of this marginal loan discounted by β. The first term of this payoff is the bank interest margin. The bank make profits by lending money at a rate greater than that at which it is itself remunerated, i.e., $\rho^\ell > \rho^d$. The second and third terms are the bank values of the financial relationships. With a probability of $(1-\delta^x)$ the financial relationship on market $x=\{\ell, d\}$ is not destroyed and the commercial bank retains the value of this relationship in the next period. Combining the first order conditions (13)-(14) determines the equilibrium search effort of banks

$$\frac{c_t^\ell}{q\left(\alpha_t^\ell\right)}+\frac{c_t^d}{q\left(\alpha_t^d\right)}=\beta\left[\left(\rho_{t+1}^\ell-\rho_{t+1}^d\right)+\left(1-\delta^\ell\right)\frac{c_{t+1}^\ell}{q\left(\alpha_{t+1}^\ell\right)}+\left(1-\delta^d\right)\frac{c_{t+1}^d}{q\left(\alpha_{t+1}^d\right)}\right] \tag{15}$$

Definition 1 *The equilibrium for commercial banks is* $\left\{\alpha_t^\ell, \alpha_t^d\right\}$ *consistent with the maximisation of profits by commercial banks, Equation (15), and the participation of households to the banking industry, Equation (6), given: the interest rates* $\left\{\rho_t^\ell, \rho_t^d\right\}$*, the marginal search costs* $\left\{c_t^\ell, c_t^d\right\}$*, the destruction rates of financial relationships* $\left\{\delta^\ell, \delta^d\right\}$*, the household preferences* $\{\beta, \rho_t^c\}$*, and the specifications (1)-(2) of the matching functions.*

3.4.2 The case of a specialised industry: narrow banks and finance houses

Narrow banks specialise in deposits, and supply funds on an interbank market at the unitary price ρ_t^b. The representative narrow bank maximises the following value function

$$\begin{aligned} B^d\left(n_t^d\right) &= \max_{v_t^d, n_{t+1}^d}\left\{\rho_t^b n_t^d-\rho_t^d n_t^d-c\left(v_t^d\right)+\beta B^d\left(n_{t+1}^d\right)\right\} \\ &\quad -\lambda_t^d\left[n_{t+1}^d-\left(1-\delta^d\right)n_t^d-q\left(\alpha_t^d\right)v_t^d\right] \end{aligned} \tag{16}$$

whose solution satisfies

$$\frac{c_t^d}{q\left(\alpha_t^d\right)}=\beta\frac{\partial B^d\left(n_{t+1}^d\right)}{\partial n_{t+1}^d}=\beta\left[\rho_{t+1}^b-\rho_{t+1}^d+\left(1-\delta^d\right)\frac{c^d}{q\left(\alpha_{t+1}^d\right)}\right] \tag{17}$$

Finance houses specialise in making loans, and buy funds on the interbank market. The representative finance house maximises the following value function

$$B^{\ell}\left(n_t^{\ell}\right) = \max_{v_t^{\ell}, n_{t+1}^{\ell}} \left\{ \left(\rho_t^{\ell} - \rho_t^{b}\right) n_t^{\ell} - c\left(v_t^{\ell}\right) + \beta B\left(n_{t+1}^{\ell}\right) \right\} - \lambda_t^{\ell} \left[n_{t+1}^{\ell} - \left(1 - \delta^{\ell}\right) n_t^{\ell} - q\left(\alpha_t^{\ell}\right) v_t^{\ell} \right] \tag{18}$$

whose solution satisfies

$$\frac{c_t^{\ell}}{q\left(\alpha_t^{\ell}\right)} = \beta \frac{\partial B^{\ell}\left(n_{t+1}^{\ell}\right)}{\partial n_{t+1}^{\ell}} = \beta \left[\rho_{t+1}^{\ell} - \rho_{t+1}^{b} + \left(1 - \delta^{\ell}\right) \frac{c^{\ell}}{q\left(\alpha_{t+1}^{\ell}\right)} \right] \tag{19}$$

Definition 2 *The equilibrium for specialised banks is* $\left\{\alpha_t^{\ell}, \alpha_t^{d}, \rho_t^{b}\right\}$ *consistent with the maximisation of profits by narrow banks, Equation (17), by finance houses, Equation (19), and the participation of households to the banking industry, Equation (6), given: the interest rates* $\left\{\rho_t^{\ell}, \rho_t^{d}\right\}$*, the marginal search costs* $\left\{c_t^{\ell}, c_t^{d}\right\}$*, the destruction rates of financial relationships* $\left\{\delta^{\ell}, \delta^{d}\right\}$ *, the household preferences* $\left\{\beta, \rho_t^{c}\right\}$ *, and the specifications (1)-(2) of the matching functions.*

4 Equilibrium

This section analyses the properties of the steady-state equilibria. It is first shown that the competition experienced by commercial and specialised banks each leads to the same allocation of resources if interest rates are exogenous. This competitive allocation is then compared with the optimal allocation that is the solution to the social planner's problem. Finally, I discuss the conditions under which this optimal allocation could be decentralised through Nash bargaining for interest rates.

4.1 Competitive allocations

The steady-state allocations of non-financial agents are defined as functions of market tightness. The steady-state mass of financed, productive entrepreneurs is given by the function $n^{\ell}\left(\alpha^{\ell}\right)$, which is the steady-state solution of (11)

$$n^{\ell}\left(\alpha^{\ell}\right)=\frac{\overline{m}^{\ell}\left(\alpha^{\ell}\right)^{\varepsilon}}{\delta^{\ell}+\overline{m}^{\ell}\left(\alpha^{\ell}\right)^{\varepsilon}}\overline{n}^{\ell} \tag{20}$$

for the specifications (1)-(2) of the matching function. The size of the production sector n^{ℓ} is positively correlated with the total loan demand $\overline{n}^{\ell}$, negatively correlated with the destruction rate of matches δ^{ℓ}, positively correlated with the scale parameter of the matching function $\overline{m}^{\ell}$, and positively correlated with the endogenous loan market tightness α^{ℓ}. A high value of $\alpha^{\ell}=v^{\ell}/\left(\overline{n}^{\ell}-n^{\ell}\right)$ implies a high proportion of entrepreneurs financed by banks $(n^{\ell}/\overline{n}^{\ell})$ and therefore a high level of production. Because each unit of loans is financed by one unit of deposits, the steady-state mass of matched households is $n^{d}\left(\alpha^{\ell}\right)=n^{\ell}\left(\alpha^{\ell}\right)$. The steady-state solution of (11) for the deposit market is the mass of searching households, which depends on the two tightness variables

$$u^{d}\left(\alpha^{d},\alpha^{\ell}\right)=\frac{\delta^{d}}{\overline{m}^{d}\left(\alpha_{t}^{d}\right)^{\varepsilon}}n^{\ell}\left(\alpha^{\ell}\right) \tag{21}$$

for the specifications (1)-(2) of the matching function.

