Simultaneous Search and Network Efficiency*

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Abstract

When workers send applications to vacancies they create a bipartite network. Coordination frictions arise if workers and firms only observe their own links. We show that those frictions and the wage mechanism are in general not independent. Wage mechanisms that give rise to ex ante wage dispersion are inefficient in terms of network formation and only wage mechanisms that allow for ex post competition generate the maximum matching on a realized network. Finally, we provide a decentralized wage mechanism that implements the social planner's solution and generates the maximum expected number of matches given market tightness and search intensity.

Keywords: Efficiency, network clearing, random network formation, simultaneous search.

JEL-Classifications: D83, D85, E24, J64

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1 Introduction

When workers apply to one or more jobs, a network arises where each application establishes a link between a worker and a firm. In such a decentralized environment there are two coordination frictions, (i) workers do not know where other workers apply to and (ii) firms do not know which workers are considered by other firms. We can think of the first coordination friction as referring to random network formation, while the second coordination friction affects network clearing (the number of matches on a given network). Treating the job search process as a matching on a bipartite graph (network) gives new insights into one of the key questions in the labor-search literature namely, under which conditions is the decentralized market outcome constrained efficient? With constrained efficiency we mean that the market outcome is identical to the outcome of a hypothetical social planner who maximizes social welfare given the fundamental frictions (i) and (ii).

The main contribution of our paper is that it shows how under directed search (workers observe the posted wages before applying), the wage mechanism affects frictions through network formation and clearing.[1] We find that efficient network formation requires that identical vacancies have the same application arrival rate (this implies no ex ante wage dispersion) and that efficient network clearing requires ex post competition between firms that consider the same candidate. The efficiency condition in Kircher (2009), where workers send multiple applications and firms can contact all workers, imposes however that some vacancies should have a higher probability to receive an application than others. The difference between our efficiency condition and Kircher's occurs because he places more restrictions on the planner's network clearing mechanism.

Wage mechanisms that allow for ex post competition generate the maximum number

[1]Coles and Eeckhout (2003) and Eeckhout and Kircher (2010) show that the number of matches in a model with identical workers is independent of the posted wage mechanism. We show that this no longer holds if workers send multiple applications. When workers apply to only one job, only the first coordination friction occurs, since all firms that receive at least one application can be sure that their selected candidate has no competing offer from another firm, see Burdett, Shi and Wright (2001). In the random search models of Diamond (1982), Mortensen (1982) and Pissarides (2000) the wage determination process and the matching process are fully independent. In Moen's (2000) competitive search model, workers can sort in submarkets which are characterized by different wage and market tightness pairs. Within each submarket, given market tightness, the number of matches does not depend on wages.

of possible matches on a realized network and are therefore socially efficient. This happens because firms can respond to a particular realization of the network by increasing their posted wages. Specifically, firms that have n candidates who are collectively linked to more than n firms will bid more aggressively than firms with n candidates who are collectively linked to less than n firms. Finally, we show how in a decentralized economy, workers and firms can reach the maximum number of matches through offers and counter offers. The mechanism only requires the agents to know their own links and not the entire network. The efficiency results for network formation and network clearing are for given labor market tightness and search intensity. At the end we briefly discuss why the decentralized economy will not be socially efficient in terms of entry and search intensity. Combining our and Kircher's (2009) results, suggests that there may not exist a decentralized wage mechanism that is efficient in all dimensions.

Our paper is the first one that analyzes how standard decentralized wage mechanisms affect network formation and network clearing in a decentralized search model with complete recall where workers only know to which firms they applied and firms only know which workers applied to them. The only other paper that we are aware of that considers a search model with multilateral negotiations where workers and firms do not know the entire network is Elliot (2011). He focuses on the efficiency of entry and search intensity and allows for heterogeneity. In his wage mechanism, the bargaining power is assumed to be independent of the type of subgraph an agent is in, while we show that the type of subgraph determines the agents' payoff.[2] Manea (2011) considers a framework where agents who are connected in a network are randomly selected to bargain. During the bargaining game they are not able to contact other connected agents. His random selection setting implies that a firm with many candidates has a stronger bargaining position, because it is more likely to be selected. In our model it is not the number of candidates that matters but whether a firm is located in a subgraph with more firms than workers.

Part of the network literature has analyzed different pricing mechanisms and has studied whether these price mechanisms lead to an efficient matching of sellers and buyers.

[2] Following Corominas-Bosch (2001) each graph can be decomposed into worker subgraphs with an excess number of workers, firm subgraphs with an excess number of firms and even subgraphs with an equal number of workers and firms.

Kranton and Minehart (2001) show for example that a public ascending price auction ensures efficient network clearing. Corominas-Bosch (2004) shows for identical sellers and buyers that an alternating-offers game where all sellers (or buyers) of a subgraph simultaneously announce prices, leads to a maximum matching. This literature, however, assumes that once a network has been formed, all agents know the complete network (or the entire subgraph of the network they are in).[3] This knowledge allows sellers and buyers to determine their exact outside option (trading partners and trading prices). We show that ex post competition achieves the maximum matching, even if agents do not know the network structure. Another part of the network literature uses the set-valued approach, i.e., it either starts with a set of competitive price vectors and shows that the resulting matches are pairwise stable and maximize aggregate welfare (see Kranton and Minehart, 2000), or it starts by assuming that pairwise stable matches must arise and then analyses wage formation (see Elliott, 2011). Those papers do not layout the game that leads to a competitive price vector or a pairwise stable matching like we do. Moreover, pairwise stable matchings are not necessarily maximum matchings (i.e., Kircher, 2009) but a maximum matching is always stable since an improvement of one agent must make another agent worse off. Finally, there is a growing number of papers that combine insights from search and network theory.[4] Those papers focus mainly on how social networks of workers can pass information of the location of jobs on to each other, which is very different from the bipartite network (between workers and firms) framework that we consider here.

The paper is organized as follows. We start in section 2 with a 3-by-3 example that illustrates our main point that wage dispersion leads to less efficient networks and ex post competition generates a maximum matching on a given network while wage commitment does not. Sections 3 and 4 consider a large labor market. In section 3 we present a wage game with multi-round offers and counter offers. Section 4 introduces some insights from graph theory to derive two important general results. First, in section 4.2 we show that ex post competition with complete recall gives the maximum matching on a given network

[3] Galeotti et al. (2010) analyse network games with limited information. However, they only consider one type of agents, i.e., they do not consider vacancies and workers or sellers and buyers in a bipartite network.

[4] Example include, Boorman (1975), Calvó-Armengol and Jackson (2004), Calvó-Armengol and Zenou (2004), Fontaine (2004).

and wage mechanisms without ex-post competition do not. In section 4.4 we show that in terms of network formation, workers should apply to each vacancy with equal probability. This only occurs, if all firms post the same wage or if search is random (workers do not observe the wage ex ante). In addition, it shows that the wage mechanism of section 3 decentralizes the Planner's solution in terms of network formation and clearing. Finally section 5 concludes.

2 An example

This section illustrates our main points that (i) ex ante wage dispersion leads to less efficient network formation and that (ii) ex post competition generates a maximum matching. We look at the following two dimensions corresponding to random network formation and network clearing; (1) random search (workers apply to each vacancy with equal probability) versus directed search, and (2) ex post competition versus wage commitment.

Consider a simple economy with 3 unemployed workers and 3 firms, each with one vacancy, $(u = v = 3)$ and where workers send two applications $(a = 2)$.[5] First, we look at network formation and assume that network clearing generates the maximum number of matches. Then, we look at which wage mechanisms are most efficient in terms of network clearing. Efficient network clearing implies that the number of matches is equal to 3, if each of the three vacancies receives at least one application, and equal to 2, if only two vacancies receive applications. Note that these are the only two possible outcomes, since no worker sends both applications to the same firm. Let ξ_i be the probability that a worker sends one of her two applications to vacancy i. Under the assumption that network clearing is efficient, the expected number of matches is,

$$M = \sum_{i=1}^{3} \left(1 - (1 - \xi_i)^3\right), \text{ with } \sum_{i=1}^{3} \xi_i = 2,$$

where $(1 - \xi_i)^3$ equals the probability that vacancy i does not get any application. Since the function $\left(1 - (1 - \xi_i)^3\right)$ is concave in ξ_i, Jensen's inequality implies that the number of matches is maximized, if all vacancies have the same probability to receive an application,

[5] If workers send 1 application or 3 applications, the number of matches generated is independent of the wage mechanism used.

i.e., if $\xi_i = 2/3$. Thus, only wage mechanisms that generate no ex ante wage dispersion (which is equivalent to random search) can lead to the maximum number of matches, $M = 26/9 \approx 2.889$. In all four cases of random versus directed search and ex post competition versus wage commitment we allow for complete recall (firms can go back and forth between their candidates) and fully characterize equilibrium wages (see the technical Appendix) and the expected matching rates.[6] It turns out that in this example, there is always ex ante wage dispersion under directed search although with ex post competition the dispersion is negligible (and we show later that it disappears if the market gets larger and firms cannot affect market utility). In the directed search with ex post competition case, only an equilibrium with one high wage and two low wage firms exists. The high wage firm has an application probability of $\xi_h \approx 0.722$ and the low wage firms a probability of $\xi_l \approx 0.639$ (the equilibrium is fully characterized in the technical appendix D.3). The total number of matches is given by $M \approx 2.884$. Under directed search with wage commitment, there is more wage dispersion; the high wage firm has an application probability of $\xi_h \approx 0.956$ and the low wage firms of $\xi_l \approx 0.522$ (details are in technical appendix D.4). As we will show below, in the case of directed search with wage commitment, both network formation and network clearing are inefficient. To isolate the effect of the wage mechanism on network formation we calculated the total number of matches, imposing efficient network clearing (which in the absence of ex post competition does not occur in equilibrium). In that case, $M \approx 2.781$. Summing up, directed search with ex post competition generates more efficient networks than without ex post competition, because the latter case generates more ex ante wage dispersion.

Next, consider network clearing. Efficient network clearing requires that the number of matches is equal to 3, if all three vacancies are collectively linked to all three workers, and that the number of matches is equal to 2, if only two vacancies are collectively linked to all three workers. Network clearing is in general not efficient, if firms commit to their

[6] With the exception of Kircher (2009), who studies directed search with wage commitment, all those cases have been studied with limited recall. For directed search with ex-post Bertrand competition, see Albrecht et al. (2006), for random search with ex post competition see Gautier and Wolthoff (2009), for directed search with commitment and no ex post competition see Galeanos and Kircher (2009) and for random search with commitment, see Gautier and Moraga Gonzalez (2004) (all those papers have no complete recall except the last one, which considers complete recall in a 3 by 3 example).

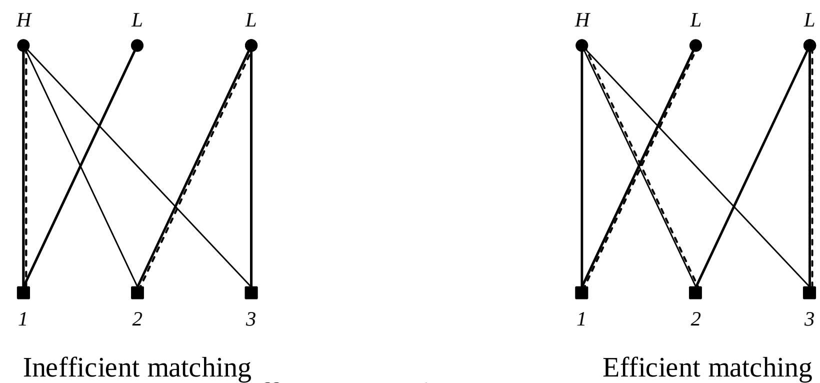

Figure 1: Inefficiency without ex post competition

posted wages. To see this, consider the graph in Figure 1, which pictures a particular realization of the case where each worker sends one application to the high-wage firm and one to one of the two low-wage firms (thick lines).

The number of matches (dashed lines) now depends on which worker is chosen by the high-wage firm. If the high-wage firm offers the job to one of the workers who are linked to the low-wage firm with two applicants, i.e., to worker 2 or 3 in Figure 1, the number of matches is equal to the maximum number of matches (3). If the high-wage firm offers the job to the worker linked to the low-wage firm with only one applicant, i.e. to worker 1 in Figure 1, there are only two matches, since the low-wage firm with only one applicant will remain unmatched. If this firm *could* ex post increase its initial offer it would bid the high wage plus epsilon and hire worker 1 while the high-wage firm would hire one of its other candidates. It is easy to show that in this example, allowing for ex post competition always leads to the maximum number of matches. The expected number of matches in a model with directed search and wage commitment is therefore lower than with ex post competition (in the technical Appendix D.4 we derive the equilibrium wages and show that $M \approx 2,538$). Finally, under random search with commitment, $M \approx 2,703$, see Gautier and Moraga-Gonzalez (2004).[7] In this case, network formation is efficient because all firms have the same application-arrival rate but network clearing is not efficient because of the lack of ex post competition.