The deposit market tightness is determined by the free entry condition on the banking industry for households, that is the steady-state solution of (6)

$$\alpha^{d}=\left(\frac{1/\beta-1+\delta^{d}}{c^{h}\left(\rho^{d}-\rho^{c}\right)}\frac{1}{\overline{m}^{d}}\right)^{1/\varepsilon^{d}} \tag{22}$$

for the specifications (1)-(2) of the matching function. The deposit market tightness α^{d} is positively correlated with the discount rate $1/\beta$, positively correlated with the destruction rate of matches δ^{d}, negatively correlated with the scale parameter of the matching function $\overline{m}^{d}$, and negatively correlated with the gap in marginal utility $\left(\rho^d - \rho^c\right)$. A higher deposit interest rate ρ^d implies a low value of $\alpha^d = v^d / \left(\overline{n}^d - n^d\right)$ because this stimulates the entry of households to the deposit market, and households accept higher matching costs as compensation for the higher payoff of matching. The deposit market tightness affects the size of the production sector through the search effort of banks on the loan market as explained in the following lemma.

Lemma 1 *For exogenous interest rates, the tightness of the financial markets and the level of production are the same for* (*i*) *an integrated banking industry with commercial banks, and* (*ii*) *a specialised banking industry with narrow banks and finance houses.*

Proof. *Let us first consider the commercial banks. The competitive search effort made by commercial banks is determined by Equation (15), which depends on the two tightness variables* α^d *and* α^ℓ. *Because the behaviour of households determines the deposit market tightness, see (22), the steady-state expression of Equation (15) yields the value of* α^ℓ *as a function of* α^d *and other structural parameters.*

$$\left(\alpha^\ell\right)^{1-\varepsilon} = \frac{1}{1/\beta - 1 + \delta^\ell} \frac{\overline{m}^\ell}{c^\ell} \left[\left(\rho^\ell - \rho^d\right) - c^d \frac{1/\beta - 1 + \delta^d}{\overline{m}^d} \left(\alpha^d\right)^{1-\varepsilon} \right] \tag{23}$$

for the specifications (1)-(2) of the matching function. I now show that this condition still holds for specialised banks. In order to show that the credit market tightness is the same, the two first order conditions (17) and (19) are summed. After simplification, the two terms for price ρ^b *cancel and the new equation is identical to (23) at the steady-state. The level of production is also identical, and equal to* zn^ℓ, *because the same degree of loan market tightness leads to an identical mass of financed productive entrepreneurs, which is determined by (20). The single difference with the equilibrium for commercial banks is that for specialised banks one equation remains to determine the price* ρ^b, *i.e.*

$$\rho^b = \rho^d + \left(1/\beta - 1 + \delta^d\right) \frac{c^d}{\overline{m}^d} \left(\alpha^d\right)^{1-\varepsilon} \tag{24}$$

using (17). Finally, the condition of existence for the equilibrium $\left\{\alpha^{\ell}, \alpha^{d}, n^{\ell}\right\}$ *is* $\alpha^{\ell} > 0$*, i.e.*

$$\left(\rho^{\ell} - \rho^{d}\right) > c^{d}\left(1/\beta - 1 + \delta^{d}\right)^{1/\varepsilon}\left(\overline{m}^{d}\right)^{-1/\varepsilon}\left(\rho^{d} - \rho^{c}\right)^{-(1-\varepsilon)/\varepsilon} \tag{25}$$

given (23). ■

The search behaviour of banks leads to the negative relationship between the two market tightness variables defined by (23) . For commercial banks, this negative relationship between the two tightness variables may be deduced directly from the optimality condition (15). Commercial banks consider the full cost of financial intermediation, which is the sum of the search costs on the deposit and loan markets. For a given payoff of a financial intermediation, banks adjust their search efforts such that the search costs of their financial intermediation remain unchanged. If α^{d} increases, the search cost for banks on the deposit market is higher, and α^{ℓ} must fall in order to reduce the search cost of banks on the loan market. On the other side of the loan market, these actions increase the average duration of the search for entrepreneurs and therefore cause the production sector to contract. The mechanism is similar for specialised banks except that it is transmitted via the interbank market price ρ^{b}. If α^{d} increases, the costs of funds for finance houses on the interbank market rises, see (24), and α^{ℓ} must fall to reduce the search costs of finance houses on the loan market.

4.2 Efficiency

The competitive allocations characterised in Lemma 1 relate to exogenous interest rates. In this section, I define the system of interest rates that decentralises the optimal allocation.

Lemma 2 *There exists a system of interest rates* $\left\{\widetilde{\rho}^{d}, \widetilde{\rho}^{\ell}\right\}$ *that makes the competitive allocation optimal for the two banking systems (integrated and specialised).*

Proof. *The optimal equilibrium* $\left\{\widetilde{\alpha}^{\ell}, \widetilde{\alpha}^{d}\right\}$ *solution of the social planner maximisation* *program solves*

$$\widetilde{\alpha}^{d}=\frac{\varepsilon}{(1-\varepsilon)\,c^{h}c^{d}} \tag{DD-O}$$

$$\frac{c^{\ell}}{\overline{m}^{\ell}}\left(\widetilde{\alpha}^{\ell}\right)^{1-\varepsilon}\left(1/\beta-1+\delta^{\ell}\right)+\left(1-\varepsilon^{\ell}\right)c^{\ell}\widetilde{\alpha}^{\ell}=\varepsilon\,(z-\rho^{c})-\frac{c^{d}}{\overline{m}^{d}}\left(\widetilde{\alpha}^{d}\right)^{1-\varepsilon}\left(1/\beta-1+\delta^{d}\right) \tag{LL-O}$$

see Appendix A.1 for details. The efficient interest rate on saving $\widetilde{\rho}^{d}$ is such that the α^{d} solution of (6) is equal to the optimal value $\widetilde{\alpha}^{d}$ given by (DD-O), or in symbols

$$\widetilde{\rho}^{d}=\rho^{c}+\left(1/\beta-1+\delta^{d}\right)\frac{1}{c^{h}\overline{m}^{d}}\left(\frac{\varepsilon}{(1-\varepsilon)\,c^{h}c^{d}}\right)^{-\varepsilon} \tag{26}$$

Then, $\widetilde{\rho}^{\ell}$ is such that the values for the α^{ℓ} solution of (15) and (LL-O) are identical (assuming that the deposit market tightness is at its optimal level $\alpha^{d}=\widetilde{\alpha}^{d}$). In symbols,

$$\widetilde{\rho}^{\ell}=\widetilde{\rho}^{d}+\varepsilon\,(z-\rho^{c})-(1-\varepsilon)\,c^{\ell}\widetilde{\alpha}^{\ell} \tag{27}$$

■

The efficient deposit interest rate, as defined by (26), is equal to the sum of the marginal utility of the asset outside the banking industry and the discounted matching costs for households, which then exactly compensate for the search costs. This ensures the optimality of the deposit market tightness α^{d}. The efficient interest rate for loans defined by (27) is equal to the cost of funds ρ^{d} plus an average weighted by ε and $(1-\varepsilon)$ of the contribution of the loan to production $(z-\rho^{c})$ and search activities $c^{\ell}\alpha^{\ell}$. This internalises the trading externalities associated with the matching friction in the interest rates and therefore ensures the optimality of the loan market tightness α^{ℓ}, provided that the deposit market tightness α^{d} is optimal.