The following table summarizes the expected number of matches that are realized in equilibrium for the different search environments and wage mechanisms.

[7] Wages are determined as in Burdett Judd (1983) with rationing.

	random search	directed search
ex post competition	$2,889$	$2,884$
wage commitment	$2,703$	$2,538$

Table 1: Expected number of matches under different search and wage mechanisms

This illustrates that the wage mechanism and the matching process are not independent. Different search environments generate different distributions of networks and whether the wage mechanism allows for ex post competition or not affects the number of matches for a given network.

3 Framework

Before presenting our main results for a large labor market, we first lay out the precize setting and the timing of events. Consider v identical firms with one vacancy each and u identical risk neutral unemployed workers, who can send $a \leq v$ applications to different firms. Workers have a reservation wage of 0 and a matched firm-worker pair produces 1. As is standard in the directed search literature we impose both symmetry and anonymity. Symmetry implies that identical workers play identical strategies while anonymity implies that firms must treat identical workers similarly and vice versa (see Burdett, Shi and Wright, 2001). In our directed search framework we allow firms to post a wage with the possibility to compete ex-post. Below, we give the exact timing of the game. Random search can be easily incorporated in this framework by noting that in that case stage 1 is irrelevant (workers just randomize over vacancies). Finally, under directed search with commitment firms are not allowed to increase their offers in stage 5.

1. Firms post a wage $\underline{w}$ and all wages are observed by the workers.
2. Workers observe all posted wages $\underline{w}$ and send out $a \geq 2$ applications.
3. Each firm selects one worker (if present) and commits to offer that worker a wage $w \geq \underline{w}$ as long as this worker does not ask for a higher offer.
4. If a worker receives one or more offers $\{w^1, w^2, ..., w^j\}$, she can either (i) accept one offer and reject all other offers, or (ii) communicate her current offers to the firms

she applied to by asking them for a better offer (a better offer should be a discrete amount higher, i.e., at least one cent). The offers are verifiable.

5. If the worker accepts an offer, the matched worker-firm pair leaves the market. If the worker asks for a better offer, the firm is no longer bound by its previous offer and can offer one of its other candidate(s) (or the same candidate) the job at a wage $w \geq \underline{w}$. If the worker does not ask for a better offer, the firm must continue to offer the job to the same worker for at least the same wage.

6. The workers and firms that remain in the market go back to stage 4. The game ends after T rounds.

Note that workers and firms do not observe the entire network but only their own links. Firms only know how many workers applied to them and whether a worker is willing to work for the offered wage. A firm fails to hire, if it has no applicants or if all its candidates choose other firms. A worker remains unemployed, if she received no offers.

In a perfect Bayesian equilibrium (PBE), firms maximize expected profits by posting a wage $\underline{w}$ given the wage offers by other firms (the wage offer distribution $F(\underline{w})$) and in each round they update their beliefs about whether their candidates will receive other offers or not according to Bayes' law. Workers maximize their expected payoffs by choosing where to send their a applications after observing the wages posted by each firm. The equilibrium of the network-clearing subgame is given by a set of wages paid by firms (the wage earnings distribution $G(w)$) and a matching M (worker-firm pairs) such that the set of wages paid in equilibrium maximizes each firm's profit given the workers' communication and acceptance decisions and such that it maximizes each workers' utility given the firms' job offer decisions. Finally, the matching M must be stable in equilibrium.

The game is solved by backward induction. In section 4.2 we derive the network clearing subgame for any realized network (stages 3 to 6) with potentially T rounds. We assume that T is sufficiently large for all necessary communications to take place.[8]. In section 4.4.2 we solve the network formation stages 1 and 2 and show that this wage mechanism is socially efficient in terms of network formation and network clearing.

[8] In section 4.2 we derive the minimum necessary value of T.

We take the number of applications that workers send out and market tightness as given. The main reason for this is that the conditions for efficient entry and the number of applications are well known and have been studied before.[9] This allows us to focus on the efficiency of random network formation and network clearing.

4 General results on random network formation and network clearing

The example of section 2 suggests that ex ante wage dispersion is inefficient in terms of random network formation and that we need ex post competition in order to get efficient network clearing. In this section we use some results from graph theory to show that those results hold in general. In section 4.2 we show that maximum matching requires ex post competition and in section 4.4 we show that all firms post a wage equal to the reservation wage, which leads to efficient network formation by ensuring that all vacancies have the same application arrival rate.

4.1 Berge's Theorem

We first briefly describe some basic concepts of graph theory that are relevant for our environment. When workers apply to jobs, each of their applications is a link (or edge) in a bipartite network (or graph). The wage mechanism and search environment determine both the distribution of networks that can arise and the matching on a given network. The networks (or graphs) in our environment are simple (workers do not send multiple applications to the same firm), undirected (if worker i is linked to firm j, then firm j is linked to worker i) and bipartite ($G = \langle u \cup v, L \rangle$ consists of a set of nodes formed by two different kind of agents, i.e., by workers and vacancies, and a set of links L where each link connects a worker to a firm, so workers are not linked to other workers and firms are not linked to other firms).

[9] Gautier and Moraga-Gonzalez (2005) and Albrecht et al. (2006) find without recall, that workers send too many applications (due to rent seeking and congestion externalities) and that entry is excessive, because firms have too much market power. Kircher (2009) shows that with directed search, wage commitment and full recall, entry and search intensity are socially efficient. Elliot (2011) finds that firm entry is never excessive but can be too small and that workers send too many applications.

Definition 1 *A matching M in a graph G is a set of links such that every node of G is in at most one link of M.*

Central to our result that a maximum matching requires ex-post competition is the following theorem by Berge.

Berge's Theorem (1957):

A matching M in a graph G is a maximum matching if and only if G contains no M-augmenting path.

In our bipartite graph environment an M-augmenting path is defined as a path where

1. worker-firm links that are part of the matching M alternate with worker-firm links that are not part of the matching M (definition of an M-alternating path) and

2. neither the origin (firm or worker) nor the terminus (worker or firm) of the path is part of the matching M.

Figure 2 depicts an M-alternating path and an M-augmenting path in a particular network. The dots represent vacancies and the squares unemployed workers. The solid lines represent applications ($a = 2$) and the dashed lines represent matched worker-firm pairs. The M-alternating path in the first panel ($A - 1 - B - 2 - C - 4$) starts with the matched vacancy A and ends at the matched worker 4. The M-augmenting path ($A - 1 - B - 2 - C - 4$) in the second panel of Figure 2 starts with an unmatched vacancy, A, and ends with an unmatched worker, 4.

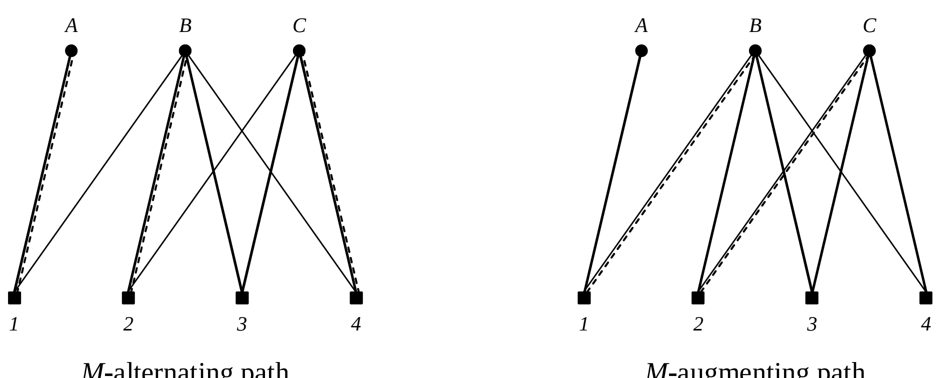

Figure 2: M-alternating path and M-augmenting path

Berge's Theorem, translated to our setting, implies that a maximum matching in a graph is only guaranteed, if an unmatched firm is not linked to an unmatched worker via an M-augmenting path. The reason that a matching is not optimal, if an M-augmenting path exists, is that one could create one more match by switching the links. Then, the unmatched firm at the start of the M-augmenting path and the unmatched worker at the end of the M-augmenting path will both be matched at the expense of one match in the middle. Comparing the two paths in the second panel of Figure 2 illustrates this. The matching $M = \{1 - B, 2 - C\}$ in an M-augmenting path can always be increased by *switching* the dashed and solid links resulting in an extra link, i.e., $M = \{A - 1, B - 2, C - 4\}$.

What remains to be shown is that if a matching M has no M-augmenting paths, it is a maximum matching. This can be proven by contradiction. Suppose that in a particular graph in our setting there is a matching M for which there are no M-augmenting paths but that (contrary to Berge's Theorem) this matching is not a maximum matching. Then there is a matching N (i.e. $A - 1, B - 2, C - 4$; dashed lines in Figure 3) with more links than M (i.e. $1 - B, 2 - C$; dotted lines in Figure 3), $|N| > |M|$. Now consider the symmetric difference $N \Delta M$ defined as the set of links that is either in N or M but not in both (the sum of dashed and dotted lines in Figure 3, $A - 1$, $B - 2$, $C - 4$, $1 - B$, $2 - C$). Each worker or firm can have at most 2 links in $N \Delta M$ because he is hired by at most one firm in M and at most one firm in N. Moreover, the links of the paths alternate between being in M and being in N, because by the definition of a matching, no node can have two links in M or two links in N. Since by assumption N is strictly bigger than M there must be at least one path in $N \Delta M$ with an odd number of links that starts with a firm

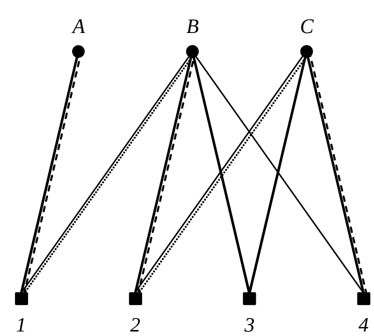

Figure 3: Symmetric difference operation ($N \Delta M$)

(worker) in N and ends with a worker (firm) in N (i.e., $A - 1 - B - 2 - C - 4$). But then this is an M-augmenting path because the firm and worker at the start and end of the path are (by the symmetric difference operation) not in M. This gives us the desired contradiction, because we started by assuming that M has no M-augmenting paths.

Thus, in order to show that ex-post competition leads to a maximum matching we need to rule out that an M-augmenting path exists.

4.2 Maximum matching requires ex post competition

In this section we show that for a given network, ex post competition with complete recall generates a maximum matching. In order to do so, we start with presenting the optimal strategies of the network-clearing subgame (stages 3 to 6) of section 3.

First note that a worker remains unemployed if he has not accepted any job offer until round T. Thus, in stage 4 of round T, all workers will have accepted their highest offer above their reservation wage. Consider now the optimal strategies in round $T-1$. A firm's optimal strategy in stage 5 of round $T-1$ depends on the beliefs on whether its candidates will get a competing offer in round $T-1$. If firm i believes that its candidate with the lowest wage request $w^l_{T-1} = \arg\min \left\{w^1_{T-1}, w^2_{T-1}, ..., w^j_{T-1}\right\}$ will not get a higher offer from a competing firm in round $T-1$, it will offer him the job at the wage $w^i_{T-1} = w^l_{T-1} + \varepsilon$. If a firm beliefs that its candidates will receive better offers with some positive probability in the final round, the optimal wage offer involves mixing, see Burdett and Judd (1983) and Gautier and Moraga-Gonzalez (2004). If a firm expects that all its candidates will receive a competing offer in $T-1$, the firm will offer $w = 1$, since any lower offer would imply that another firm hires the worker. Note, that we will choose T sufficiently large such that all firms that enter ex post competition will have bid their wages up to 1 in round $T-2$. Consequently, all remaining firms can infer that their candidate with the lowest wage request will not get a higher offer from a competing firm. We derive the minimum necessary value of T below.

Consider the optimal strategies in any previous round $T-t$. In stage 4 of round $T-t$, a worker who receives the highest possible wage offer ($w = 1$) will accept it. For a worker who has not yet received a wage of one, it is optimal to communicate his highest wage offer that he received so far ($w^h_{T-t-1} = \arg\max \left\{ w^1_{T-t-1}, w^2_{T-t-1}, ..., w^j_{T-t-1} \right\}$) to the other firms he is linked to and ask them for a better offer $w^i_{T-t} > w^h_{T-t-1}$. It is also optimal for a worker to not ask the firm that offered the highest wage so far to increase its wage offer, because then the firm could reject this request and the worker would run the risk of remaining unmatched. The optimal strategy for a firm that is asked to increase its offer in stage 5 of round $T-t$ does not depend on its beliefs about whether or not its candidates will get a competing offer in the *same* round, because it can always react to a competing offer in a later round. A firm that receives a request to increase its last offer will make an offer to the candidate with the lowest request equal to $w^i_{T-t} = w^l_{T-t} + \varepsilon$.

In stage 3, each firm that received at least one application selects a worker randomly and offers him the wage $w^i = \underline{w}^i$.

Next, we prove in four simple steps that the network-clearing subgame generates a maximum matching on any given network.

Lemma 1 *If a firm remains unmatched after $2u \times (1/\varepsilon)$ rounds, then all workers along an M-alternating path that starts with the unmatched firm must earn a wage equal to the marginal product, i.e., $w = 1$.*

Proof: We start with showing that $T - 2 = 2u\,(1/\varepsilon)$ and then prove that all workers along this M-alternating path receive $w = 1$. Since there are more firms than workers, there is at least one firm in each round whose candidate will request a higher offer. The requests and the subsequent offers lead to a communication along the path and transports the higher wage requests. The longest possible path contains all u workers. Thus, $2u$ is the maximum number of rounds it takes to process the information from the start of the path to the end and back. After $2u$ rounds each firm along the M-alternating path will have found it optimal to increase its initial offer (if it did not post a sufficiently high wage initially). The wage increases are at least ε for each of the $2u$ rounds. Thus, it takes at most $2u\,(1/\varepsilon)$ rounds for all wages along the M-alternating path that starts with an unmatched firm to be equal to 1.

To see why all workers along the M-alternating path receive $w = 1$, first note that if a firm with candidates (firm A) remains unmatched after $2u\,(1/\varepsilon)$ rounds, then all its applicants must have accepted a wage $w = 1$ (since if at least one of its candidates would earn $w < 1$, firm A would have offered that worker $w < 1$ and make positive profits). But then the other candidate of the next firm along the M-alternating path (firm B) that hired A's candidate must also receive $w = 1$ otherwise firm B would have hired that worker at a $w < 1$. Repeating this argument implies that all firms along the M-alternating path pay a wage of 1. ■

Lemma 1 implies that all workers in M-alternating paths that start with an unmatched firm have been offered a wage equal to 1 and have left the market after $T - 1$ rounds.

Lemma 2 *If a worker remains unmatched after* $2u(1/\varepsilon) + 1$ *rounds, each firm along an* M*-alternating path that starts with the unmatched worker pays no more than the highest posted wage.*

Proof: The firm (firm A) to which the unmatched worker (worker 1) applied will offer the worker who it hired (worker 2) at most its posted wage otherwise it could have offered the job to the unmatched worker 1. But then the worker (worker 3) who is hired by the next firm along the M-alternating path (firm B) must also earn weakly less than the highest posted wage, else his firm (B) would have hired worker 2. Repeating this argument implies that all firms along the M-alternating path that starts with an unmatched worker pay a wage less than the highest posted wage. ■

Lemma 2 implies that all workers in M-alternating paths that start with an unmatched worker have been offered a wage no higher than the highest posted wage after $T-1$ rounds. Lemma 4 below shows that this holds for all M-alternating paths that do not start with an unmatched firm. It follows that all workers who requested a wage above the highest posted wage have left the market after $T - 1$ rounds (Lemma 1). Thus, $T = 2u(1/\varepsilon) + 2$ is sufficiently large to ensure that all remaining firms can infer that their candidates with the lowest wage request will not get a higher offer from a competing firm in round $T - 1$.

Lemma 3 *The highest posted wage is strictly smaller than* 1*.*

Proof: Under directed search, any firm that offers a wage equal to 1 makes no profit and could increase its profits by offering a wage strictly less than one since there is a positive probability that one of its candidates receives no better offers and accepts this lower offer in round T. ■

According to Berge's Theorem a maximum matching exists if and only if there is no M-alternating path that starts with an unmatched worker and ends with an unmatched firm, i.e., if and only if there is no M-augmented path. Given the wage pattern in an M-alternating path that starts with an unmatched worker (Lemma 2) or with an unmatched firm (Lemma 3), we can write down our main Proposition.

Proposition 1 *Ex-post competition leads to a maximum matching in any realized network.*

Proof: Suppose it would not lead to a maximum matching. In that case there would exist an M-augmenting path with at least one unmatched worker and one unmatched firm. But then Lemma 1, 2 and 3 imply that all firms along the M-augmenting path (that is also an M-alternating path) offer both a wage less than 1 and a wage equal to 1, which is a contradiction. ■

Note that this result is very general. If firms can only interview a subset of their workers as in Wolthoff (2011) or one as in Albrecht et al. (2006) and Galenianos and Kircher (2009), the realized network will be different but Proposition 1 still holds. The same is true, if workers have for example different search costs and consequently send out different numbers of applications. Also, if firms can create shortlists of at most n candidates, our result holds. This just requires an intermediate step where all firms with more than n candidates must eliminate (at random) a number of links. After this intermediate step, a new network arises for which the same results on maximum matching hold as above.

The flexibility to adjust wages ex post is central to achieve efficiency in network clearing. If firms commit to their posted wages and do not adjust their wages ex post, we can typically observe different wages along an M-alternating path. If both end nodes of the M-alternating path are unmatched, i.e., if we have an M-augmenting path, there is no mechanism inherent in the matching process associated with wage commitment that can induce the matched firm-worker pairs to rematch with the unmatched firm and worker at the end of the M-augmenting path. Thus, the inefficient network clearing result of wage commitment from the 3 by 3 example of section 2 holds in general. Therefore, Berge's Theorem also implies the following Corollary,

Corollary 1 *If firms commit not to increase their posted wages ex-post, network clearing is generally inefficient and the maximum matching is not realized.*

Corollary 1 shows that directed search models with fixed posted wages are not able to solve the second coordination friction (firms do not know which workers are considered by other firms). Thus, although directed search with fixed posted wages is constraint efficient in terms of firm entry and number of applications that workers send, see Kircher (2009), it generally does not generate the maximum matching that is possible given the network that is formed between firms and their applicants.

Proposition 1 also implies that a social planner would never want to give one subgroup of firms the right to match first. Such a property arises, if some firms offer higher wages than others and wages cannot be raised ex-post as in Kircher (2009).

Corollary 2 *It is socially inefficient to have a subgroup of firms that matches first.*

Corollary 2 implies that it is socially inefficient to have a subgroup of high wage firms that match first and a subgroup of low wage firms that match only if their candidate(s) receive no offers at a high wage firm.[10]

4.3 Wages

Lemmas 1 to 3 are also informative about the payoffs that workers and firms receive. According to Lemma 1, all workers that are part of an M-alternating path that starts with an unmatched firm will be matched and earn a wage equal to the marginal product, i.e., $w = 1$. Thus, these M-alternating paths are characterized by an excess number of firms. Similarly, there are M-alternating paths that are characterized by an excess number of workers. According to Lemma 2 all firms that are part of such an M-alternating path are matched and pay a wage no higher than the highest posted wage. Lemmas 1 to 3 also allow for M-alternating paths with equal number of workers and firms where all workers and firms are matched. In order to determine the wages paid in such even subgraphs we use the properties of the Decomposition Theorem by Corominas-Bosch (2004) (details

[10] Note, that Kircher's (2009) equilibrium is constrained efficient because the planner takes the existence of a subset of firms that match first as given, whereas here this is not part of the planner's constraint.

in Appendix A.1), which – in terms of our terminology – decomposes a network into firm-, worker- and even subgraphs. A firm subgraph contains more firms than workers and workers are paid their marginal product. A worker subgraph contains more workers than firms and workers are paid a wage no higher than the highest posted wage. In even subgraphs, the number of workers equals the number of firms.

Such a decomposition into firm, worker and even subgraphs plus some extra links can be obtained by an algorithm introduced by Corominas-Bosch (2004), see step 2 in Appendix A.1 for the exact algorithm. The algorithm first looks for firm subgraphs and separates all of them from the network. Then it identifies worker subgraphs and removes all of them from the network. The remaining subgraphs are even subgraphs. The decomposition is not unique but the Decomposition Theorem states that any firm and any worker will always belong to the same type of subgraph, a property important to guarantee that the different possible decompositions are payoff equivalent.

Figure 4 illustrates the Decomposition Theorem. The algorithm starts with the first firm and identifies a set of firms as firm subgraph if it has less neighbors (more precisely, if it is jointly linked to less neighbors, i.e., $|F| > |N(F)|$). In order to ensure that the maximum matching is found, the algorithm has to start with $|F| = 1$. The number $|F|$ increases by one once all firm combinations with $|F|$ have been considered (Hall's Theorem, 1935). The first subgraph in Figure 4 is the unmatched firm G. The firm subgraph G_1^f is removed before the algorithm continues. Since there are no firm subgraphs with $|F| = 2$, the next firm subgraph has three firms, i.e., $|F| = 3$, The three firms A, B and C in this subgraph are collectively linked to workers 1 and 2, i.e., $N(\{A, B, C\}) = \{1, 2\}$ and $|N(\{A, B, C\})| = 2$. Once the firm-subgraph G_2^f is removed, it is easy to identify that the remaining sets of firms are collectively linked to more neighbors, i.e., $|F| \leq |N(F)|$. Hence, there are no further firm subgraphs. The algorithm continues by looking for worker subgraphs in the same way as it looked for firm subgraphs. At $|W| = 4$, the algorithm identifies a worker subgraph with $N(\{3, 4, 5, 6\}) = \{D, E, F\}$ and $|N(\{3, 4, 5, 6\})| = 3$. Once the worker subgraph G_1^w is removed, and no further worker subgraphs are found the algorithm stops by identifying all remaining subgraphs as even subgraphs, i.e., in Figure 4 the remaining subgraph G_1^e is an even subgraph with $N(\{7, 8\}) = \{H, I\}$ and $|N(\{7, 8\})| = 2 = |\{H, I\}|$.

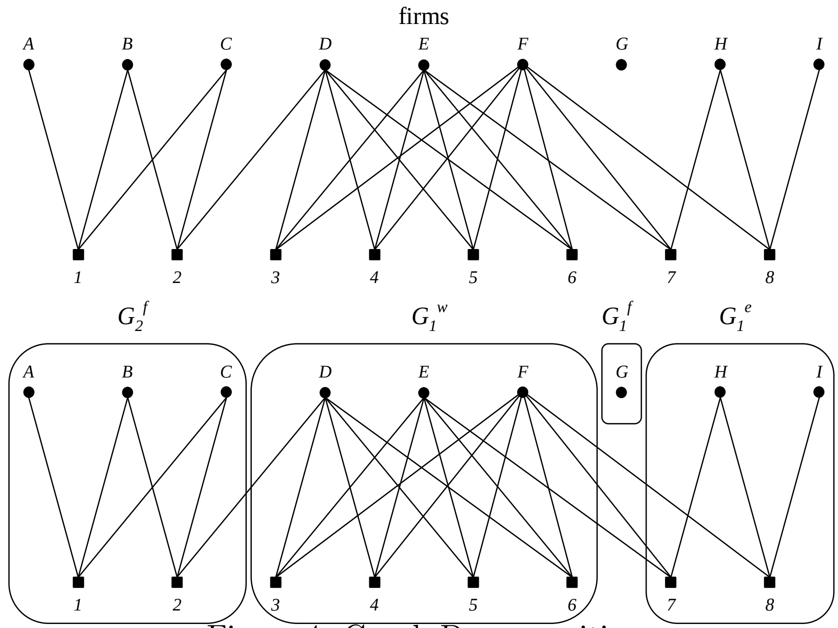

Figure 4: Graph-Decomposition

The decomposition Theorem of Corominas-Bosch (2004) is also useful for the analysis of network formation that we discuss in the next section, because it allows us to determine which kind of links formed by an additional application will result in an extra match. Since all firms in even subgraphs and worker subgraphs are matched, only applications from workers in worker subgraphs (which includes unmatched workers) to firms in firm subgraphs (which include firms without any application) will result in additional matches.