4.3 Bargained Interest Rates

This section extends the competitive model of bank competition to the case of endogenous interest rates.

4.3.1 The bargaining processes

As commonly assumed in the literature on matching frictions, interest rates are the outcome of a Nash bargaining program:

$$\rho_t^x = \arg\max \ (X_t^m - X_t^u)^\eta (\Delta B_t^x)^{1-\eta} \tag{28}$$

where $(1-\eta)$ measures the bargaining power of the bank and η the bargaining power of the non-financial agent. The contributions of the non-financial agent to the Nash criteria are $(X_t^m - X_t^u)$ for $X = \{D, L\}$. For the household, this is

$$D_t^m - D_t^u = \rho_t^d - \rho_t^c + 1/c^h + \left(1 - \delta^d - p\left(\alpha_t^d\right)\right) \beta \left(D_{t+1}^m - D_{t+1}^u\right) \tag{29}$$

using (4)-(5). For the entrepreneur, the equivalent expression is

$$L_t^m - L_t^u = z_t - \rho_t^\ell + \left(1 - \delta^\ell - p\left(\alpha_t^\ell\right)\right) \beta \left(L_{t+1}^m - L_{t+1}^u\right) \tag{30}$$

using (7)-(8).

The contributions of the bank ΔB_t^x depend on the system of organisation of the banking industry. The contribution of the narrow bank to the Nash criteria is

$$\Delta B^d = \rho^b - \rho^d + \left(1 - \delta^d\right) \lambda_t^d \tag{31}$$

or $\partial B\left(n_t^d\right)/\partial n_t^d$ using the definition of the value function in (16). The contribution of the finance house to the Nash criteria is

$$\Delta B^\ell = \rho^\ell - \rho^b + \left(1 - \delta^\ell\right) \lambda_t^\ell \tag{32}$$

or $\partial B\left(n_t^\ell\right)/\partial n_t^\ell$ using the definition of the value function in (18). The key difference between a commercial bank and a specialised bank in the search-theoric approach is that the commercial bank faces a *double* matching process whereas the specialised bank faces a *single* matching process. Because the balance sheet of the commercial bank is $n_t^d = n_t^\ell$, the contribution of one additional unit of loan or of deposit is the same from the bank's point of view: $\Delta B^\ell = \Delta B^d = \Delta B$

$$\Delta B = \rho^\ell - \rho^d + \left(1 - \delta^\ell\right) \lambda_t^\ell + \left(1 - \delta^d\right) \lambda_t^d \tag{33}$$

or $\partial B \left(n^\ell\right) / \partial n^\ell$ using the definition of the value function in (12).[16] The provision of one marginal unit of loan to an entrepreneur generates a flow of revenues that is equal to the loan interest rate ρ^ℓ. In addition, this loan will not be destroyed in the following period with probability $\left(1 - \delta^\ell\right)$, and the bank will keep its value equal to λ_t^ℓ. To provide this unit of loan, however, the commercial bank cannot use a frictionless market as the specialised bank can, instead it must use one unit of deposit collected on the frictional deposit market, which is remunerated at an interest rate of ρ^d, and remains on the bank's balance sheet at the next period with a probability of $\left(1 - \delta^d\right)$ for a value of λ^d. The solutions of (28) solve

$$(1 - \eta) \left(X_t^m - X_t^u\right) = \eta \Delta B_t^x \tag{34}$$

for $x = \{\ell, d\}$ and for the contributions (29)-(30)-(31)-(32)-(33) to the Nash criteria.

4.3.2 Normative implications

The three following propositions present the normative implications of the search model.

Proposition 1 *Interest rates bargained by narrow banks and finance houses lead to the optimal financial markets' tightness under the conditions of Hosios (1990), that is $\varepsilon = 1 - \eta$.*

Proof. *See Appendix A.2.* ■

[16] The contribution of the commercial bank, defined by (33), is the sum of the contributions of the commercial bank and of the finance house, defined by (31)-(32) respectively.

The condition of Hosios (1990) is well-known to internalise efficiently the trading externalities of the matching process in the Nash bargaining process, see also Pissarides (2000). When banks are specialised, this condition ensures the efficiency of the competition. The following two propositions show that this is not the case for commercial banks.

Proposition 2 *For an exogenous loan interest rate, the bargained deposit interest rate between households and commercial banks is inefficient. Under the Hosios (1990) condition,* $\varepsilon = (1-\eta)$, *the equilibrium is unique and characterised by an insufficient market tightness for deposits and loans* ($\alpha^x < \widetilde{\alpha}^x$ *for* $x = \{d, \ell\}$). *If* $\eta < (1-\varepsilon)$, *multiple equilibria may exist.*

Proof. *The Nash solution (34) can be expressed as follows*

$$\frac{1}{c^h p\left(\alpha^d\right)} \frac{1}{\beta} = \left(\frac{\eta}{1-\eta}\right) \Delta B = \left(\frac{\eta}{1-\eta}\right) \frac{1}{\beta} \left(\frac{c^d}{q\left(\alpha^d\right)} + \frac{c^\ell}{q\left(\alpha^\ell\right)}\right) \tag{35}$$

using Equations (6)-(13)-(14). For the specifications (1)-(2) of the matching function, it may be re-expressed as follows

$$\left(\alpha^\ell\right)^{1-\varepsilon} = \frac{\overline{m}^\ell}{c^\ell} \left[\left(\frac{1-\eta}{\eta}\right) \frac{1}{c^h \overline{m}^d \left(\alpha^d\right)^\varepsilon} - \frac{c^d}{\overline{m}^d} \left(\alpha^d\right)^{1-\varepsilon}\right] \tag{DD-S}$$

which satisfies $\partial\alpha^\ell / \partial\alpha^d < 0$, *and the associated locus of (DD-S) is decreasing in the plane* $\left(\alpha^d, \alpha^\ell\right)$. *Because* $\alpha^\ell > 0$, *there is an upper limit for* α^d, *denoted* $\alpha^d_{\max}$ *and equal to*

$$\alpha^d < \alpha^d_{\max} = \left(\frac{1-\eta}{\eta}\right) \frac{1}{c^h c^d} \tag{36}$$

The upper limit of α^d *for commercial banks is the equilibrium value for narrow banks, defined by (A.7), which is the optimal value* $\widetilde{\alpha}^d$ *under the Hosios (1990) condition* $\varepsilon = (1-\eta)$. *The value of* ρ^d *is deduced from the free entry condition of households, Equation (22)*

$$\rho^d = \rho^c + \left(1/\beta - 1 + \delta^d\right) \frac{1}{c^h \overline{m}^d \left(\alpha^d\right)^\varepsilon} \tag{37}$$