An alternative way to interpret the decomposition is in terms of splitting firms and workers into "strong", "weak" and "even" firms and workers depending on their capability to extract the maximum surplus from their matched partners (see Corominas-Bosch, 2004, p. 51). Workers in firm subgraphs are "strong" nodes, since they earn a wage equal to their marginal product. Similarly, firms in worker subgraphs are "strong" nodes, since they are able to extract the maximum surplus conditional on the posted wage. Contrary, workers in worker subgraphs and firms in firm subgraphs are "weak" nodes and workers and firms in even subgraphs are even nodes. The first part of the Decomposition Theorem states that "weak" nodes can only be linked to "strong" nodes and that "even" nodes cannot be linked to "weak" nodes. The latter implies in terms of our model that firms in even subgraphs cannot be linked to workers in worker subgraphs or that workers in even subgraphs cannot be linked to firms in firm subgraphs. Thus, the outside option of workers in even subgraphs is at most the highest posted wage, since they can only be linked to firms in even subgraphs or worker subgraphs. This last property is important to determine the wages in even subgraphs.

Lemma 4 *(i) Firms in firm subgraphs pay a wage equal to the marginal product.*
(ii) Firms in worker subgraphs pay a wage no higher than the highest posted wage.
(iii) Firms in even subgraphs pay a wage no higher than the highest posted wage.

Proof: (i) and (ii) follow immediately from Lemmas 1 to 3. To prove (iii) consider the following properties of an even subgraph. In an even subgraph that results from the decomposition algorithm introduced by Corominas-Bosch (2004), workers are either linked to firms in even or in worker subgraphs. Part (ii) of the Lemma implies that the wage offers made by firms in worker subgraphs to workers in even subgraphs are no higher than the highest posted wage. To establish part (iii) it remains to be shown that firms in even subgraphs never have an incentive to offer a wage above the highest posted wage given the game outlined in section 3. A firm only increases its wage offer above the highest posted wage, if all workers that are linked to it ask for a wage above the highest posted wage. At stage 4 of the game, a worker will only ask for a wage above the highest posted wage, if she has received an offer above the highest posted wage. But the first firm that offers a wage above the highest posted wage either does not maximize profits or is not part of the even subgraph. ■

Note that directed search with ex post competition generates ex post wage dispersion similar to Albrecht, Gautier and Vroman (2006). The knowledge about wages paid in the different subgraphs allows us also to gain some insight into the payoffs that firms get in different subgraphs. This will be useful for analyzing efficiency in network formation.

4.4 Efficient network formation given efficient network clearing

In our setting, network formation is random. The symmetry and anonymity assumptions do not allow workers to identify certain firms and to condition their application decision on firms' names. Workers do observe the posted wage, and can condition their application decision on that.

The game of section 3 implies an urn-ball model of network formation, (see Albrecht, Gautier and Vroman, 2004) where workers randomly send out a applications to different firms.[11] Each application can be thought of as creating a link in a bipartite graph. This process differs from the seminal Erdös and Rényi (1960) random network formation model where each link is formed with a certain probability and the number of applications that a worker sends is a random variable.[12] In our framework the number of applications that each worker sends is given and the randomness comes from the fact that workers do not know where other workers apply. The number of applications that a firm receives is therefore a random variable. Under directed search, the expected number of applications a firm receives will of course depend on the wage (or more generally on the wage mechanism) it posts. If we make the labor market large by letting N be an arbitrary large finite number and $v \to N$ with $v/u = \theta$, the number of applications are approximately distributed according to a Poisson distribution with mean a/θ.

4.4.1 Social planner's problem

An unconstrained social planner will trivially assign each unemployed worker to a vacancy such that the number of matches equals the short side of the market. If workers send out multiple applications, the same first best assignment can be achieved, if the social planner partitions the labor market into submarkets where the number of firms and workers in each submarket is no higher than the number of applications. However, if the social planner faces the same coordination frictions as the market, he must assign symmetric strategies to identical workers, implying that he can only decide on the probability with which a worker has to send an application to a subgroup of firms.

We constrain the social planner to choose the set of firm subgroups C (where each subgroup c is defined by a certain color), the measure of vacancies v_c within each subgroup c and the probability $p_{c,i}$ that a worker sends its i-th application to subgroup $c \in C$. The expected number of applications sent to subgroup c is equal to

$$a_c = u \sum_{i=1}^{a} p_{c,i}.$$

[11] See also Kircher (2009) and Galeanos and Kircher (2009) and Fontaine (2004).

[12] See Bollobas (2001) for a bipartite version.

The total number of workers that apply to subgroup c equals $u_c = \left(1 - \prod_{i=1}^{a}(1 - p_{c,i})\right) u$, where $\prod_{i=1}^{a}(1 - p_{c,i})$ is equal to the probability that a given unemployed worker does not send any application to subgroup c. While vacancies can by definition only be part of one subgroup, workers can be linked to at most a different subgroups depending on where they send their applications to. Workers are, however, only part of one subgraph (worker, firm or even). Subgraphs can, therefore, contain vacancies of different subgroups, if the workers that belong to that subgraph are linked to vacancies in different subgroups.

The maximum matching that is achieved by ex-post competition implies that the number of matches within each subgroup c equals the number of workers in firm subgraphs u_c^f, the number of firms in worker subgraphs v_c^w and the number of firms (or workers) in even subgraphs v_c^e or (u_c^e), i.e., $M_c = u_c^f + v_c^w + v_c^e$. Using the fact that the sum of vacancies equals the sum of vacancies in firm, worker and even subgraphs, i.e., $v_c = v_c^f + v_c^w + v_c^e$, we can rewrite the expected number of matches in a subgroup c as the number of vacancies in subgroup c minus the number of vacancies in subgroup c in firm subgraphs that are not matched, i.e.,

$$M_c = v_c - \left(v_c^f - u_c^f\right). \tag{1}$$

Coromina-Bosch's Decomposition Theorem allows us also to derive the first derivatives of the matching function with respect to an additional application.[13] Since all firms in worker and even subgraph are matched, only applications to firms in firm subgraphs can result in additional matches. Furthermore, an application will only lead to an *additional* match, if the worker who sends the application is part of a worker subgraph. The probability that a vacancy is part of a firm subgraph in subgroup c is v_c^f / v_c and the probability that a worker is part of a worker subgraphs is u^w / u, where u^w is the number of unemployed workers in all worker subgraphs. An additional application of a randomly selected worker therefore leads with the following probability to an additional match,

$$\frac{\triangle M_c}{\triangle a_c} = \frac{v_c^f}{v_c} \frac{u^w}{u}. \tag{2}$$

Any additional match that is formed by a link of a vacancy in a firm subgraph and a worker in a worker subgraph decreases the excess number of firms in firm subgraphs, i.e.,

[13] Note, that a marginal increase in the expected number of applications results form a marginal increase in the application probability $p_{c,i}$.

decreases $v_c^f - u_c^f$. If the excess number of firms in a particular firm subgraph is equal to one, then this additional match turns vacancies located in firm subgraphs into vacancies in even subgraphs. Thus, an additional link decreases the expected number of firms in firm subgraphs. Furthermore, any additional match that is formed by a link between a vacancy in a firm subgraph and a worker in a worker subgraph reduces the number of workers in worker subgraphs u^w. This implies that the number of matches in any subgroup c is a concave function of the number of applications, i.e.,

$$\frac{\triangle^2 M_c}{\triangle a_c^2} = \frac{1}{v_c}\frac{u^w}{u}\frac{\triangle v_c^f}{\triangle a_c} + \frac{v_c^f}{v_c}\frac{1}{u}\frac{\triangle u^w}{\triangle a_c} < 0, \text{ since } \frac{\triangle v_c^f}{\triangle a_c} < 0 \text{ and } \frac{\triangle u^w}{\triangle a_c} < 0. \tag{3}$$

Although we do not know the exact form of the matching function, these properties of the matching function are sufficient to characterize the necessary and sufficient conditions for efficient network formation.

We allow the social planner to be able to generate the maximum matching on each realized network (this is reasonable given that in the previous section we described a wage mechanism that generates this). The planner also chooses the set of firm subgroups C, the measure of firms v_c within each subgroup c and the total number of applications a_c that unemployed workers send to vacancies in each subgroup c, i.e.,

$$\max_{C, v_c, a_c} \sum\nolimits_{c \in C} M_c.$$

Note, that choosing the total number of applications, a_c, is (by the law of large numbers) equivalent to choosing the probability, $p_{c,i}$, that a worker sends its i-th application to subgroup c, since symmetry requires that all workers use the same application strategy.

Proposition 2 *(i) Network formation is efficient, if and only if*

$$\frac{v_c^f}{v_c} = \frac{v^f}{v} \quad \textit{for all } \ c \in C. \tag{4}$$

which is equivalent to having the same application intensity in each subgroup, i.e.,

$$\frac{v_c}{a_c} = \frac{v}{au} \quad \textit{for all } \ c \in C. \tag{5}$$

(ii) Efficient network formation is independent of the set C of subgroups and the number of vacancies v_c in each subgroup.

Proof: We prove part (i) by showing that the number of matches is only maximized, if $v_c^f/v_c = v^f/v$ for all $c \in C$. Suppose that the probability of a firm being in a firm subgraph is higher in the red subgroup $r \in C$ than in the blue subgroup $b \in C$, i.e. $v_r^f/v_r > v_b^f/v_b$.[14] According to equation (2), this implies the following relationship for the marginal contribution of an additional application, i.e.,

$$\frac{v_r^f}{v_r} > \frac{v_b^f}{v_b} \Longleftrightarrow \frac{\triangle M_r}{\triangle a_r} > \frac{\triangle M_b}{\triangle a_b}.$$

Given that the matching function is concave in the number of applications, see equation (3), the total number of matches in subgroups r and b can be increased by redirecting applications from subgroup b to subgroup r, until

$$\frac{\triangle M_r}{\triangle a_r} = \frac{\triangle M_b}{\triangle a_b} \Longleftrightarrow \frac{v_r^f}{v_r} = \frac{v_b^f}{v_b}.$$

Since the same argument applies for all $c \in C$, condition (4) must hold in order to maximize the total number of matches for a given set of subgroups C.

Condition (4) holds, because the number of applications a_c directed to each subgroup is adjusted accordingly. This implies that the number of applications to each subgroup is proportional to the number of vacancies in each subgroup, i.e.,

$$\frac{a_c}{v_c} = \frac{au}{v} \quad \text{for all} \quad c \in C.$$

To prove part (ii) we show that conditional on $v_c^f/v_c = v^f/v$ and $a_c/v_c = au/v$ for all $c \in C$, the total number of matches is independent of the number of subgroups C and the number of vacancies v_c within each subgroup. If market tightness is the same in all subgroups, i.e., condition (5) holds by symmetry, the number of unemployed workers matched with vacancies in each subgroup must also be proportional to the number of vacancies in each subgroup. This is also true for each subtype of matched workers, i.e., for workers in worker, firm and even subgraphs. Thus, the number of matched workers u_c^f that are part of firm subgraphs in subgroup c are proportional to the number of vacancies in subgroup c, i.e.,

$$\frac{u_c^f}{v_c} = \frac{u^f}{v} \quad \text{for all} \quad c \in C.$$

[14] Note, if no worker applied to subgroup c, then all firms in subgroup c are in firm-subgraphs, i.e. $v_c^f/v_c = 1$.

Using this last equality and condition (4) implies that the total number of matches is independent of the set C of subgroups and the number of vacancies v_c in each subgroup, i.e.,

$$\begin{aligned}
\sum\nolimits_{c\in C} M_c &= \sum\nolimits_{c\in C} \left[v_c - \left(v_c^f - u_c^f\right)\right] \\
&= \sum\nolimits_{c\in C} v_c \left[1 - \left(\frac{v_c^f}{v_c} - \frac{u_c^f}{v_c}\right)\right] \\
&= \left[1 - \left(\frac{v^f}{v} - \frac{u^f}{v}\right)\right] \sum\nolimits_{c\in C} v_c \\
&= v - \left(v^f - u^f\right).
\end{aligned}$$

where the third step applies equality (4). ■

The efficiency condition for network formation in Proposition 2 implies that all vacancies should have the same probability to be contacted by a worker. This makes the network as balanced as possible and therefore minimizes the fraction of firms that are not matched. Shimer (2005) derives a similar condition for a directed search environment where workers can apply to only one firm. In the setting by Galenianos and Kircher (2009), where workers can send more than one application but firms can contact only one worker, the total number of matches is also maximized, if all firms have the same probability to be contacted by a worker. In contrast, the efficiency condition in Kircher (2009), where workers send multiple applications and firms can contact all workers, differs from our efficiency condition, because he constrains the social planner to let a subgroup of firms always match first (i.e., be in a high location). Those firms in a high location should be more likely to be contacted by a worker, since this reduces the probability that a worker is not available for hiring at a firm in a low location (where firms can only match, if their candidates do not have an offer from a firm in a high location). Allowing the social planner to also choose the network clearing mechanism, Corollary 2 shows that it is not optimal to let a subgroup of firms match first. Thus, Kircher's (2009) efficiency result differs from our efficiency result, because he restricts the social planner to use a network clearing mechanism that does not allow for ex post competition.