This expression for ρ^d is then introduced to the equilibrium condition on loan market tightness, Equation (23), to yield

$$\left(\alpha^{\ell}\right)^{1-\varepsilon} = \frac{1}{1/\beta - 1 + \delta^{\ell}} \frac{\overline{m}^{\ell}}{c^{\ell}} \left[\rho^{\ell} - \rho^{c} - \left(1/\beta - 1 + \delta^{d}\right)\left(\frac{1}{c^{h}\overline{m}^{d}\left(\alpha^{d}\right)^{\varepsilon}} + \frac{c^{d}}{\overline{m}^{d}}\left(\alpha^{d}\right)^{1-\varepsilon}\right)\right] \quad \text{(LL-S)}$$

In the plane $\left(\alpha^d, \alpha^\ell\right)$, the locus associated with (LL-S) increases for values of α^ℓ below its optimal value $\widetilde{\alpha}^d$ and decreases for values of α^ℓ above this value. More precisely, it satisfies

$$\frac{\partial \alpha^{\ell}}{\partial \alpha^{d}} \begin{cases} < 0, \text{ if } \alpha^{d} > \widetilde{\alpha}^{d} \\ = 0, \text{ if } \alpha^{d} = \widetilde{\alpha}^{d} \\ > 0, \text{ if } \alpha^{d} < \widetilde{\alpha}^{d} \end{cases} \tag{38}$$

where $\widetilde{\alpha}^d$ is defined by (DD-O). The equilibrium allocation $\left\{\alpha^d, \alpha^\ell\right\}$ is at the intersection of the two loci associated with (DD-S) and (LL-S). Under the Hosios (1990) condition, the upper-limit of α^d is its optimal value: $\alpha^d_{\max} = \widetilde{\alpha}^d$. Therefore, the equilibrium, if it exists, is necessarily on the increasing part of the locus associated with (LL-S) according to (38), where it intersects the decreasing locus associated with (DD-S). Multiple equilibria may occur if the Hosios (1990) condition is not satisfied and if the upper limit of α^d is greater than its optimal value $\widetilde{\alpha}^d$, i.e. $\eta < (1-\varepsilon)$. In this case, the locus associated with (LL-S) increases and decreases for $\alpha^d \in \left[0, \alpha^d_{\max}\right]$ and it can cross the locus associated with (DD-S) for a second time. In this second equilibrium, α^d is higher and α^ℓ is lower than in the first. ■

The household receives a share $\eta/(1-\eta)$ of the marginal value of the deposit for the bank, equal to $c^d/q\left(\alpha^d\right)$ for both commercial and narrow banks as is commonly the case in models that use search frictions. Here, the novelty is that the value is shared across the market for commercial banks. Indeed, households also receive a share $\eta/(1-\eta)$ of the marginal value of the loan equal to $c^\ell/q\left(\alpha^\ell\right)$, see (35) for commercial banks.[17] Because the deposit is allocated

[17] This is not the case for narrow banks; see (A.6) in the Appendix A.2.

by a commercial bank on the market with matching frictions (the loan market), whereas the narrow bank allocates this deposit on a frictionless market (the interbank market), there is a cross-market sharing mechanism that distorts the efficiency of the bargaining process.

In order to determine the equilibrium values of (α^d, α^ℓ), let us consider the first equilibrium condition (DD-S), which decreases in the plane (α^d, α^ℓ). Households have a stronger incentive to enter the banking industry by doing business with commercial banks in preference to narrow banks because they get a share of a higher rent value, which includes the bank value of the loan made by the deposit. A high value of α^ℓ is associated with a high marginal value of loans, which is equal to the average matching costs $c^\ell/q(\alpha^\ell) = (c^\ell/\overline{m}^\ell)(\alpha^\ell)^{1-\varepsilon}$. This therefore stimulates the entry of households, which accept higher matching costs, accompagnied by a lower equilibrium value for the tightness α^d. This mechanism explains why the locus associated with the equilibrium condition (DD-S) decreases in the plane (α^d, α^ℓ). The locus associated with the second equilibrium condition (LL-S) is hump-shaped in the plane (α^d, α^ℓ), whereas it strictly decreases for the exogenous interest rate. The endogenous reaction of the deposit interest rate explains why this locus increases for low values of α^d. Higher values of α^d reduce the deposit interest rate through (37) and increase the bank interest rate margin, which improves the bank's incentives to search on the loan market. For $\varepsilon \geq (1-\eta)$, multiple equilibria cannot exist because the slope of the locus (LL-S) becomes negative for values of α^d outside the domain of definition $[0, \alpha^d_{\max}]$. For $\varepsilon < (1-\eta)$, multiple equilibria may occur, characterised by a first equilibrium where α^ℓ is high and α^d is low and a second where α^ℓ is low and α^d is high.[18]

Proposition 3 *For an optimal deposit market tightness and an efficient deposit interest rate, the bargained loan interest rate between entrepreneurs and commercial banks is generally inefficient and, under the Hosios (1990) condition, makes the loan market tightness insufficient.*

Proof. *For the Nash solution (34) and the competitive behaviour of commercial banks*

[18] The two equilibria are stable because the dynamic optimality conditions do not depend on the state variables; see (6)-(15). Therefore, the economy can instantaneously select one of two equilibria and thereafter remains locked in it.

(15), the payoff of entrepreneurs (30) is

$$L^m - L^u = z - \rho^\ell + \left(1 - \delta^\ell - p\left(\alpha^\ell\right)\right)\frac{\eta}{1-\eta}\left(\lambda^\ell + \lambda^d\right) \tag{39}$$

The loan interest solution of (34) for the Nash contributions (33) and (39) is

$$\rho^\ell = \widetilde{\rho}^d + \left(1 - \eta^\ell\right)(z - \rho^c) - \eta^\ell c^\ell \alpha^\ell - \eta\left(\delta^\ell + \overline{m}^\ell\left(\alpha^\ell\right)^\varepsilon + \frac{1-\beta}{\beta}\right)\frac{c^d}{\overline{m}^d}\left(\widetilde{\alpha}^d\right)^{1-\varepsilon} \tag{40}$$

assuming $\rho^d = \widetilde{\rho}^d$ and $\alpha^d = \widetilde{\alpha}^d$. For the bargained loan interest ρ^ℓ given by (40), the optimality condition (23) becomes

$$\begin{aligned} & \frac{c^\ell}{\overline{m}^\ell}\left(\alpha^\ell\right)^{1-\varepsilon^\ell}\left(1/\beta - 1 + \delta^\ell\right) + \eta c^\ell \alpha^\ell \qquad \textit{(LL-S')} \\ = \; & (1-\eta)(z-\rho^c) - c^d \frac{1/\beta - 1 + \delta^d}{\overline{m}^d}\left(\widetilde{\alpha}^d\right)^{(1-\varepsilon)} - \eta\left(\delta^\ell + \overline{m}^\ell\left(\alpha^\ell\right)^\varepsilon + \frac{1-\beta}{\beta}\right)\frac{c^d}{\overline{m}^d}\left(\widetilde{\alpha}^d\right)^{1-\varepsilon} \end{aligned}$$

It is clear that for $\varepsilon = (1-\eta)$, which holds under the Hosios (1990) condition, if $\widetilde{\alpha}^d > 0$ the competitive tightness is below its optimal value $\alpha^\ell < \widetilde{\alpha}^\ell$ because of the last term on the RHS of (LL-S'), which is absent in the solution of the social planner (LL-O). ■

The contribution of the commercial bank to the Nash criteria includes the value of the deposit associated with the loan. The entrepreneur receives a share $\eta/(1-\eta)$ of the sum of the marginal values of a loan and a deposit, which is $\left(\lambda^\ell + \lambda^d\right)$ in (39). As for the deposit interest rate, there is a cross-market sharing of the values of the financial services. This is not the case for specialised banks.[19] Therefore, entrepreneurs pay lower interest rates with commercial banks than with finance houses, but this also implies a reduced search effort on the loan market for commercial banks than for finance houses. Finally, the loan market tightness is generally suboptimal and unambiguously below its optimal value under the Hosios (1990) condition.