The second part of Proposition 2 also implies that the total number of matches does not change, if there are no firm subgroups. The simulated examples in Appendix A.2

show that this property only holds for a large number of workers and firms. If the labor market is small, the expected number of matches decreases if firms are partitioned into different subgroups.

4.4.2 Decentralized wage setting

In this section we show that the decentralized game of section 3 not only leads to efficient network clearing but also to efficient network formation.

Proposition 3 *The unique directed-search equilibrium of the game in section 3, is for firms to post* $\underline{w} = 0$ *and for workers to randomize between firms. The equilibrium is socially efficient in terms of network formation and clearing.*

Proof. See Appendix B.1.

The proof in Appendix B.1 shows that given any candidate equilibrium where firms post positive wages, it is always individually profitable to deviate and offer a lower wage. We focus on the case where in the candidate equilibrium all firms post $w > 0$. In a technical Appendix C we consider a candidate equilibrium where firms mix and offer ex ante different wages and show that it is also individually profitable to deviate and post $\underline{w} = 0$. In that case there also always exists a profitable downward deviation. Below, we give some intuition for Proposition 3.

Consider a candidate equilibrium where all firms post a wage $\underline{w}$. The market-utility condition then implies that workers apply with equal probability to each vacancy. This wage mechanism generates a particular (random) network. Now consider a deviating firm that offers a lower wage $w^{d\downarrow} < \underline{w}$. The market-utility condition requires that workers must be indifferent between applying to the deviating firm or to any of the non-deviating firms. So for small deviations, workers will apply to the deviating firm with a positive probability. To show that this deviation is profitable, we first establish that, conditional on being matched, the probability to earn $w^{d\downarrow}$ for a worker who applied to the deviating firm is smaller than the probability that the deviating firm pays $w^{d\downarrow}$. If the deviant firm ends up in a firm subgraph then any worker who applies there will earn a wage of 1. If the deviant firm ends up in a worker or even subgraph, two cases can occur. First, consider the case that the deviant has to compete for its candidate and must offer $\underline{w}$ to hire him.

In this case all workers who applied to the deviant must earn $\underline{w}$. Next, consider the case that the deviant firm does not have to compete so it pays $w^{d\downarrow}$. Conditional on the deviant paying $w^{d\downarrow}$, a worker who applied to the deviant may still receive $\underline{w}$, if he is hired by an other firm. Thus, conditionally on being hired, the probability that a worker who applies to the deviating firm is paid $w^{d\downarrow}$ is smaller than the probability that the deviating firm pays $w^{d\downarrow}$. Finally, note that the lower $w^{d\downarrow}$ the larger is the hiring probability for a worker who applies to the deviant (this follows from the market-utility condition). All of this implies that lowering the wage will benefit the deviating firm more than it harms the workers who apply to the deviating firm. Thus, at any candidate equilibrium where all firms post a positive wage, a firm that deviates and offers a lower wage gives the same expected utility to its applicants as non-deviating firms, but generates higher profits. Therefore, the only equilibrium that can exist is one where all firms offer $\underline{w} = 0$. Using a similar argument, we can show that it is also not profitable to deviate and offer a higher wage $w^{d\uparrow} > 0$ if all other firms offer $\underline{w} = 0$.

The possibility to compete ex post for a worker weakens the firm's incentive to post a higher initial wage in order to attract more applicants ex ante. Workers are still willing to apply to firms who offer a zero wage because it increases the probability to end up in a firm subgraph and to receive the full match surplus. Albrecht et al. (2006) show that this also holds if firms can only consider one candidate.

The fact that all firms post the same initial wage establishes our main result that our decentralized game is efficient in terms of network formation and clearing. Thus, the first coordination friction that workers do not know where other workers apply is minimized in the decentralized economy. In addition, ex post competition generates the maximum matching. It therefore eliminates the second coordination friction between firms, i.e., the friction that firms do not know which other workers are considered by other firms. Therefore, the directed search model generates the same expected number of matches as a random search model where all firms also post the reservation wage.

Corollary 3 *In the decentralized economy with ex post competition random search is equally efficient as directed search.*

Corollary 3 follows trivially from the fact that under random search firms will always start with offering the worker her reservation wage.

Finally, we can show that wage mechanisms that generate ex ante wage dispersion like Kircher (2009) are not efficient in terms of network formation.

Corollary 4 *If equally productive firms post different wages, network formation is not efficient.*

Proof. See Appendix B.2.

The intuition is simple. Wage dispersion implies that a subset of firms have a higher expected arrival rate of applicants. This creates unbalanced networks and leads to inefficient network formation.

5 Final remarks

This paper contributes to one of the fundamental question in economics namely under which conditions do decentralized markets generate constraint efficient outcomes. Our focus is on the labor market where it is common that unemployed workers simultaneously send multiple applications. This creates a bipartite network between workers and firms. In such an environment there are two coordination frictions, (i) workers do not know where other workers apply to and (ii) firms do not know which workers are considered by other firms. We show that the second coordination friction between firms can be eliminated if wages in the decentralized market are determined by ex post competition and if firms can go back and forth between their applicants. In that case, the number of matches on a given network equals the maximum number of possible matches. The first coordination friction is minimized if the decentralized market ensures that workers apply to each vacancy with equal probability. This implies that an equilibrium with ex ante wage dispersion is inefficient in terms of network formation. We show that in a directed search equilibrium with complete recall, all firms post a wage equal to the reservation wage, which implies that the decentralized market equilibrium is equal to the social planner's solution.

An interesting question is what happens if firms can commit to richer contracts, do we get a maximum matching in that case? We believe that this requires that firms must be able to post wages conditional on each possible network realization. Given the huge number of possibilities, this is impossible in practice.

Although our wage mechanism is efficient in terms of network formation and network clearing, it may not be efficient in other dimensions like vacancy creation and search intensity. Kircher (2009) shows for example that wage commitment without ex post competition implies wage dispersion and that the resulting equilibrium is efficient in terms of search intensity and firm entry. Combining those results suggests that there may not exist a simple wage mechanism that by itself generates the constrained efficient outcome along all dimensions. It is unlikely that workers send the socially desirable number of applications because there are many externalities. For example, since workers' payoffs are independent of whether they are in a worker or even subgraph, they do not take into account that an additional application might turn a worker subgraph into an even subgraph and thereby generate an additional match. Moreover, there is a rent seeking externality, since an additional application can increase the chances of turning an even subgraph into a firm subgraph, which then implies that the surplus goes to the workers and no longer to firms. The probabilities that these events occur and consequently the expected payoffs will depend on market tightness and the aggregate search intensity. Depending on which one is more likely, workers either send too many or too few applications. Concerning firm entry, firms fail to take into account that when they enter they also destroy the expected payoffs of other firms by making it more likely that these firms end up in firm subgraphs.[15] Finally, the interaction between the application and entry decisions will matter. We plan to investigate this in future work.

Another important and interesting extension for future research is to allow for heterogeneity in firm and or worker types, see Shimer (2005) and Elliot (2011). We conjecture that this makes ex post competition equally desirable as in a homogenous firm world, because high productive firms should be able to outbid low productive firms. Furthermore,

[15] Consider for example the case where there are 10 unemployed workers sending out 10 applications to 10 vacancies. The social contribution of an additional firm is zero but the private contribution is positive, because there is a positive probability that the entrant ends up in an even subgraph.

this will make directed search more desirable than in our setting because high productive firms should be able to signal their types in order to get matched with a higher probability. Finally, it would be interesting to consider limited interview capacity. We expect that our main results still hold if firms can interview at most n workers. In the network formation process this makes it less attractive to offer high wages. Since we already find that with full recall, posted wages equal the reservation wage, we expect our results to hold there as well.

References

[1] Albrecht, J., P.A. Gautier, S. Tan and S. Vroman, (2004), Matching with multiple applications, *Economic Letters*, vol. 84(3), pp. 311-314.

[2] Albrecht, J., P.A. Gautier and S. Vroman, (2006), Equilibrium directed search with multiple applications, *Review of Economic Studies*, vol. 73(4), pp. 869-891.

[3] Boorman, S. (1975), A combinatorial optimization model for transmission of job information through contact networks, *Bell Journal of Economics*, vol. 6, 216-249.

[4] Burdett, K. and K.L. Judd, (1983), Equilibrium price dispersion, *Econometrica*, vol. 51(4), pp. 955-969.

[5] Burdett, K., S. Shi, and R. Wright, (2001), Pricing and matching with frictions, *Journal of Political Economy*, vol. 109(5), pp. 1060-1085.

[6] Calvó-Armengol, A. and M.O. Jackson, (2004), The effects of social networks on employment and inequality, *American Economic Review*, vol. 94(3), 426-454.

[7] Calvó-Armengol, A. and Y. Zenou, (2004), Job matching, social network and word-of-mouth communication, *Journal of Urban Economics*, vol. 57, 500-522.

[8] Chade, H. and L. Smith, (2006), Simultaneous search, *Econometrica*, vol. 74(5), pp. 1293-1307.

[9] Coles, M. and J. Eeckhout, (2003), "Indeterminacy and Directed Search", *Journal of Economic Theory*, vol. 111, pp. 265-276.

[10] Corominas-Bosch, M., (2004), Bargaining in a network of buyers and sellers, *Journal of Economic Theory*, vol. 115, pp. 35-77.

[11] Diamond, P.A. (1982), Aggregate demand management in search equilibrium, *Journal of Political Economy*, 90, 881-94.

[12] Eeckhout, J. and P. Kircher, (2010), Sorting vs Screening – Search Frictions and Competing Mechanisms, *Journal of Economic Theory*, vol. 145, pp. 1354-1385.

[13] Elliott, M., (2011). Search with multilateral bargaining, mimeo, Stanford University.

[14] Erdös, P. and A. Rényi, (1960), On the evolution of random graphs, *Publication of the mathematical institute of the Hungarian academy of sciences*, vol. 5, 17-61.

[15] Fontaine, F., (2004), Why are similar workers paid differently? The role of social networks, IZA discussion paper 1786, Bonn.

[16] Galenianos, M. and P. Kircher, (2009), Directed search with multiple job applications, *Journal of Economic Theory*, vol. 114(2), pp. 445-471.

[17] Galeotti, A., (2010), Strategic Information Transmission in Networks, mimeo, University of Essex.

[18] Gautier, P.A. and J.L. Moraga-Gonzalez, (2004), *Strategic wage setting and random search with multiple applications*, Tinbergen Institute discussion paper 04-063/1, Tinbergen Institute.

[19] Gautier, P.A. and R. Wolthoff, (2009), Simultaneous search with heterogeneous firms and ex-post competition, *labor Economics*, vol. 16(3), 311-19.

[20] Ioannides, Y.M., (2004), Random Graphs and Social Networks: An Economics Perspective, mimeo Tufts University.

[21] Julien, B., J. Kennes and I. King, (2000), Bidding for labor, *Review of Economic Dynamics*, vol. 3(4), pp. 619-649.

[22] Kircher, P., (2009), Efficiency of simultaneous search, *Journal of Political Economy*, vol. 117(5), pp. 861- 913.

[23] Kranton, R.E. and D. F. Minehart, (2000), Competition for goods in buyer-seller networks, *Review of Economic Design*, vol. 5, pp. 301-331.

[24] Kranton, R.E. and D. F. Minehart, (2001), A theory of buyer-seller networks, *American Economic Review*, vol. 91(3), 485-508.

[25] Manea, M., (2011), Bargaining in stationary networks, *American Economic Review*, forthcoming.

[26] Moen, E., (1997), Competitive search equilibrium, *Journal of Political Economy*, vol. 105(2), pp. 385-411.

[27] Mortensen, D., (1982), Property rights and efficiency of mating, racing, and related games. *American Economic Review* 72 (5), pp. 968–79.

[28] Motwani, R., R. Panigrahy and Y. Xu (2006), Fractional Matching Via Balls-and-Bins, *Approximation, Randomization, and Combinatorial Optimization. Algorithms and Techniques,* Lecture Notes in Computer Science, pp. 487-498.

[29] Pissarides, C.A., (2000), *Equilibrium unemployment theory*, 2nd edition, MIT Press, Cambridge.

[30] Shi, S., (2002), A directed search model of inequality with heterogeneous skills and skill-biased technology, *Review of Economic Studies*, vol. 69, pp. 467-491.

[31] Shimer, R., (2005), The assignment of workers to jobs in an economy with coordination frictions, *Journal of Political Economy*, vol. 113(5), pp. 996-1025.

6 Appendix

A Simulation algorithm and decomposing a graph into subgraphs

A.1 Simulation algorithm

In our simulations, we apply the following algorithm where step 2 follows Corominas-Bosch (2004) which is based on Hall's marriage theorem.