[19] With finance houses, the entrepreneur receives only a share $\eta/(1-\eta)$ of the marginal value of the loan, i.e. λ^ℓ in (A.9), see Appendix A.2.

4.4 Numerical Illustrations

Using numerical examples, this section illustrates the normative properties of the search model of banking competition. The model is calibrated for the optimal allocation (solution of the social planner or of the competition of specialised banks under the Hosios (1990) condition). One unit of time corresponds to one month. The discount rate is $\beta = 0.999$. The average duration of a financial relationship is set to six months for households ($\delta^d = 1/6$), less than the average duration for entrepreneurs, which is set to three years ($\delta^\ell = 1/36$). The average duration of search is also shorter for households (one month and one week) than for entrepreneurs (three months), i.e. $p\left(\alpha^d\right) = 1/1.25$ and $p\left(\alpha^\ell\right) = 1/3$. The corresponding rate of matching for entrepreneurs is $n^\ell = 0.92$ under the normalisation $\overline{n}^\ell = 1$. The values of the scale parameters of the matching functions are deduced to be $\overline{m}^d = 0.71$ and $\overline{m}^\ell = 0.29$. The productivity of loans is set to $z = 2$ and the search costs are $c^h = 0.8$ and $c^d = c^\ell = 1$. The marginal utilities for households are set to $\rho^c = 0.5$ and $\rho^\ell = 1.15$, and I deduce the value of $\rho^d = 0.762$.

The model is first calibrated under the Hosios (1990) condition: $\varepsilon = (1-\eta) = 1/2$. Figure 1 shows how the bargaining process on the loan interest rate by commercial banks leads to an insufficient loan market tightness even if the deposit market tightness is optimal. Figure 2 shows how the bargaining process on the deposit interest rate by commercial banks (for an exogenous loan interest rate) makes the tightness insufficient on both markets: the average cost of matching is excessively high for the two non-financial agents. The model is not then calibrated under the Hosios (1990) condition: $\varepsilon = \eta = 0.21 < (1-\eta)$. Figure 3 shows how multiple equilibria emerge. In the first equilibrium (on the left hand side of the figure), the tightness is above its optimal value for the loan market and below it for the deposit market. For the second equilibrium (at the right hand side of the figure), the position is reversed: tightness is excessive on the deposit market and insufficient on the loan market.

5 Conclusion

This article contributes to the debate on the organisation of the banking industry by providing a new mechanism in favour of specialised banks and against integrated banks. This mechanism proceeds from the existence of search frictions in the deposit and loan markets. In the search environment, commercial banks search in both markets, whereas specialised banks only search in one market. If interest rates are exogenous, this difference does not matter and has no consequences on the allocation of resources. However, this difference becomes crucial when endogenous interest rates are considered, because the value of financial services is not shared by specialised and integrated banks in the same way. Because commercial banks consider the value of financial services in both markets, households are allowed to obtain a share of the value associated with the lending relationship and, reciprocally, entrepreneurs are allowed a share of the rent value associated with the deposit-taking relationship. This cross-market sharing distorts the efficiency of the bargaining processes on interest rates and may be at the origin of excessive credit rationing and multiple equilibria.

These findings supplement the literature in that they provide a mechanism in support of the narrow banking proposal and against commercial banking, although no attempt has been made to set this mechanism against the other potential advantages of commercial banks. This consideration is beyond the scope of the present paper. It would nevertheless be of interest to extend the present arguments by considering search frictions in the interbank market, in the spirit of the approaches of Rocheteau and Weill (2011), rather than the frictionless interbank market consider herein. In this sense, the distortion of the Nash bargaining processes induced by commercial banks could be compared to the saving of search costs on the interbank market that permits the banking industry to be organised in this way.

References

Berger A. N., R. .S. Demsetz, P. E. Strahan, 1999. The consolidation of the financial services industry: Causes, consequences, and implications for the future, Journal of Banking & Finance 23, 135–194.

Bossone B., 2002. "Should Banks be Narrowed?," The Public Policy Brief Series of the Levy Institute n°69.

Becsi Z., V.E. Li, and P. Wang, 2005. "Heterogeneous borrowers, liquidity, and the search for credit," Journal of Economic Dynamics and Control 29(8), 1331-1360.

Calomiris C. W.,C. M. Kahn, 1991. "The Role of Demandable Debt in Structuring Optimal Banking Arrangements," The American Economic Review 81(3), 497-513.

Chamley C., C. Rochon, 2011. "From Search to Match: When Loan Contracts Are Too Long," Journal of Money, Credit and Banking, 43, 385–411.

Chamley C., L. J. Kotlikoff, 2009. "Limited Purpose Banking – Putting An End to Financial Crises," The Financial Times, January.

Chow J.T.S., J. Surti, 2011. "Making Banks Safer: Can Volcker and Vickers Do It?," IMF Working Paper 11/236.

Craig B. R, J. G. Haubrich, 2006. "Gross loan flows," Working Paper 06-04, Federal Reserve Bank of Cleveland.

De Grauwe P., 2009. "Lessons from the Banking Crisis: A Return to Narrow Banking," CESifo DICE Report 7(2), 19-23.

Degryse H., S. Ongena, 2008. "Competition and Regulation in the Banking Sector: A Review of the Empirical Evidence on the Sources of Bank Rents, " in Handbook of financial intermediation & banking, edited by Anjan V. Thakor, Arnoud Boot, Elsevier publications.

Dell'Ariccia G., P. Garibaldi, 2005. "Gross Credit Flows," Review of Economic Studies 72(3), 665-685.

Den Haan W.J., G. Ramey, and J. Watson, 2003. "Liquidity flows and fragility of business enterprises," Journal of Monetary Economics 50(6), 1215-1241.

Diamond P. A., 1982. "Aggregate Demand Management in Search Equilibrium," Journal of Political Economy 90(5), 881-94.

Diamond D. W., P. H. Dybvig, "Bank Runs, Deposit Insurance, and Liquidity," Journal of Political Economy 91, 401–419.

Duffie D., N. Gârleanu, L. Pedersen, 2005. "Over-the-counter Markets,"Econometrica 73, 1815–47.

Hosios A., 1990. "On the Efficiency of Matching and Related Models of Search and Unemployment," Review of Economic Studies 57(2), 279-298.

Kashyap A. K., R. Rajan R, J. C. Stein, 2002. "Banks as Liquidity Providers: An Explanation for the Coexistence of Lending and Deposit-Taking," Journal of Finance, 57(1), 33-73.

Kay J., 2009. Narrow Banking: The Reform of Banking Regulation, Centre for the Study of Financial Innovation.

Kiser E.H., 2002. "Predicting Household Switching Behavior and Switching Costs at Depository Institutions," Review of Industrial Organization 20, 349–365.

Laeven L., R. Levine, 2007. "Is there a diversification discount in financial conglomerates?," Journal of Financial Economics 8, 331–367.

Litan R. E., 1987. "What Should Banks Do?" Washington D.C. The Brooking Institution.

Mishkin F. S., 1999. "Financial consolidation: Dangers and opportunities," Journal of Banking & Finance 23, 675–691.

Moen E. R., 1997. "Competitive Search Equilibrium," Journal of Political Economy 105(2), 385-411.

Mortensen, D. T., C.A. Pissarides, 1994. "Job Creation and Job Destruction in the Theory of Unemployment," Review of Economic Studies 61(3), 397-415.

Petrosky-Nadeau N., E. Wasmer, 2011. "Macroeconomic Dynamics in a Model of Goods, Labor and Credit Market Frictions," IZA Working Paper DP 5763.

Phillips R. J., 1995. "Narrow banking reconsidered: The functional approach to financial reform," The Public Policy Brief Series of the Levy Institute n°17.

Phillips R. J., A. Roselli, 2009. "How to Avoid the Next Taxpayer Bailout of the Financial System: The Narrow Banking Proposal," Networks Financial Institute Policy Briefs, 2009.

Pissarides C.A., 2000. Equilibrium unemployment theory (Second edition). MIT Press.

Pulley L. B., D. B. Humphrey, 1993. "The Role of Fixed Costs and Cost Complementarities in Determining Scope Economies and the Cost of Narrow Banking Proposals," The Journal of Business 66(3), 437-462.

Rocheteau G., P.-O. Weill, 2011. "Liquidity in Frictional Asset Markets," Journal of Money, Credit and Banking, 43(s2), 261–282.

Sharpe S. A., 1997. "The Effect of Consumer Switching Costs on Prices:A Theory and its Application to the Bank Deposit Market," Review of Industrial Organization 12, 79–94.

Shy O, R. Stenbacka, 2007. "Liquidity provision and optimal bank regulation," International Journal of Economic Theory 3(3), 219–233.

Shy O., 2002. "A quick-and-easy method for estimating switching costs," International Journal of Industrial Organization 20, 71–87.

Strahan P. E., 2008. "Bank Structure and Lending: What We Do and Do Not Know," in Handbook of financial intermediation & banking, edited by Anjan V. Thakor, Arnoud Boot, Elsevier publications.

Vander Vennet R., 2002. "Cost and Profit Efficiency of Financial Conglomerates and Universal Banks in Europe," Journal of Money, Credit and Banking, 34(1), 254-282.

Vissing-Jørgensen, 2002." Limited Asset Market Participation and the Elasticity of Intertemporal Substitution," Journal of Political Economy, 110(4), 825-853.

Wallace N., 1996. "Narrow Banking Meets the Diamond-Dybvig Model," Federal Reserve Bank of Minneapolis Quarterly Review 20(1), 3–13.

Wasmer E. and P. Weil, 2004. "The Macroeconomics of Labor and Credit Market Imperfections," American Economic Review 94(4), 944-963.

Zephirin M. G., 1994. "Switching Costs in the Deposit Market," The Economic Journal 104(423), 455-461.

A Appendix

A.1 Proof of Lemma 2

The optimal allocation is the solution of the program of the social planner defined as follows

$$
\begin{aligned}
& P\left(n_t^\ell\right) && \text{(A.1)}\\
= & \max_{n_{t+1}^\ell, v_t^d, v_t^\ell, u_t^d} \left\{ n_t^\ell z_t + \left(\overline{n}_t^d - n_t^\ell - u_t^d\right)\rho_t^c + u_t^d\left(\rho_t^c - \frac{1}{c^h}\right) - c\left(v_t^d\right) - c\left(v_t^\ell\right) + \beta P\left(n_{t+1}^\ell\right)\right\} \\
& -\lambda_t^\ell\left[n_{t+1}^\ell - \left(1-\delta^\ell\right)n_t^\ell - m^\ell\left(\overline{n}_t^\ell - n_t^\ell, v_t^\ell\right)\right] \\
& -\lambda_t^d\left[n_{t+1}^\ell - \left(1-\delta^d\right)n_t^\ell - m^d\left(u_t^d, v_t^d\right)\right]
\end{aligned}
$$

The first order conditions are

$$v_t^x : c_t^x = \lambda^x m_2^x\left(u_t^x, v_t^x\right); \text{ for } x = \{\ell, d\} \tag{A.2}$$

$$u_t^d : \frac{1}{c^h} = \lambda_t^d m_1^d\left(u_t^d, v_t^d\right) \tag{A.3}$$

$$n_{t+1}^\ell : \lambda_t^\ell + \lambda_t^d = \beta\frac{\partial P\left(n_{t+1}^\ell\right)}{\partial n_{t+1}^\ell} \tag{A.4}$$

where $m_i^x\left(u_t^x, v_t^x\right)$ is the partial derivative of the function $m^x(\cdot,\cdot)$ with respect to the i-th argument for $i = 1, 2$. The marginal contribution of the loan to the social planner value function is

$$\beta\frac{\partial P\left(n_{t+1}^\ell\right)}{\partial n_{t+1}^\ell} = \beta\left\{\begin{array}{c} z_{t+1} - \rho_{t+1}^c + \left(1-\delta^\ell\right)\lambda_{t+1}^\ell \\ + \left(1-\delta^d\right)\lambda_{t+1}^d - m_1^\ell\left(\overline{n}_{t+1}^\ell - n_{t+1}^\ell, v_{t+1}^\ell\right)\lambda_{t+1}^\ell \end{array}\right\} \tag{A.5}$$

The partial derivatives of the matching functions (1) satisfy $m_1^x\left(u_t^x, v_t^x\right) = (1-\varepsilon)\,p\left(\alpha^x\right)$ and $m_2^x\left(u_t^x, v_t^x\right) = \varepsilon q\left(\alpha^x\right)$. The value of the multipliers given by (A.2) are therefore $\lambda^x = c_t^x / \left(\varepsilon q\left(\alpha^x\right)\right)$. The introduction of these expressions for the multipliers into the first order condition (A.3) yields (DD-O). Equation (LL-O) may then be deduced from equation (A.5).