Decomposition Theorem (Corominas-Bosch, 2004):

(1) Every graph G can be decomposed into a number of firm subgraphs (G_1^f,..., $G_{n_f}^f$), worker subgraphs (G_1^w,..., $G_{n_w}^w$) and even subgraphs (G_1^e,..., $G_{n_e}^e$) in such a way that each node (firm or worker) belongs to one and only one subgraph and any firm (worker) in a firm-(worker-) subgraph $G_{i,}^f(G_{i,}^w)$ is only linked to workers (firms) in a firm-(worker-)subgraph $G_j^f(G_j^w)$.
(2) Moreover, a given node (firm or worker) always belongs to the same type of subgraph for any such decomposition. We will write $G = G_1^f \cup ... \cup G_{n_f}^f \cup G_1^w \cup ... \cup G_{n_w}^w \cup G_1^e \cup ... \cup G_{n_e}^e$, with the union being disjoint.

The decomposition algorithm of Corominas-Bosch (2004) works as follows:

Step a: Eliminate all vacancies that did not receive any applicants.

Step b: For $k = 2, ..., v$, identify the groups of k vacancies that are jointly linked to less than k workers. Remove and collect them. We refer to those subgraphs as firm subgraphs.

Step c: Repeat step b but now reverse the role of workers and vacancies. The resulting subgraphs are called worker subgraphs.

Step d: When all worker subgraphs are removed, the remaining ones are balanced (or even) subgraphs (with an equal number of workers and firms).

Denote the total number of firm subgraphs by F, the total number of worker subgraphs by W, and the number of even subgraphs by E, u_i^f (u_i^e) is number of workers in firm (even) subgraph i, v_i^w is number of firms in worker subgraph i. v_i^f is the number of firms in firm subgraph i, u_i^w is number of workers in worker subgraph i. The number of matches, M, is then given by,

$$M = \sum\nolimits_{i=1}^{F} u_i^f + \sum\nolimits_{i=1}^{W} v_i^w + \sum\nolimits_{i=1}^{E} u_i^e,$$

the fraction of firms in firm subgraphs and the fraction of workers in worker subgraphs by,

$$\frac{v^f}{v} = \sum\nolimits_{i=1}^{F} \frac{v_i^f}{v} \quad \text{and} \quad \frac{u^w}{u} = \sum\nolimits_{i=1}^{W} \frac{u_i^w}{u}.$$

A.2 Simulation examples

To illustrate that having equal application arrival rates is desirable when agents do not know the network we numerically compare equal application rates with the case where a subset of vacancies has a higher application arrival rate. Take a, u, v as given and let $a < v$. First, we color a fraction q of the vacancies blue and a fraction $(1-q)$ green and let each worker send one application to a blue vacancy and the other $(a-1)$ applications to a green one. Each blue vacancy receives an application from worker 1 with probability $1/qv$ and the same for workers 2,..., u. For the $a = 3$ example, each green vacancy gets with probability, $(a-1)/(1-q)v$, the second application of worker 1 and if it did not get the second one, it gets the third one with probability $(a-2)/((1-q)v-1)$ etc. The same holds for the other workers. For $q = 1/a$, the arrival rate at each firm is the same and the only difference with full equalization of application rates is that the market is partitioned. Since we want to focus on network formation here, we assume maximum matching on each realized network and use the Decomposition algorithm given above. Let p_n be the probability that a firm receives no workers, let var(M) be the variance of applicants that

a	p_n	$E(M)$	var(M)	v^f/v	u^w/u
joint					
2	1.343	10.416	0.812	0.012	0.061
3	0.377	11.554	0.382	0.045	0.343
6	0.003	11.997	0.003	0.000	0.003
partitioned ($q = \frac{1}{3}$)					
2	1.748	10.064	0.875	0.187	0.684
3	**0.405**	**11.533**	**0.387**	**0.046**	**0.347**
6	0.124	11.876	0.111	0.010	0.122
partitioned ($q = \frac{1}{6}$)					
2	2.851	9.137	0.945	0.242	0.675
3	0.719	11.206	0.540	0.075	0.510
6	**0.005**	**11.995**	**0.005**	**0.000**	**0.005**

Table 2: Simulation results for $v = u = 12$

a particular firm receives. Recall that the fraction of firms in firm subgraphs is v^f/v and the fraction of workers in worker subgraphs is u^w/u. The matlab code is available upon request.

In Table 2 we present simulation results for $v = u = 12$. We generate a sample of 1000 networks for each case. Table 2 presents those variables for different values of a, q. We see that partitioning the market reduces the expected number of matches, $E(M)$, but that for $q = \frac{1}{a}$ (those rows are in bold), the arrival rate at each firm is the same and the difference in the expected number of matches $E(M)$ with the non partitioning case is relatively small. We also see that if a is large relatively to v, that partitioning hardly matters.

B The decentralized directed search equilibrium

B.1 Proof of Proposition 3

Consider a candidate equilibrium where all firms post a wage $\underline{w}$. The market-utility condition then implies that workers apply with equal probability to each vacancy. This generates a particular network (random) graph. Denote the resulting probability that a vacancy is in a worker or even subgraph by $\rho_{\underline{w}}^w$ or $\rho_{\underline{w}}^e$. Since firms in a firm subgraph pay the marginal product, we can write the profit of a firm offering $\underline{w}$ as,

$$\Pi\left(\underline{w}\right) = \left(\rho_{\underline{w}}^w + \rho_{\underline{w}}^e\right)\left(1 - \underline{w}\right). \tag{6}$$

The probabilities that a worker is part of a worker, firm or even subgraph are denoted by $\phi_{\underline{w}}^w$, $\phi_{\underline{w}}^f$, and $\phi_{\underline{w}}^e$, respectively. Note, that a worker who is part of a worker subgraph matches only with probability $\rho_{\underline{w}}^w v/\phi_{\underline{w}}^w u < 1$, i.e., with the probability that he is matched with one of the vacancies in a worker subgraph. Workers who are part of a firm or even subgraph match with probability one. For even subgraphs we get $\rho_{\underline{w}}^e v = \phi_{\underline{w}}^e u$, since the number of vacancies equals the number of workers. The expected utility of a worker is therefore given by,

$$U\left(\underline{w}\right) = \left(\phi_{\underline{w}}^w \frac{\rho_{\underline{w}}^w v}{\phi_{\underline{w}}^w u} + \phi_{\underline{w}}^e\right)\underline{w} + \phi_{\underline{w}}^f = \frac{v}{u}\left(\rho_{\underline{w}}^w + \rho_{\underline{w}}^e\right)\underline{w} + \phi_{\underline{w}}^f. \tag{7}$$

Now consider a deviating firm that offers a lower wage $w^{d\downarrow} < \underline{w}$. The market-utility condition implies that workers must be indifferent between applying to the deviating firm or to any of the non-deviating firms. We will now show that a firm that deviates and offers a lower wage makes a higher profit then the firms that continue to offer $\underline{w}$. Let the probabilities that a deviating firm that offers a lower wage $w^{d\downarrow} < \underline{w}$ ends up in a worker or even subgraph be ρ_d^w and ρ_d^e. The deviating firm's profit is given by,

$$\Pi\left(w^{d\downarrow}|\underline{w}\right) = \left(\rho_d^w + \rho_d^e\right)\left(1 - \alpha\underline{w} - (1-\alpha)\,w^{d\downarrow}\right), \tag{8}$$

where α denotes the probability (conditional on being in a worker or even subgraph) that the deviating firm has to increase its initial offer to $\underline{w}$ in order to attract a worker in an even or worker subgraph. Note that $\alpha > 0$, since the deviating firm offers a lower wage. Also note that for large markets the expected payoff of a non-deviating firm does not change by the action of a single deviant. The same is true for workers who send all their applications to non deviating firms and who receive market utility.

Denote the probabilities that workers, who send one of their applications to the deviant, are part of a worker, firm or even subgraph by ϕ_d^w, ϕ_d^f, and ϕ_d^e, respectively. Let γ denote the probability that a worker is in a subgraph with the deviating firm and $(1-\gamma)$ the probability that the worker is in a subgraph with only non-deviating firms. Note that a worker and a firm that are linked via an application do not need to be in the same subgraph (compare the Decomposition Theorem). A worker, who applies to the deviating firm that offers a lower wage than other firms, is sometimes paid a higher wage. Let β denote the probability that a worker is paid a wage $\underline{w}$ despite the fact that he is part of a worker or even subgraph with the deviating firm conditional on being in a worker or even subgraph with the deviating firm and conditional on being hired. The utility of a worker that sends one application to the deviating firm can then be written as,

$$\begin{aligned} U\left(w^{d\downarrow}|\underline{w}\right) &= \gamma\left[\frac{v}{u}\left(\rho_d^w+\rho_d^e\right)\left(\beta\underline{w}+(1-\beta)\,w^{d\downarrow}\right)+\phi_d^f\right] \\ &\quad +(1-\gamma)\left[\frac{v}{u}\left(\rho_{\underline{w}}^w+\rho_{\underline{w}}^e\right)\underline{w}+\phi_{\underline{w}}^f\right]. \end{aligned} \tag{9}$$

A worker who applied to the deviant and who is in a worker or even subgraph with the deviating firm receives (conditional on being hired) $\underline{w}$ for sure if the deviating firm must increase its initial offer to $\underline{w}$ (which happens with probability α). The worker also receives $\underline{w}$ if he is hired by one of the non-deviating firms he applied to despite the fact that he is in a worker or even subgraph with the deviating firm. This happens with some positive probability δ if the worker is not the only applicant at the deviating firm. Note that $\delta > 0$, since the probability that the deviating firm receives more than one application is positive. Thus, the probability β is given by

$$\beta = \alpha + (1-\alpha)\,\delta. \tag{10}$$

The fact that workers must be indifferent between applying to the deviating and non-deviating firms, i.e. using (7) and (9) and $U\left(w^{d\downarrow}|\underline{w}\right) = U\left(\underline{w}\right)$, implies,

$$\left(\rho_d^w+\rho_d^e\right)\left(\beta\underline{w}+(1-\beta)\,w^{d\downarrow}\right)+\frac{u}{v}\phi_d^f = \left(\rho_{\underline{w}}^w+\rho_{\underline{w}}^e\right)\underline{w}+\frac{u}{v}\phi_{\underline{w}}^f. \tag{11}$$

Note, that because $U\left(w^{d\downarrow}|\underline{w}\right) = U\left(\underline{w}\right)$, the terms that are multiplied by γ and $(1-\gamma)$ in (9) are equal. The expected number of matches in the candidate equilibrium where all firms offer $\underline{w}$ is given by,

$$M_{\underline{w}} = v\left(\rho_{\underline{w}}^w+\rho_{\underline{w}}^e\right)+u\phi_{\underline{w}}^f,$$

and if a firm deviates by,

$$M_d = v\left[\gamma\left(\rho_d^w+\rho_d^e\right)+(1-\gamma)\left(\rho_{\underline{w}}^w+\rho_{\underline{w}}^e\right)\right]+u\left[\gamma\phi_d^f+(1-\gamma)\,\phi_{\underline{w}}^f\right].$$

Given the law of large numbers the expected number of matches with and without a firm deviating is the same, i.e., $M_{\underline{w}} = M_d$. Rearranging implies

$$\left(\rho_{\underline{w}}^w+\rho_{\underline{w}}^e\right)-\left(\rho_d^w+\rho_d^e\right) = \frac{u}{v}\left[\phi_d^f-\phi_{\underline{w}}^f\right].$$

Substituting $\rho_{\underline{w}}^w + \rho_{\underline{w}}^e$ out (using the workers' indifference condition, 11) implies,

$$\frac{u}{v}\left[\phi_d^f - \phi_{\underline{w}}^f\right] = \left(\rho_d^w + \rho_d^e\right)\left(\beta + (1-\beta)\frac{w^{d\downarrow}}{\underline{w}}\right) + \frac{u}{v}\left[\phi_d^f - \phi_{\underline{w}}^f\right]\frac{1}{\underline{w}} - \left(\rho_d^w + \rho_d^e\right),$$

or

$$\frac{u}{v}\left[\phi_d^f - \phi_{\underline{w}}^f\right]\frac{1-\underline{w}}{\underline{w}} = \left(\rho_d^w + \rho_d^e\right)(1-\beta)\frac{\underline{w} - w^{d\downarrow}}{\underline{w}}. \tag{12}$$

Given the above results we can now investigate whether offering a lower wage $w^{d\downarrow} < \underline{w}$ is profitable. Taking the difference between the profit of a deviating and a non-deviating firm gives,

$$\begin{aligned}
\Pi\left(w^{d\downarrow}|\underline{w}\right) - \Pi\left(\underline{w}\right) &= \left(\rho_d^w + \rho_d^e\right)\left(1 - \alpha\underline{w} - (1-\alpha)w^{d\downarrow}\right) - \left(\rho_{\underline{w}}^w + \rho_{\underline{w}}^e\right)\left(1-\underline{w}\right), \\
&= \left(\rho_d^w + \rho_d^e\right)\left(1 - \alpha\underline{w} - (1-\alpha)w^{d\downarrow}\right) \\
&\quad - \left[\left(\rho_d^w + \rho_d^e\right)\left(\beta\underline{w} + (1-\beta)w^{d\downarrow}\right) + \frac{u}{v}\left[\phi_d^f - \phi_{\underline{w}}^f\right]\right]\frac{1-\underline{w}}{\underline{w}},
\end{aligned}$$

where the second equality follows from substituting $\rho_{\underline{w}}^w + \rho_{\underline{w}}^e$ out using the workers' indifference condition (11). Rearranging implies

$$\begin{aligned}
&\Pi\left(w^{d\downarrow}|\underline{w}\right) - \Pi\left(\underline{w}\right) \\
&= \left(\rho_d^w + \rho_d^e\right)\left[(1-\alpha)\underline{w} + (1-\beta)(1-\underline{w})\right]\frac{\underline{w} - w^{d\downarrow}}{\underline{w}} - \frac{u}{v}\left[\phi_d^f - \phi_{\underline{w}}^f\right]\frac{1-\underline{w}}{\underline{w}}.
\end{aligned}$$

Using the equality (12) to substitute $\phi_d^f - \phi_{\underline{w}}^f$ gives,

$$\begin{aligned}
&\Pi\left(w^{d\downarrow}|\underline{w}\right) - \Pi\left(\underline{w}\right) \\
&= \left(\rho_d^w + \rho_d^e\right)\left[(1-\alpha)\underline{w} + (1-\beta)(1-\underline{w})\right]\frac{\underline{w} - w^{d\downarrow}}{\underline{w}} - \left(\rho_d^w + \rho_d^e\right)(1-\beta)\frac{\underline{w} - w^{d\downarrow}}{\underline{w}} \\
&= \left(\rho_d^w + \rho_d^e\right)[\beta - \alpha]\left(\underline{w} - w^{d\downarrow}\right).
\end{aligned}$$

The fact that lowering the wage will benefit the deviating firm more than it costs workers who apply to the deviating firm, i.e., $\beta > \alpha$, implies that lowering the wage is always profitable, i.e., $\Pi\left(w^{d\downarrow}|\underline{w}\right) > \Pi\left(\underline{w}\right)$. In order to show that such an equilibrium, where all firms offer $\underline{w} = 0$, exist, we need to show that it is not profitable for a firm to deviate and offer a wage $w^{d\uparrow} > 0$ if all other firms offer $\underline{w} = 0$.

The profit of a firm that offers $\underline{w} = 0$ is according to equation (14) given by $\Pi(0) = \left(\rho_0^w + \rho_0^e\right)$. The profit of a deviating firm that offers a higher wage $w^{d\uparrow} > 0$ is given by $\Pi\left(w^{d\uparrow}|0\right) = \left(\rho_d^w + \rho_d^e\right)\left(1 - w^{d\uparrow}\right)$. Note, that the deviant firm never has to increase its initial wage offer in an even or firm subgraph, since it offers a higher wage. The utility of a worker that applies only to firms offering $\underline{w} = 0$ is according to equation (9) given by $U(0) = \phi_0^f$. The utility of a worker, who sends one application to the deviating firm, is given by,

$$U\left(w^{d\uparrow}|0\right) = \gamma\left[\frac{v}{u}\left(\rho_d^w + \rho_d^e\right)(1-\delta)w^{d\uparrow} + \phi_d^f\right] + (1-\gamma)\phi_0^f.$$

The workers' indifference condition between sending one or no application to the deviating firm is therefore given by,

$$\frac{v}{u}\left(\rho_d^w+\rho_d^e\right)(1-\delta)\,w^{d\uparrow}+\phi_d^f=\phi_0^f.$$

The law of large number again implies $M_0 = M_d$, or,

$$\left(\rho_0^w+\rho_0^e\right)-\left(\rho_d^w+\rho_d^e\right)=\frac{u}{v}\left[\phi_d^f-\phi_0^f\right].$$

Using the workers' indifference condition to substitute $\phi_d^f - \phi_0^f$ and rearranging implies,

$$\left(\rho_d^w+\rho_d^e\right)\left(1-(1-\delta)\,w^{d\uparrow}\right)=\left(\rho_0^w+\rho_0^e\right). \tag{13}$$

Since $\Pi\left(w^{d\uparrow}|0\right)$ is smaller than the lhs and $\Pi(0)$ equal to the rhs of equality (13), it follows that offering a wage $w^{d\uparrow} > 0$ reduces profits, i.e., $\Pi\left(w^{d\uparrow}|0\right) < \Pi(0)$. Thus, $\underline{w} = 0$ is the only symmetric equilibrium that exists.

B.2 Proof of Corollary 4

In order to prove that **ex ante** wage dispersion leads to inefficient network formation we start by showing that the equal profit condition (which must hold if equally productive firms post different wages) implies $v_L^f/v_L > v_H^f/v_H$, if $w_H > w_L$. Lemma 2 implies that all firms earn zero profit, if they are part of a firm subgraph, since they pay a wage equal to the marginal product. High wage firms in even or worker subgraphs earn $1 - w_H$. Low wage firms earn more. If they are part of an even (or worker-)subgraph, they pay with probability α their low posted wage w_L and with probability $1-\alpha$ the high posted wage w_H. Note that the appropriate probabilities satisfy $\alpha > 0$, since there exists a positive probability that a low wage firm does not have to compete with a high wage firm for a worker in an even or a worker subgraph.

The equal profit condition of high and low wage firms is, therefore, given by

$$\frac{v_H^f}{v_H}[1-1]+\left[\frac{v_H^e}{v_H}+\frac{v_H^w}{v_H}\right][1-w_H]=\frac{v_L^f}{v_L}[1-1]+\left[\frac{v_L^e}{v_L}+\frac{v_L^w}{v_L}\right][1-\alpha w_L-(1-\alpha)\,w_H]$$

Rearranging and noting that $\frac{v_c^f}{v_c}+\frac{v_c^e}{v_c}+\frac{v_c^w}{v_c}=1$ implies

$$\left[\frac{v_L^f}{v_L}-\frac{v_H^f}{v_H}\right][1-w_H]=\left[\frac{v_L^w}{v_L}+\frac{v_L^e}{v_L}\right]\alpha\,[w_H-w_L].$$

Since $w_H > w_L$, it follows immediately that $v_L^f/v_L > v_H^f/v_H$. ■

C Technical Appendix to Gautier and Holzner: Decentralized equilibrium allowing for wage dispersion (for on line publication)

Consider a candidate equilibrium where firms offer N different wages $\underline{w}_i \in W^N$ with $i = 1, 2, ..., N$ (note that N can also be equal to one). We will show that deviating and offering a wage $\underline{w}_0 = 0$ increases profits for the firm offering the highest wage $\underline{w}_N$. Let us rank the posted wages $\underline{w}_i$ such that $\underline{w}_1 < \underline{w}_2 < \cdots < \underline{w}_N$. The market-utility condition then implies that workers apply with some positive probability to each vacancy (firms will make sure their posted wages are high enough to attract applicants with positive probability) and the network formation process generates a (random) network. Denote the probability that a vacancy is in an even or worker subgraph by ρ_i^e and ρ_i^w. Since firms offer different wages, firms with low initial wage offers might have to increase their wages in order to attract a worker in even or worker subgraphs. We denote the probability that a firm that offers $\underline{w}_i$ has (conditional on being in an even or worker subgraph) to increase its offer to the wage $\underline{w}_j > \underline{w}_i$ by $\alpha_{i,j}$. The firm that posts the highest wage $\underline{w}_N$ never has to increase its initial offer in a worker or even subgraph, i.e., $\alpha_{N,j} = 0$ by definition. The profit of a firm offering $\underline{w}_i$ is therefore given by,

$$\Pi\left(\underline{w}_i\right) = \left(\rho_i^w + \rho_i^e\right)\left(1 - \sum_{j=i}^{N-1} \alpha_{i,j+1}\underline{w}_{j+1} - \sum_{j=i}^{N-1}\left(1 - \alpha_{i,j+1}\right)\underline{w}_i\right), \tag{14}$$

The profit of a deviating firm that offers $\underline{w}_0 = 0$ equals equation (14) for $i = 0$.

Denote by γ_i the probability that a worker is in a subgraph where the lowest offered wage is $\underline{w}_i$, with $\sum_{i=1}^N \gamma_i = 1$. Given that a worker is in a subgraph where the lowest offered wage is $\underline{w}_i$, denote the probability that the worker is part of a worker, firm or even subgraph by ϕ_i^w, ϕ_i^f, and ϕ_i^e, respectively. Note, that a worker who is part of a worker subgraph matches only with probability $\rho_i^w v/\phi_i^w u < 1$, i.e., with the probability that he is matched with one of the vacancies in a worker subgraph. Workers who are part of a firm or even subgraph match with probability one, which implies, $\phi_i^e = \rho_i^e v/u$. The utility of a worker who applies with a positive probability to all wages $\underline{w}_i \in W^N$ is thus given by,

$$U\left(W^N\right) = \sum_{i=1}^N \gamma_i \left[\frac{v}{u}\left(\rho_i^w + \rho_i^e\right)\left(\sum_{j=i}^{N-1} \beta_{i,j+1}\underline{w}_{j+1} + \sum_{j=i}^{N-1}\left(1 - \beta_{i,j+1}\right)\underline{w}_i\right) + \phi_i^f\right], \tag{15}$$

where $\beta_{i,j}$ denotes the probability that a worker is paid the wage $\underline{w}_j$ conditional on being hired in a worker or even subgraph where the lowest offered wage is $\underline{w}_i$. Note, that the worker can be paid the wage $\underline{w}_j$ by any firm offering $\underline{w}_k \le \underline{w}_j$ as longs as he applied to the firm. The worker is paid $\underline{w}_j$, if the firm that he applied to has to increase its initial offer from $\underline{w}_k$ to $\underline{w}_j$ (which happens with probability $\alpha_{k,j}$). Even if the firm where he applied to pays $\underline{w}_k$ (which happens with probability $1 - \alpha_{k,j}$), the worker can earn $\underline{w}_j$ if another firm where he applied to offers him the wage $\underline{w}_j$. This happens with some positive probability, if the worker is not the only applicant at the firm offering $\underline{w}_k$. Thus, the probability $\beta_{i,j}$ satisfies $\beta_{i,j} > \alpha_{i,j}$ with $\beta_{N,j} = 0$ by definition.

The utility of a worker that applies with a positive probability to the deviating firm that offers $\underline{w}_0 = 0$ is also given by equation (15) with $i = 0$ and with γ_i being replaced by γ_i^0 with $\gamma_i^0 = \gamma_i / (1 + \gamma_0^0)$ such that $\sum_{i=0}^{N} \gamma_i^0 = 1$.

The set of wages W^N constitutes an equilibrium, if workers are indifferent between applying to any firm posting $\underline{w}_i \in W^N$. The workers' indifference condition implies for all $i, k \in \{1, 2, ..., N\}$,

$$\begin{aligned} & (\rho_i^w + \rho_i^e) \left(\sum_{j=i}^{N-1} \beta_{i,j+1} \underline{w}_{j+1} + \sum_{j=i}^{N-1} (1 - \beta_{i,j+1}) \underline{w}_i \right) + \frac{u}{v} \phi_i^f \\ = \; & (\rho_k^w + \rho_k^e) \left(\sum_{j=k}^{N-1} \beta_{k,j+1} \underline{w}_{j+1} + \sum_{j=k}^{N-1} (1 - \beta_{k,j+1}) \underline{w}_k \right) + \frac{u}{v} \phi_k^f. \end{aligned} \tag{16}$$

The market-utility condition implies that workers must also be indifferent between applying to the deviating firm or to any of the non-deviating firms. The workers' indifference condition is again given by equation (16) with $i = 0$.