A.2 Proof of Lemma 1

The Nash solution (34) for narrow banks can be expressed as follows

$$\frac{1}{c^h p\left(\alpha^d\right)} \frac{1}{\beta} = \left(\frac{\eta}{1-\eta}\right) \Delta B_t^d = \left(\frac{\eta}{1-\eta}\right) \frac{1}{\beta} \frac{c^d}{q\left(\alpha^d\right)} \tag{A.6}$$

using the free entry condition of households of Equation (6), and the optimal bank search effort on the deposit market of Equation (17). The equilibrium deposit market tightness α^d solution of (A.6) is

$$\alpha^d = \frac{1-\eta}{\eta c^h c^d} \tag{A.7}$$

which corresponds exactly to $\widetilde{\alpha}^d$ for $\varepsilon = (1-\eta)$, i.e. the Hosios (1990) condition. The equilibrium deposit interest rate is deduced from the Nash solution (34) for the Nash contributions of the household, (29), and of the bank, (31). In symbols, this is

$$\rho^d = \rho^c + \left(1/\beta - 1 + \delta^d\right) \frac{1}{c^h \overline{m}^d} \left(\frac{1-\eta}{\eta c^h c^d}\right)^{-\varepsilon} \tag{A.8}$$

using (24) for the value of ρ^b, and (A.7) for the value of α^d– under the Hosios (1990) condition: $\varepsilon = (1-\eta)$. The payoff of entrepreneurs (30) is

$$L^m - L^u = z - \rho^\ell + \left(1 - \delta^\ell - p\left(\alpha^\ell\right)\right) \frac{\eta}{1-\eta} \lambda^\ell \tag{A.9}$$

using the Nash solution (34) and the optimality condition on the search behaviour of finance houses, namely (19). The loan interest solution of the Nash solution (34) for the Nash contributions (A.9) and (32) is

$$\rho^\ell = (1-\eta) z + \eta \rho^d + \eta \left(1/\beta - 1 + \delta^d\right) \frac{c^d}{\overline{m}^d} \left(\alpha^d\right)^{1-\varepsilon} - \eta^\ell c^\ell \alpha^\ell \tag{A.10}$$

using the value of ρ^b given by (24). This expression for ρ^ℓ can be simplified as follows considering (A.6) and (A.8)

$$\rho^\ell = \rho^d + (1-\eta)(z-\rho^c) - \eta c^\ell \alpha^\ell \tag{A.11}$$

which correspond to the efficient loan interest rate $\widetilde{\rho}^\ell$ for $\varepsilon = (1-\eta)$, i.e. the Hosios (1990) condition.

B Figures

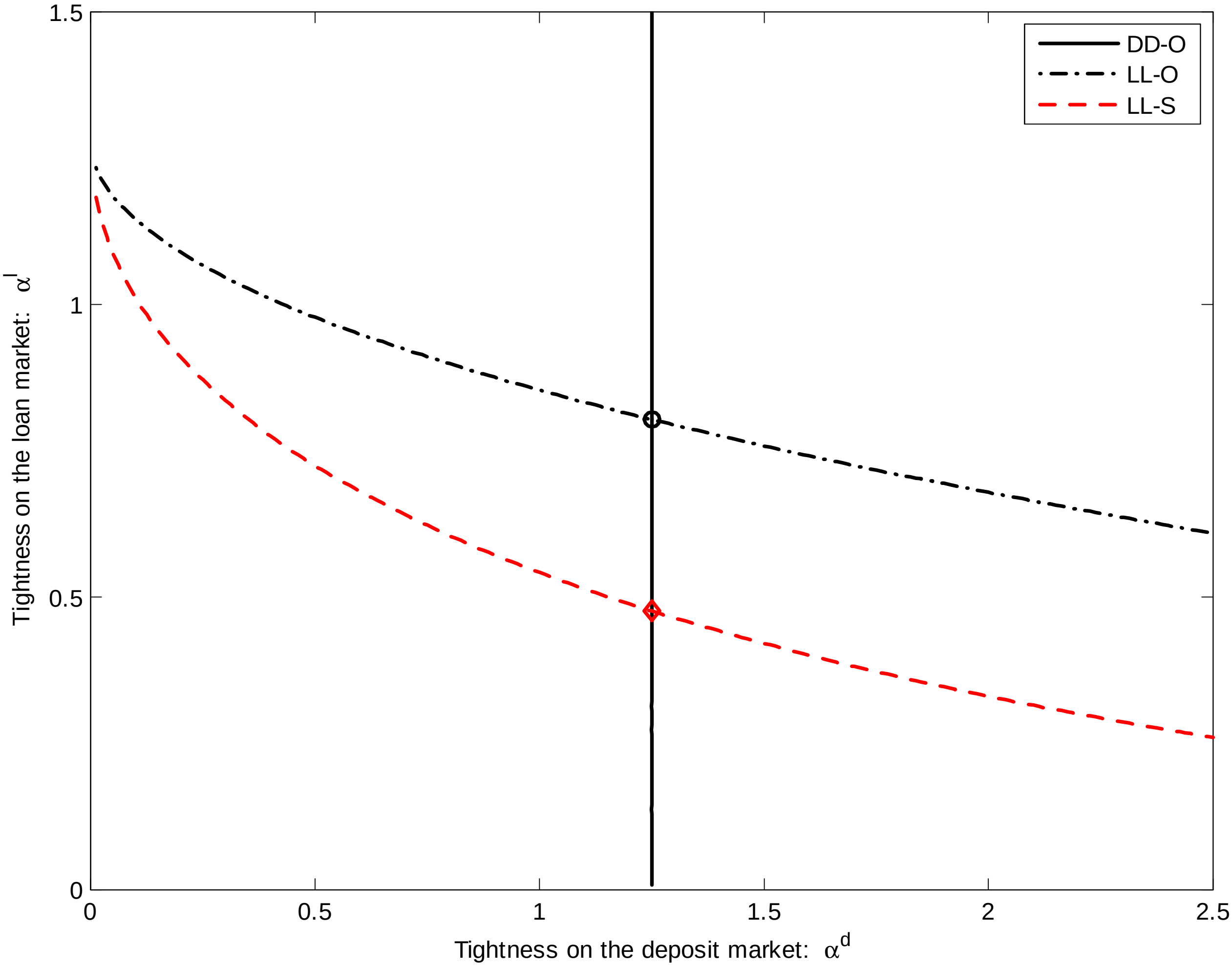

Figure 1: Optimal equilibrium (circle-mark) and competitive equilibrium (diamond-mark) for commercial banks with bargained loan interest rate (the deposit interest rate is set to its optimal value).

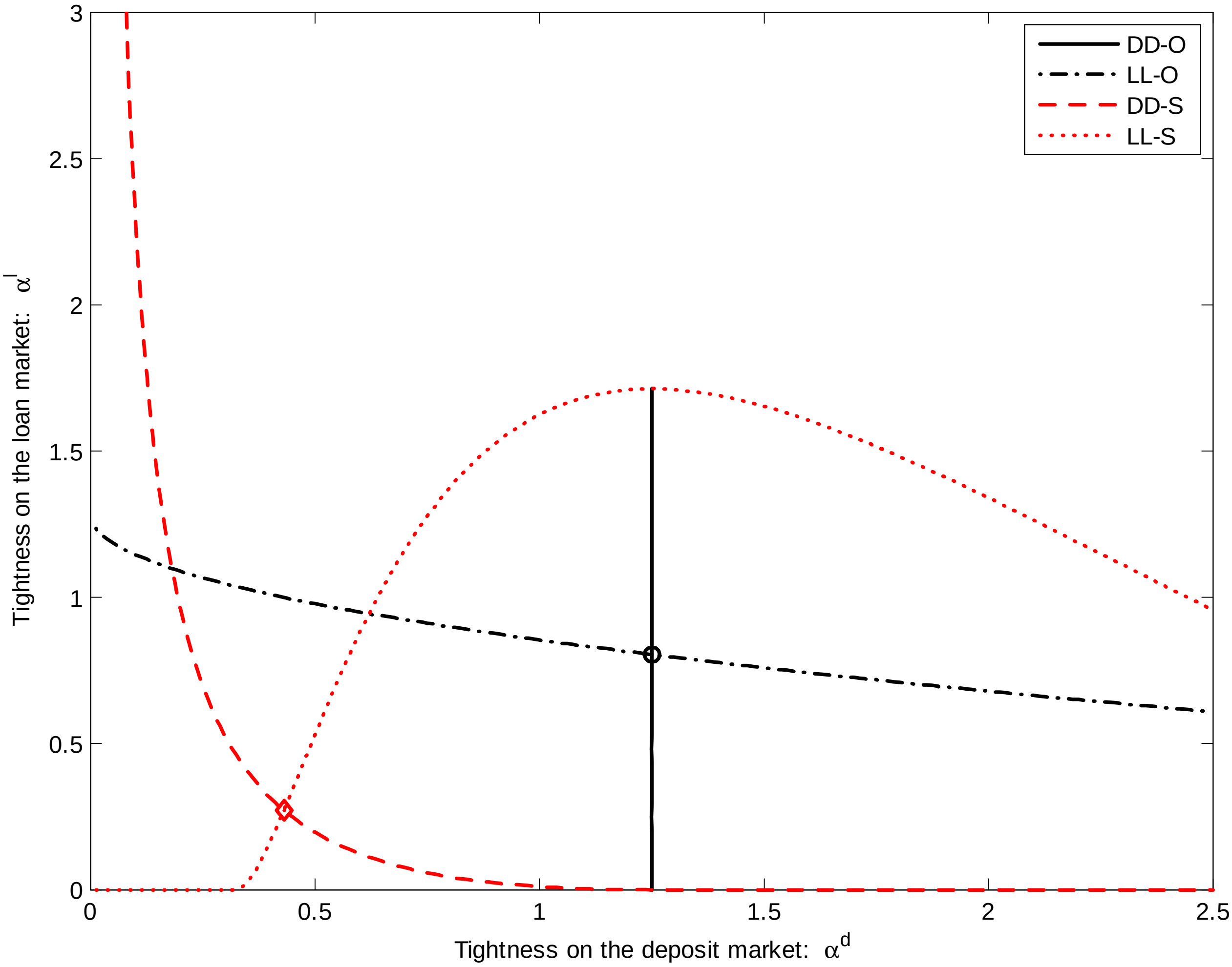

Figure 2: Optimal equilibrium and equilibrium for commercial banks with bargained deposit interest rate under the Hosios condition (the loan interest rate is exogenous).

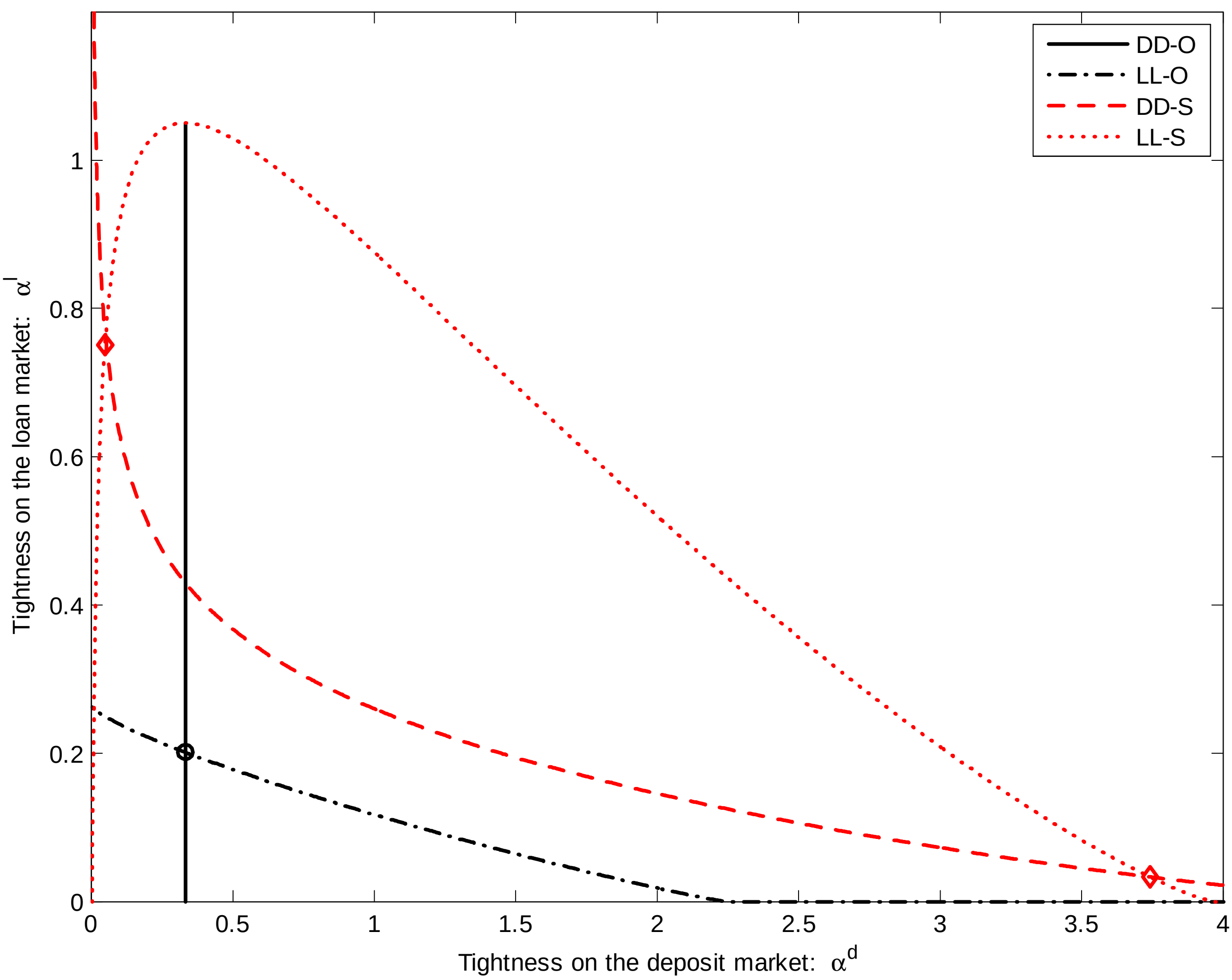

Figure 3: The optimal equilibrium (circle-mark) and multiple equilibria (diamond-marks) for commercial banks with bargained deposit interest rate for $\varepsilon < (1 - \eta)$ (the loan interest rate is exogenous).