The number of matches in the considered equilibrium is given by,

$$M_{W^N} = \sum_{i=1}^{N} \gamma_i \left[v (\rho_i^w + \rho_i^e) + u \phi_i^f \right], \tag{17}$$

and the number of matches with the deviating firm is again given by equation (17) with $i = 0$. Due to the law of large numbers the number of expected matches satisfy $M_{W^N} = M_d$, which implies, $M_{W^N} = v (\rho_0^w + \rho_0^e) + u \phi_0^f$, or,

$$\sum_{i=1}^{N} \gamma_i (\rho_i^w + \rho_i^e) - (\rho_0^w + \rho_0^e) = \frac{u}{v} \sum_{i=1}^{N} \gamma_i \left[\phi_0^f - \phi_i^f \right]. \tag{18}$$

The workers' indifference condition (16) can be used to substitute $\sum_{i=1}^{N} \gamma_i (\rho_i^w + \rho_i^e)$ in equation condition (18) and obtain,

$$\begin{aligned} & \frac{u}{v} \sum_{i=1}^{N} \gamma_i \left[\phi_0^f - \phi_i^f \right] \frac{1 - \left(\sum_{j=i}^{N-1} \beta_{i,j+1} \underline{w}_{j+1} + \sum_{j=i}^{N-1} (1 - \beta_{i,j+1}) \underline{w}_i \right)}{\left(\sum_{j=i}^{N-1} \beta_{i,j+1} \underline{w}_{j+1} + \sum_{j=i}^{N-1} (1 - \beta_{i,j+1}) \underline{w}_i \right)} \\ = \; & (\rho_0^w + \rho_0^e) - (\rho_0^w + \rho_0^e) \sum_{i=1}^{N} \gamma_i \frac{\sum_{j=0}^{N-1} \beta_{0,j+1} \underline{w}_{j+1}}{\left(\sum_{j=i}^{N-1} \beta_{i,j+1} \underline{w}_{j+1} + \sum_{j=i}^{N-1} (1 - \beta_{i,j+1}) \underline{w}_i \right)}. \end{aligned} \tag{19}$$

Taking the difference of the profit of the deviating and a non-deviating firm implies,

$$\begin{aligned} \Pi (\underline{w}_0) - \Pi (\underline{w}_i) \;=\; & (\rho_0^w + \rho_0^e) \left(1 - \sum_{j=0}^{N-1} \alpha_{0,j+1} \underline{w}_{j+1} \right) \\ & - (\rho_i^w + \rho_i^e) \left(1 - \sum_{j=i}^{N-1} \alpha_{i,j+1} \underline{w}_{j+1} - \sum_{j=i}^{N-1} (1 - \alpha_{i,j+1}) \underline{w}_i \right) \end{aligned}$$

Substituting $(\rho_i^w+\rho_i^e)$ using the workers' indifference condition (16) and multiplying by $\frac{1-\left(\sum_{j=i}^{N-1}\beta_{i,j+1}\underline{w}_{j+1}+\sum_{j=i}^{N-1}\left(1-\beta_{i,j+1}\right)\underline{w}_i\right)}{1-\sum_{j=i}^{N-1}\alpha_{i,j+1}\underline{w}_{j+1}-\sum_{j=i}^{N-1}(1-\alpha_{i,j+1})\underline{w}_i}$ implies,

$$\begin{aligned}
&\left[\Pi\left(\underline{w}_0\right)-\Pi\left(\underline{w}_i\right)\right]\frac{1-\sum_{j=i}^{N-1}\beta_{i,j+1}\underline{w}_{j+1}-\sum_{j=i}^{N-1}\left(1-\beta_{i,j+1}\right)\underline{w}_i}{1-\sum_{j=i}^{N-1}\alpha_{i,j+1}\underline{w}_{j+1}-\sum_{j=i}^{N-1}\left(1-\alpha_{i,j+1}\right)\underline{w}_i}\\
=\ &\left(\rho_0^w+\rho_0^e\right)\left(1-\sum_{j=0}^{N-1}\alpha_{0,j+1}\underline{w}_{j+1}\right)\frac{1-\sum_{j=i}^{N-1}\beta_{i,j+1}\underline{w}_{j+1}-\sum_{j=i}^{N-1}\left(1-\beta_{i,j+1}\right)\underline{w}_i}{1-\sum_{j=i}^{N-1}\alpha_{i,j+1}\underline{w}_{j+1}-\sum_{j=i}^{N-1}\left(1-\alpha_{i,j+1}\right)\underline{w}_i}\\
&-\left(\rho_0^w+\rho_0^e\right)\sum_{j=0}^{N-1}\beta_{0,j+1}\underline{w}_{j+1}\frac{1-\sum_{j=i}^{N-1}\beta_{i,j+1}\underline{w}_{j+1}-\sum_{j=i}^{N-1}\left(1-\beta_{i,j+1}\right)\underline{w}_i}{\sum_{j=i}^{N-1}\beta_{i,j+1}\underline{w}_{j+1}+\sum_{j=i}^{N-1}\left(1-\beta_{i,j+1}\right)\underline{w}_i}\\
&-\frac{u}{v}\left[\phi_0^f-\phi_i^f\right]\frac{1-\sum_{j=i}^{N-1}\beta_{i,j+1}\underline{w}_{j+1}-\sum_{j=i}^{N-1}\left(1-\beta_{i,j+1}\right)\underline{w}_i}{\sum_{j=i}^{N-1}\beta_{i,j+1}\underline{w}_{j+1}+\sum_{j=i}^{N-1}\left(1-\beta_{i,j+1}\right)\underline{w}_i}.
\end{aligned}$$

Taking the weighted sum over i and noting that all firms offering a wage $\underline{w}_i\in W^N$ make equal profit, i.e., $\sum_{i=1}^{N}\gamma_i\Pi\left(\underline{w}_i\right)=\Pi\left(\underline{w}_i\right)$, implies,

$$\begin{aligned}
&\left[\Pi\left(\underline{w}_0\right)-\Pi\left(\underline{w}_i\right)\right]\sum_{i=1}^{N}\gamma_i\frac{1-\sum_{j=i}^{N-1}\beta_{i,j+1}\underline{w}_{j+1}-\sum_{j=i}^{N-1}\left(1-\beta_{i,j+1}\right)\underline{w}_i}{1-\sum_{j=i}^{N-1}\alpha_{i,j+1}\underline{w}_{j+1}-\sum_{j=i}^{N-1}\left(1-\alpha_{i,j+1}\right)\underline{w}_i}\\
=\ &\left(\rho_0^w+\rho_0^e\right)\left(1-\sum_{j=0}^{N-1}\alpha_{0,j+1}\underline{w}_{j+1}\right)\sum_{i=1}^{N}\gamma_i\frac{1-\sum_{j=i}^{N-1}\beta_{i,j+1}\underline{w}_{j+1}-\sum_{j=i}^{N-1}\left(1-\beta_{i,j+1}\right)\underline{w}_i}{1-\sum_{j=i}^{N-1}\alpha_{i,j+1}\underline{w}_{j+1}-\sum_{j=i}^{N-1}\left(1-\alpha_{i,j+1}\right)\underline{w}_i}\\
&-\left(\rho_0^w+\rho_0^e\right)\sum_{j=0}^{N-1}\beta_{0,j+1}\underline{w}_{j+1}\sum_{i=1}^{N}\gamma_i\frac{1-\sum_{j=i}^{N-1}\beta_{i,j+1}\underline{w}_{j+1}-\sum_{j=i}^{N-1}\left(1-\beta_{i,j+1}\right)\underline{w}_i}{\sum_{j=i}^{N-1}\beta_{i,j+1}\underline{w}_{j+1}+\sum_{j=i}^{N-1}\left(1-\beta_{i,j+1}\right)\underline{w}_i}\\
&-\frac{u}{v}\sum_{i=1}^{N}\gamma_i\left[\phi_0^f-\phi_i^f\right]\frac{1-\sum_{j=i}^{N-1}\beta_{i,j+1}\underline{w}_{j+1}-\sum_{j=i}^{N-1}\left(1-\beta_{i,j+1}\right)\underline{w}_i}{\sum_{j=i}^{N-1}\beta_{i,j+1}\underline{w}_{j+1}+\sum_{j=i}^{N-1}\left(1-\beta_{i,j+1}\right)\underline{w}_i}.
\end{aligned}$$

Using equality (19) in order to substitute the last term of the previous equation implies the following equality, i.e.,

$$\begin{aligned}
&\left[\Pi\left(\underline{w}_0\right)-\Pi\left(\underline{w}_i\right)\right]\sum_{i=1}^{N}\gamma_i\frac{1-\sum_{j=i}^{N-1}\beta_{i,j+1}\underline{w}_{j+1}-\sum_{j=i}^{N-1}\left(1-\beta_{i,j+1}\right)\underline{w}_i}{1-\sum_{j=i}^{N-1}\alpha_{i,j+1}\underline{w}_{j+1}-\sum_{j=i}^{N-1}\left(1-\alpha_{i,j+1}\right)\underline{w}_i}\\
=\ &\left(\rho_0^w+\rho_0^e\right)\left(1-\sum_{j=0}^{N-1}\alpha_{0,j+1}\underline{w}_{j+1}\right)\sum_{i=1}^{N}\gamma_i\frac{1-\sum_{j=i}^{N-1}\beta_{i,j+1}\underline{w}_{j+1}-\sum_{j=i}^{N-1}\left(1-\beta_{i,j+1}\right)\underline{w}_i}{1-\sum_{j=i}^{N-1}\alpha_{i,j+1}\underline{w}_{j+1}-\sum_{j=i}^{N-1}\left(1-\alpha_{i,j+1}\right)\underline{w}_i}\\
&-\left(\rho_0^w+\rho_0^e\right)\sum_{j=0}^{N-1}\beta_{0,j+1}\underline{w}_{j+1}\sum_{i=1}^{N}\gamma_i\frac{1-\sum_{j=i}^{N-1}\beta_{i,j+1}\underline{w}_{j+1}-\sum_{j=i}^{N-1}\left(1-\beta_{i,j+1}\right)\underline{w}_i}{\sum_{j=i}^{N-1}\beta_{i,j+1}\underline{w}_{j+1}+\sum_{j=i}^{N-1}\left(1-\beta_{i,j+1}\right)\underline{w}_i}\\
&-\left(\rho_0^w+\rho_0^e\right)-\left(\rho_0^w+\rho_0^e\right)\sum_{i=1}^{N}\gamma_i\frac{\sum_{j=0}^{N-1}\beta_{0,j+1}\underline{w}_{j+1}}{\sum_{j=i}^{N-1}\beta_{i,j+1}\underline{w}_{j+1}+\sum_{j=i}^{N-1}\left(1-\beta_{i,j+1}\right)\underline{w}_i}
\end{aligned}$$

Rearranging gives

$$
\begin{aligned}
& \left[\Pi\left(\underline{w}_0\right)-\Pi\left(\underline{w}_i\right)\right] \sum_{i=1}^N \gamma_i \frac{1-\left(\sum_{j=i}^{N-1} \beta_{i,j+1} \underline{w}_{j+1}+\sum_{j=i}^{N-1}\left(1-\beta_{i,j+1}\right) \underline{w}_i\right)}{1-\sum_{j=i}^{N-1} \alpha_{i,j+1} \underline{w}_{j+1}-\sum_{j=i}^{N-1}\left(1-\alpha_{i,j+1}\right) \underline{w}_i} \\
= & \left(\rho_0^w+\rho_0^e\right)\left(1-\sum_{j=0}^{N-1} \alpha_{0,j+1} \underline{w}_{j+1}\right) \sum_{i=1}^N \gamma_i \frac{1-\left(\sum_{j=i}^{N-1} \beta_{i,j+1} \underline{w}_{j+1}+\sum_{j=i}^{N-1}\left(1-\beta_{i,j+1}\right) \underline{w}_i\right)}{1-\sum_{j=i}^{N-1} \alpha_{i,j+1} \underline{w}_{j+1}-\sum_{j=i}^{N-1}\left(1-\alpha_{i,j+1}\right) \underline{w}_i} \\
& -\left(\rho_0^w+\rho_0^e\right)\left(1-\sum_{j=0}^{N-1} \beta_{0,j+1} \underline{w}_{j+1}\right)
\end{aligned}
$$

Considering the firm that offers the highest wage, i.e., $i = N$, implies

$$
\Pi\left(\underline{w}_0\right)-\Pi\left(\underline{w}_N\right)=\left(\rho_0^w+\rho_0^e\right) \sum_{j=0}^{N-1}\left(\beta_{0,j+1}-\alpha_{0,j+1}\right) \underline{w}_{j+1}
$$

Since $\beta_{i,j+1} > \alpha_{i,j+1}$ for all $j \in \{0, 1, 2..., N-1\}$, offering a wage $\underline{w}_0 = 0$ generates a higher profit than offering $\underline{w}_N$. Thus, only an equilibrium with a wage equal to $\underline{w}_0 = 0$ can exist.

We have shown in Appendix B.1 that there is no incentive for firms to deviate from an equilibrium where all firms post $\underline{w}_0 = 0$. This implies that all firms post $\underline{w}_0 = 0$ in equilibrium.